Static input allocation for reaction wheels desaturation using magnetorquers

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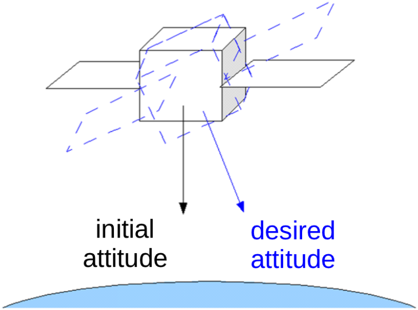
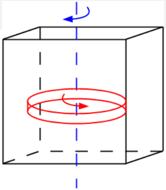
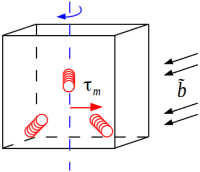
Attitude control performed with two actuators

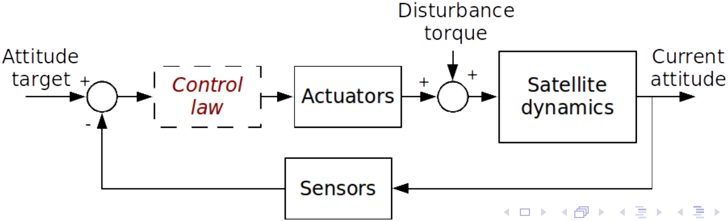




Reaction wheels



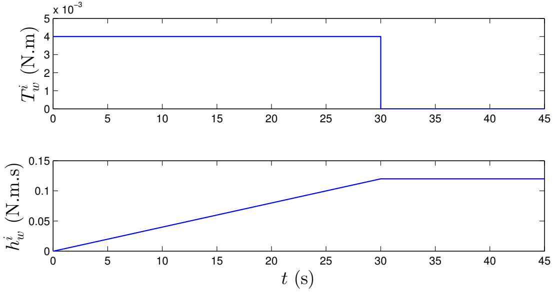
Magnetorquers



Reaction wheels suffer from total momentum problems

Reaction wheels

$$T_w = \dot{h}_w$$



Nomenclature

- $h_w \in \mathbb{R}^3$: angular momentum
- $T_w \in \mathbb{R}^3$: control torque

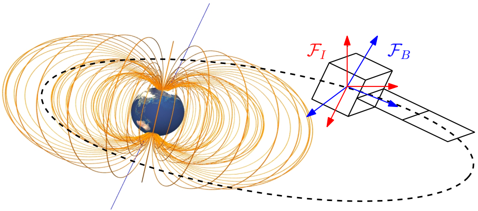
✗ The total momentum cannot be modified (wheel turns CW, satellite turns CCW)

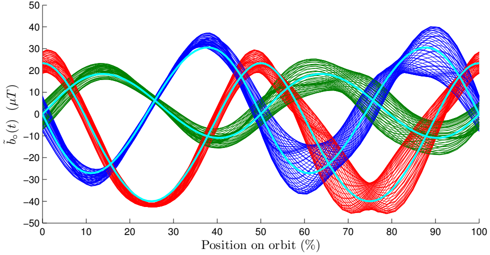
✗ risk of saturation of h_w

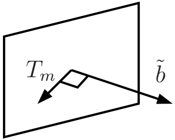
$$\Rightarrow h_w(t) = \int_0^t T_w(\tau) d\tau \text{ needs to be controlled}$$

Magnetorquers confined to exert 2D torque

$$T_m = -\tilde{b}^{\times}(t,q)\tau_m = -(R(q)\tilde{b}_{\circ}(t))^{\times}\tau_m$$







Notation

$$z^{\times} = \begin{bmatrix} 0 & -z_z & z_y \\ z_z & 0 & -z_x \\ -z_y & z_x & 0 \end{bmatrix}$$

Nomenclature

- $T_m \in \mathbb{R}^3$: control torque
- $\tilde{b} \in \mathbb{R}^3$: magnetic field
- $\tau_m \in \mathbb{R}^3$: magnetic momentum
- $q \in \mathbb{R}^4$: quaternion
- $R \in \mathbb{R}^{3\times 3}$: rotation matrix

✗ $(\ )^{\times}$: instantaneous controllability restricted to a plane ($\forall z \in \mathbb{R}^3$, $z^{\times}$ is singular)

✗ $\tilde{b}_{\circ}(t)$: almost periodic and uncertain

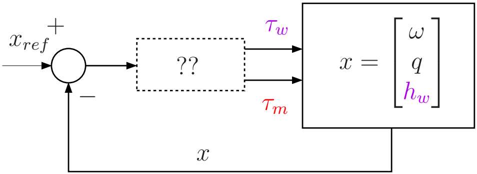
Stabilization problem requires coordination of the actuators

Equations of the attitude motion

$$J\dot{\omega} = -\omega^{\times}(J\omega + h_w) - \tau_w - \overbrace{\tilde{b}^{\times}(t,q)\tau_m}^{T_m} \quad (1a)$$

$$\dot{h}_w = \tau_w \quad (1b)$$

$$\begin{bmatrix}\dot{\varepsilon}\\ \dot{\eta}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}-\omega^{\times} & \omega\\ -\omega^{T} & 0\end{bmatrix}\begin{bmatrix}\varepsilon\\ \eta\end{bmatrix} \quad (1c)$$



Nomenclature

Satellite:

- ω: angular velocity
- $q = (\varepsilon, \eta)$: quaternion
- J: inertia matrix

Reaction wheels:

- h_w: angular momentum
- $\tau_w = T_w$: control torque

Magnetorquers:

- $\tilde{b}(t,q)$: geomagnetic field
- τ_m: magnetic momentum

➡ Stabilizing state-feedback problem: find $\tau_w(x)$ and $\tau_m(x)$ such that $x = \begin{bmatrix}\omega\\ q\\ h_w\end{bmatrix} \rightarrow \begin{bmatrix}\mathbf{0}\\ q_\circ\\ h_{ref}\end{bmatrix}$

✗ actuators may badly interact

Global attitude properties via hybrid feedback laws

Ideal attitude feedback u_{att} must be selected as a hybrid control law

$$J\dot{\omega} = -\omega^{\times} J\omega + u_{att} + d$$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega^{\times} & \omega \\ -\omega^{T} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$

- Even if $d = 0$, no time-invariant continuous selection $u_{att}(x)$ stabilizes the compact attractor $\mathcal{A} := \{\omega = \varepsilon = 0, \eta = \pm 1\}$ [Bhat et al, 2000]
- hybrid solution available in the literature [Mayhew et al, 2009]:

For any scalars $c > 0$, $\delta \in (0, 1)$ and any matrix $K_\omega \succ 0$, the attractor $\mathcal{A}$ is globally asymptotically and locally exponentially stabilized by the control law:

$$\begin{array}{ll} u_{att} := -c x_c \varepsilon - K_\omega \omega & , \\ \dot{x}_c = 0, & (q, \omega, x_c) \in C \\ x_c^+ = -x_c, & (q, \omega, x_c) \in D \end{array}$$

where the flow set C and the jump set D are defined as

$$C := \{(q, \omega, x_c) \in \mathbb{S}^3 \times \mathbb{R}^3 \times \{-1, 1\} : x_c \eta \geq -\delta\}$$
$$D := \{(q, \omega, x_c) \in \mathbb{S}^3 \times \mathbb{R}^3 \times \{-1, 1\} : x_c \eta \leq -\delta\},$$

✗ does not take into account limitations of the actuators

I. The industrial solution: "cross product control law"

Ignore the interaction of the two inputs

$$J\dot{\omega} = -\omega^{\times} J\omega \overbrace{-\tau_w - \omega^{\times} h_w}^{u_{att}(x_c,\varepsilon,\omega)} + \overbrace{T_m}^{d},$$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega^{\times} & \omega \\ -\omega^{T} & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$

- loop 1: Attitude control performed by the reaction wheels
- loop 2: Regulation of h_w by the magnetorquers
- the two loops are treated separately

The cross-product control law

$$\tau_w = -\omega^{\times} h_w - u_{att}, \qquad \tau_m = -\frac{\tilde{b}^{\times}(t)}{|\tilde{b}(t)|^2} k_p(h_w - h_{ref})$$

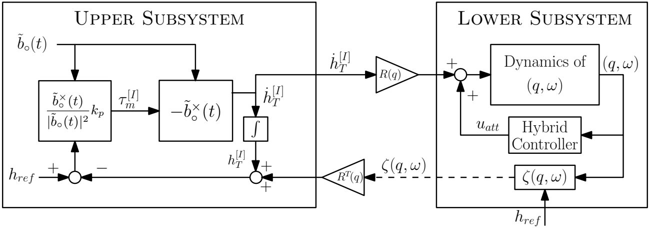
Lack of proof of stability

✗ formally proving desirable stabilization properties of the overall scheme seems hard

- frequency separation between the two loops (= very aggressive action of the attitude stabilizer) gives an engineering solution [Camillo,1980; Carrington 1981; Chen 1999]

New point of view on the classical approach

- quasi cascaded structure where $h_T^{[I]}$ refers to the total angular momentum (satellite + wheels)



A revisited version of the cross-product control law

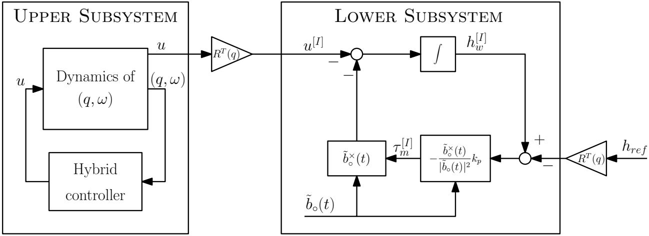
$$\tau_w = -\omega^\times h_w - u_{att}, \qquad \tau_m = -\frac{\tilde{b}^\times(t)}{|\tilde{b}(t)|^2} k_p (h_w + J\omega - R(q)h_{ref})$$

- the feedback branch (the dashed line) can be avoided by redefining τ_m
- ✓ GAS is achieved for any stabilizer u_{att} (under ISS and reasonable assumptions on $\tilde{b}_\circ(t)$)
- ✗ attitude dynamics is affected by the secondary task of momentum damping

III. New static-allocation-based controller induces desirable attitude

Allocation-based controller equations

$$\tau_w = -\omega^\times h_w - (R(q)\tilde{b}_\circ(t))^\times \tau_m - u_{att}, \qquad \tau_m = -\frac{(R(q)\tilde{b}_\circ(t))^\times}{|\tilde{b}_\circ(t)|^2} k_p(h_w - h_{ref})$$



Reversing the cascaded structure

- giving priority to the attitude control goal
- equivalent to a new different partition of the dynamics equation:

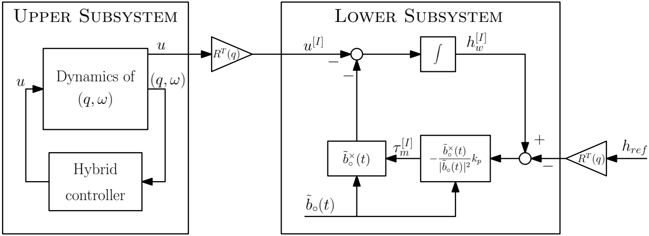
$$J\dot{\omega} + \omega^\times J\omega = \underbrace{-\tau_w - \omega^\times h_w + T_m}_{u_{att}}.$$

✓ GAS is achieved for any stabilizer u_{att} (No ISS needed but same mild assumptions on $\tilde{b}_\circ(t)$)

III. New static-allocation-based controller induces desirable attitude

Allocation-based controller equations

$$\tau_w = -\omega^\times h_w - (R(q)\tilde{b}_\circ(t))^\times \tau_m - u_{att}, \qquad \tau_m = -\frac{(R(q)\tilde{b}_\circ(t))^\times}{|\tilde{b}_\circ(t)|^2} k_p(h_w - h_{ref})$$



Proof of stability uses reduction theorem for hybrid systems

- if attractor $\mathcal{A}$ is GAS (and LES) for the upper system
- if the origin is GAS for the lower system with zero input
- if all solutions are bounded (proved with exponential convergence of u + Gronwall)

Then the attractor $\mathcal{A} \times \{h = h_{ref}\}$ is GAS for the overall system.

Simulation results reveal advantages of the proposed controller

Context of the simulations

- mission of the micro-satellite Demeter designed by CNES, the French space agency
- $\tilde{b}_\circ(t)$ evaluated by means of the IGRF (high fidelity model of the geomagnetic field)
- rest-to-rest maneuvers with non-nominal h_w

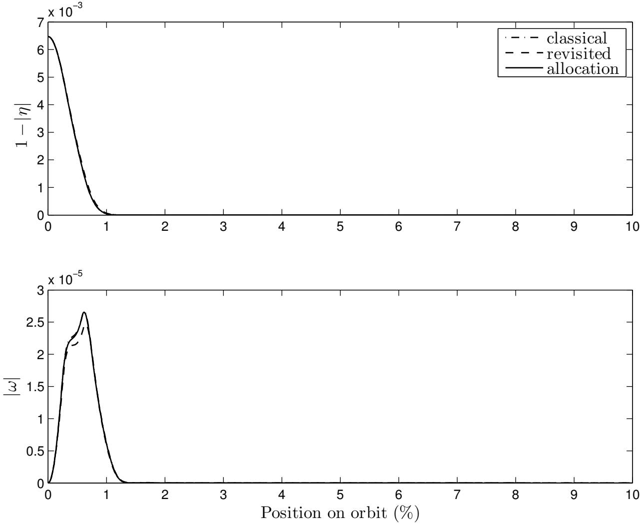
Controllers used

- **Classical** "cross product control" controller
- **Revisited** version of the classical controller
- **Allocation**-based controller

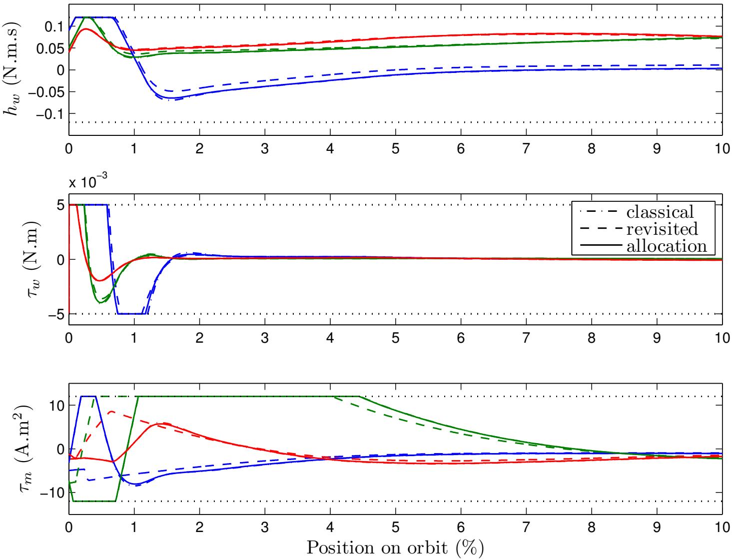
Simulation tests

- **Nominal**: Shows that the **classical** solution diverges
- **Perturbed** J: **Allocation** outperforms **Revisited**
- **Periodic disturbances**: **Allocation** outperforms **Revisited**

Aggressive attitude controller u_{att}



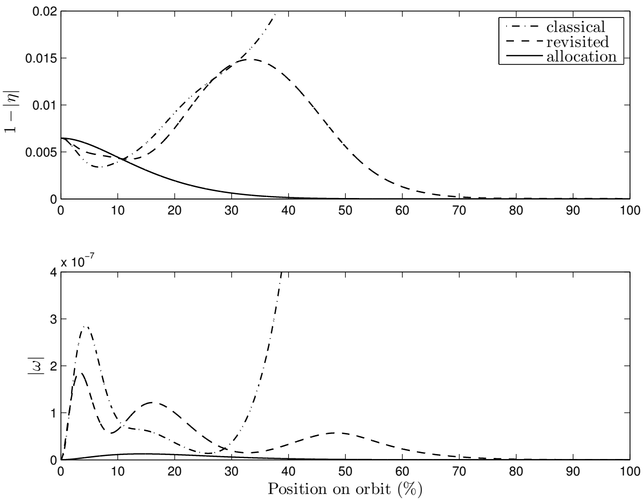
✓ **Similar results**



h_w (N.m.s)
τ_w (N.m)
x 10^{-3}
classical
revisited
allocation
τ_m (A.m^2)
Position on orbit (%)

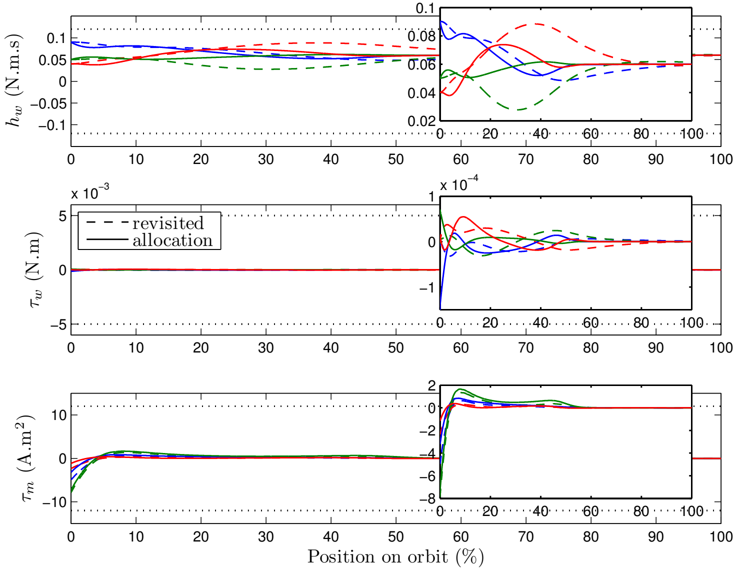
✗ **saturation**

Non-aggressive attitude controller u_{att}



✓ **revisited and allocation controllers preserve stability**

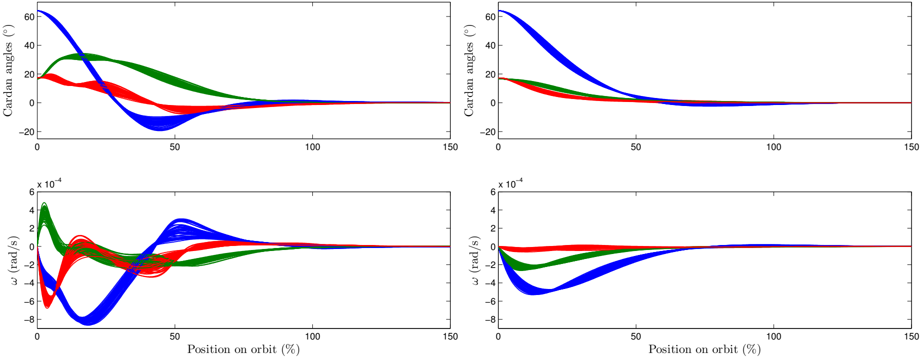
✓ **Attitude transient is more regular for the allocation-based strategy**



✓ **Actuators do not saturate**

Monte-Carlo study with uncertainties on J reveals improved transients

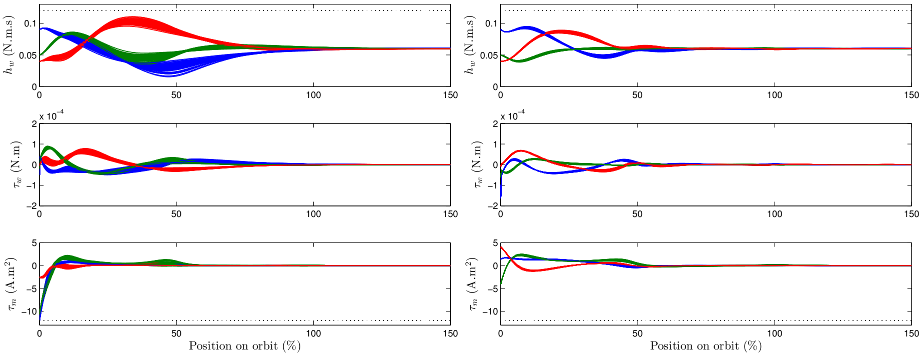
- Clear advantages emerge from swapping the cascaded structure



✓ **Improved attitude transients with allocation-based controller**

Monte-Carlo study with uncertainties on J reveals smaller inputs

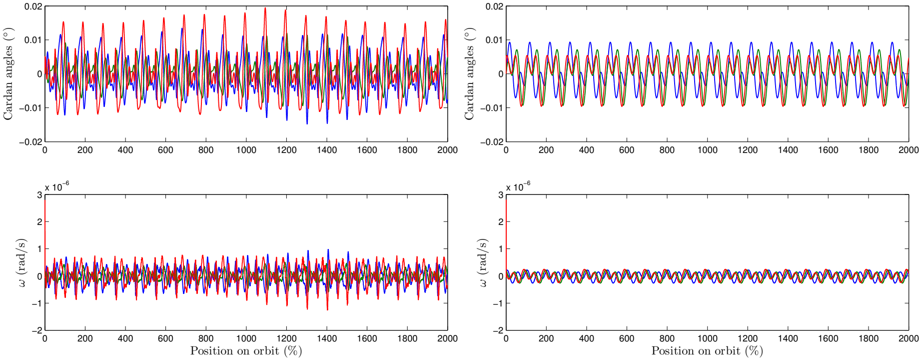
- Reduced spread and usage of the actuators efforts



✓ **Improved attitude transients with allocation-based controller**

Periodic disturbanced are best handled by allocator

- No formal analysis has been performed for this case



✓ **Improved attitude response with allocation-based controller**

Conclusions

Summary of the advantages of the new allocation-based controller

- ✓ actuators are less inclined to saturate (non-aggressive attitude stabilizers can be handled)
- ✓ attitude dynamics independent of the momentum damping
- ✓ rigorous proof of stability
- ✓ good properties of robustness w.r.t. uncertainties on $\tilde{b}_{\circ}(t)$ (according to simulation results)

Perspectives

- ✗ mean value of attitude perturbations induces a drift of the momenta of the reaction wheels [Lovera, 2001]
- ➥ How this new allocation framework can prevent these phenomena to occur?

References

- Jean-François Trégouët, Denis Arzelier, Dimitri Peaucelle and Luca Zaccarian. Static input allocation for reaction wheels desaturation using magnetorquers. In *Automatic Control in Aerospace*, volume 19, Würzburg, Germany, 2013.
- Jean-François Trégouët, Denis Arzelier, Dimitri Peaucelle, Christelle Pittet and Luca Zaccarian. Reaction wheels desaturation using magnetorquers and static input allocation. *IEEE Transactions on Control Systems Technology*, 23(2):525539, 2015.