Input driven consensus algorithm for distributed estimation and classification in sensor networks

Chiara Ravazzi

Dipartimento di Matematica
Politecnico di Torino, ITALY
http://calvino.polito.it/~ravazzi

Work in progress with F. Fagnani, S. M. Fosson

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Introduction

Given

- a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}),\ |\mathcal{V}| = N$
    - nodes in $\mathcal{V} \rightsquigarrow$ sensors
    - edges in $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \rightsquigarrow$ available communication links
- set of observations

$$y_i = \theta + T_i n_i \qquad i \in \mathcal{V}$$

- $\theta \in \mathbb{R}$ unknown continuous parameter
- T_i unknown discrete parameter $\rightsquigarrow$ status of node $i \in \mathcal{V}$

$$T_i = \begin{cases} \alpha & \text{with probability } 1 - p \\ \beta & \text{with probability } p \end{cases} \qquad \alpha \ll \beta$$

- $n_i \sim \mathrm{N}(0, 1)$ independent gaussian noise

Goal: Estimate of $T = (T_1, ..., T_N)$ and θ, starting from $y = (y_1, ..., y_N)$

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- Which estimator?

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- Which estimator?
- Which estimation algorithm?

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- Which estimator?
- Which estimation algorithm?
- Which communication graph?

Which estimator?

Which estimator?

1. "Optimal" estimators

$$\widehat{T}^\star = \underset{\widehat{T}\in\{\alpha,\beta\}^N}{\mathrm{argmin}}\ \mathbb{E}[\mathrm{d_H}(\widehat{T}, T)] \qquad \widehat{\theta}^\star = \underset{\widehat{\theta}\in\mathbb{R}}{\mathrm{argmin}}\,\mathbb{E}[|\theta - \widehat{\theta}|^2]$$

where $\mathrm{d_H}(\widehat{T}, T) = |\{i \in \mathcal{V} : \widehat{T}_i \neq T_i\}|$

- computationally untractable
- difficult to decentralize

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   where $\mathrm{d_H}(\widehat{T}, T) = |\{i \in \mathcal{V} : \widehat{T}_i \neq T_i\}|$

   - computationally untractable
   - difficult to decentralize

2. ML-estimators

$$(\widehat{\theta}_{ML}, \widehat{T}_{ML}) = \underset{\theta\in\mathbb{R}, T\in\{\alpha,\beta\}^N}{\operatorname{argmax}} p(T, \theta|y) = \underset{\theta\in\mathbb{R}, T\in\{\alpha,\beta\}^N}{\operatorname{argmax}} L(\theta, T)$$

   where

$$L(\theta, T) = -\sum_{k=1}^N \frac{(y_k-\theta)^2}{2T_k^2} - \eta\sum_{i=1}^N T_i, \quad \eta = \eta(p, \alpha, \beta)$$

   - still computationally complex
   - computation could be decentralized

Optimization problem: $$(\widehat{\theta}_{ML}, \widehat{T}_{ML}) = \underset{\theta \in \mathbb{R}, T \in \{\alpha, \beta\}^N}{\operatorname{argmax}} L(\theta, T)$$

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- No closed form solution

S1 First maximize in θ, then in T

$$\widehat{\theta}(T) = \underset{\theta \in \mathbb{R}}{\text{argmax}}\, L(\theta, T) = \frac{\sum_{k=1}^{N} \frac{y_k}{T_k^2}}{\sum_{k=1}^{N} \frac{1}{T_k^2}}$$

$$\widehat{T}_{ML} = \underset{T \in \{\alpha,\beta\}^N}{\text{argmax}} L(\widehat{\theta}(T), T) \quad \text{and} \quad \widehat{\theta}_{ML} = \widehat{\theta}(\widehat{T}_{ML})$$

S2 First maximize in T, then in θ

$$\widehat{T}_i(\theta) = \begin{cases} \alpha & \text{if } |y_i - \theta| < \delta \\ \beta & \text{otherwise} \end{cases} \qquad \delta = \delta(p, \alpha, \beta)$$

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- Which algorithm? Distributed algorithm?

Algorithms

Summary

1. Initialize $\widehat{\theta}^{(0)} = y$
2. For $t \in \mathbb{Z}_{\geq 0}$: for all $i \in \mathcal{V}$

$$\widehat{T}_i^{(t)} = \begin{cases} \alpha & \text{if } |y_i - \theta| < \delta \\ \beta & \text{otherwise} \end{cases} \qquad \delta = \delta(p, \alpha, \beta)$$

$$\widehat{\theta}^{(t+1)} = \frac{\sum_{k=1}^{N} \frac{y_k}{\hat{T}_k^2}}{\sum_{k=1}^{N} \frac{1}{\hat{T}_k^2}}$$

3. Stop criteria: $|\widehat{\theta}^{(t+1)} - \widehat{\theta}^{(t)}| < \text{toll}$

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+ Iterative algorithm: at each time

hard-decoding + convex combination

– Convergence is not guaranteed

– Requires complete communication graphs

Expectation-maximization (EM) algorithm [Dempster&al.'77]

+ Iterative algorithm: at each time
  - E-step ($\sim$ soft-decoding)
  - M-step ($\sim$ conditional expected value)
+ Convergence is guaranteed
– Requires complete communication graphs

Distributed algorithms

1. Distributed implementations of EM
   + All nodes acquire information from neighbors
   – Critical: Number of iterations for averaging
   – Convergence not guaranteed
2. Belief propagation algorithm [Saligrama&al.'05]
   + All nodes acquire information from neighbors
   – Convergence not guaranteed in general cases (trees/regular graphs)
   – Critical: setting various parameters
3. Input driven consensus algorithm (IDCA) [new!]

Summary

1. Initialize $\mu^{(0)} = 0,\ \nu^{(0)} = 0,\ \widehat{T}^{(0)} = \alpha \mathbb{1}$
2. for $t \in \mathbb{Z}_{\geq 0}$: for all $i \in \mathcal{V}$

$$\mu_i^{(t+1)} = (1-\gamma^{(t)}) \overbrace{\sum_j P_{ij}\mu_j^{(t)}}^{\text{consensus part}} + \gamma^{(t)} \overbrace{\frac{y_i}{\left(\widehat{T}_i^{(t)}\right)^2}}^{\text{input}}$$

$$\nu_i^{(t+1)} = (1-\gamma^{(t)}) \underbrace{\sum_j P_{ij}\nu_j^{(t)}}_{\text{consensus part}} + \gamma^{(t)} \underbrace{\frac{1}{\left(\widehat{T}_i^{(t)}\right)^2}}_{\text{input}}$$

$$\widehat{\theta}^{(t+1)} = \mu_i^{(t+1)} / \nu_i^{(t+1)} \qquad \widehat{T}_i^{(t+1)} = \begin{cases} \alpha & \text{if } |y_i - \widehat{\theta}^{(t+1)}| < \delta \\ \beta & \text{otherwise} \end{cases} \qquad \delta = \delta(p, \alpha, \beta)$$

- algorithm parametrized by:
  - sequence of weights $\{\gamma^{(t)}\}_{t\in\mathbb{N}}$, $\gamma^{(t)} \in (0,1)\ \forall t \in \mathbb{N}$
  - nonnegative doubly-stochastic matrix $P = P(\mathcal{G})$
- messages in memory: $\mu_i^{(t)}, \nu_i^{(t)}$, $\forall i \in \mathcal{V}$ ($\Rightarrow$ sufficient statistics)
- local information is gradually propagated through entire network

Theoretical results

Theorem 1 [Convergence to a local maximum of ML-function]

H1: $P \in \mathbb{R}_+^{N \times N}$ doubly-stochastic, irreducible and aperiodic

H2: $\gamma^{(t)} \in (0,1)\ \forall t \in \mathbb{N}$, $\gamma^{(t)} \searrow 0$ and $\sum_t \gamma^{(t)} = +\infty$

Then

T1: there exists $\widehat{T}^{(\infty)} \in \{\alpha, \beta\}^N$ such that

$$\lim_{t \to +\infty} \widehat{T}^{(t)} \overset{\text{a.s.}}{=} \widehat{T}^{(\infty)}, \quad \lim_{t \to +\infty} \widehat{\theta}^{(t)} \overset{\text{a.s.}}{=} \widehat{\theta}^{(\infty)} = \frac{\sum_{k=1}^{N} y_k \left[\widehat{T}_k^{(\infty)}\right]^{-2}}{\sum_{k=1}^{N} \left[\widehat{T}_k^{(\infty)}\right]^{-2}} \mathbb{1};$$

T2: $(\widehat{\theta}^{(\infty)}, \widehat{T}^{(\infty)})$ is a local maximum of log-likelihood function $L(\theta, T)$

Relative classification error:

$$\mathsf{P}_N(\mathsf{e}) := \frac{1}{N}\mathbb{E}\left[\mathsf{d_H}(T, \widehat{T}^\infty)\right]$$

Theorem 2 [Lower bound on $\mathsf{P}_N(\mathsf{e})$]

$$\liminf_{N\to+\infty} \ \mathsf{P}_N(\mathsf{e}) \geq P_{LB}(p, \alpha, \beta).$$

$$P_{LB}(p, \alpha, \beta) = (1-p)\mathrm{erfc}\left(\frac{\delta}{\alpha\sqrt{2}}\right) + p\left[1 - \mathrm{erfc}\left(\frac{\delta}{\beta\sqrt{2}}\right)\right]$$

Remarks:

$$\lim_{p\to 0} P_{LB}(p, \alpha, \beta) = 0$$

$$\lim_{\frac{\beta}{\alpha}\to+\infty} P_{LB}(p, \alpha, \beta) = 0 \qquad \lim_{\frac{\beta}{\alpha}\to 1} P_{LB}(p, \alpha, \beta) = 1$$

Simulations

Model: $\theta = 0$, $\alpha = 0.3$, $\beta = 10$,

$$y_i = \theta + T_i n_i \qquad p(T_i = x) = \begin{cases} 0.25 & \text{if } x = \beta \\ 0.75 & \text{if } x = \alpha \end{cases}$$

Sequence of parameters: $\gamma^{(t)} \asymp t^{-\zeta},\ \zeta \in \{0.3, 0.5, 0.7, 0.9\}$

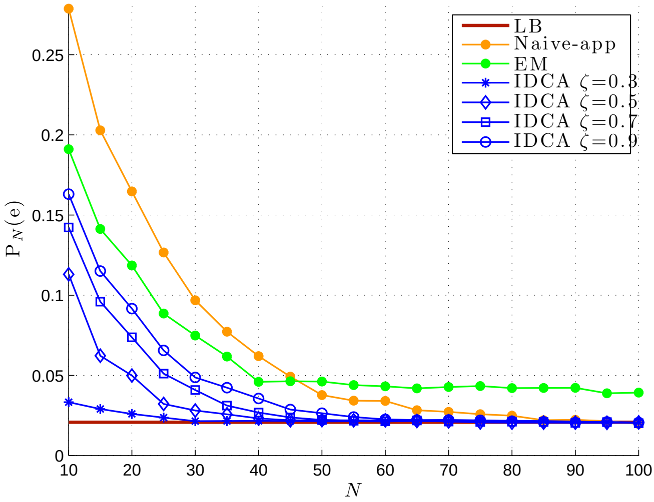
Tested communication graphs:

- Complete graph, circulant graph, 2d-grid
- Random geometric graphs ($r = 0.3$)

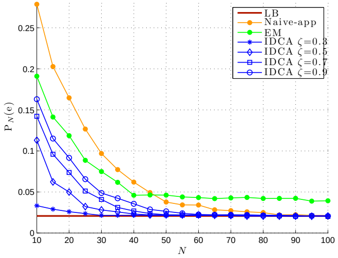
Performance metric:

- average classification error $P_N(\mathrm{e}) = \mathbb{E}\left[\frac{\mathrm{d_H}(\widehat{T}^{(\infty)}, T)}{N}\right]$
- convergence time

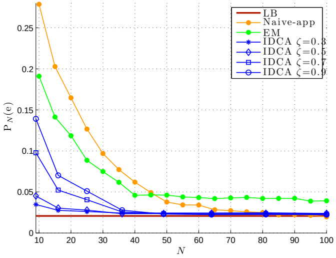
Complete graph



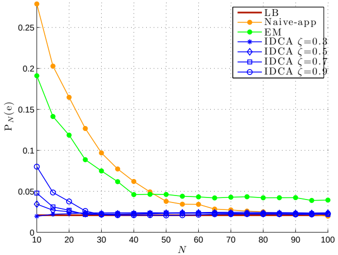
Complete graph



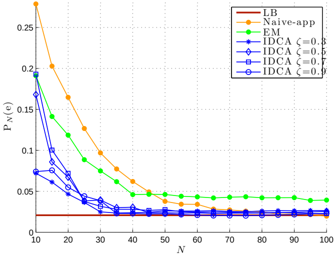
2d-grid graph



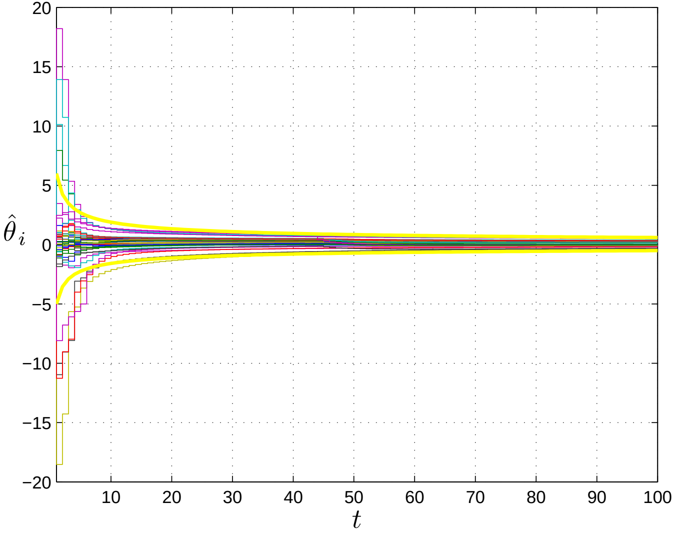
Circulant graph



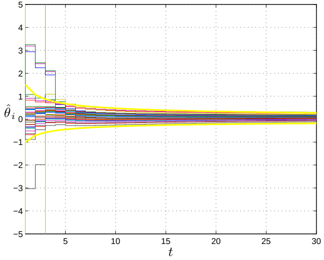
Random geometric graph



Circulant graph $\gamma^{(t)} \asymp t^{-0.5}, N = 40$



Complete graph



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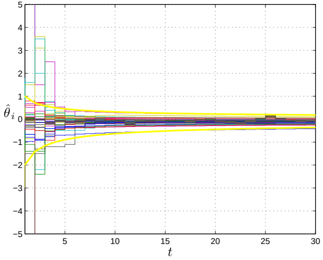

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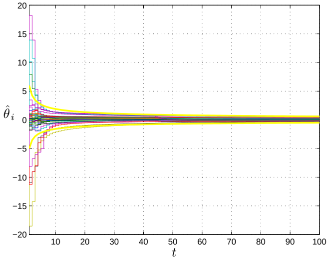

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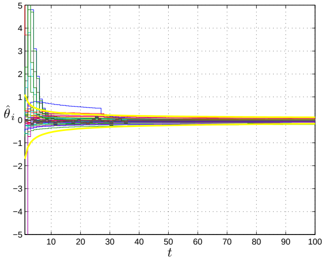

Sketch of the proofs

Dynamical system $\mu^{(0)} = y, \nu^{(0)} = \mathbb{1}, \widehat{T}^{(0)} = \alpha \mathbb{1}$
$t \in \mathbb{Z}_{t \geq 0}$

$$\mu_i^{(t+1)} = (1 - \gamma^{(t)}) \sum_j P_{ij} \mu_j^{(t)} + \gamma^{(t)} \frac{y_i}{\left(\widehat{T}_i^{(t)}\right)^2}$$
$$\nu_i^{(t+1)} = (1 - \gamma^{(t)}) \sum_j P_{ij} \nu_j^{(t)} + \gamma^{(t)} \frac{1}{\left(\widehat{T}_i^{(t)}\right)^2}$$

$$\widehat{\theta}_i^{(t+1)} = \mu_i^{(t+1)} / \nu_i^{(t+1)} \qquad \widehat{T}^{(t+1)} = \begin{cases} \alpha & \text{if } |y_i - \widehat{\theta}_i^{(t+1)}| < \delta \\ \beta & \text{otherwise} \end{cases}$$

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1. Convergence of $\mu^{(t)}, \nu^{(t)}$ ($\Rightarrow \widehat{\theta}^{(t)}$), assuming $\widehat{T}^{(t)}$ is stabilized
   - Linear systems
   - Perron-Frobenius theory of nonnegative matrices
2. Stabilization of $\widehat{T}^{(t)}$ in finite time
   - Study of discrete-time switched system, using asymptotic techniques
   - Study of geometry of candidate limit points

2. Stabilization (a.s.) of $\widehat{T}^{(t)}$

Given observations y and fixed $\omega \in \{\alpha, \beta\}^N$

$$\Theta_\omega = \{x \in \mathbb{R}^N : \ |x_i - y_i| < \delta, \text{ if } \omega_i = \alpha, \ |x_i - y_i| \geq \delta, \text{ if } \omega_i = \beta\}$$

If $\theta^{(t)} \in \Theta_\omega$

$$\mu^{(t+1)} = f_\omega(t, \mu^{(t)}) \qquad \nu^{(t+1)} = g_\omega(t, \nu^{(t)})$$

$$\widehat{\theta}_i^{(t+1)} = \mu_i^{(t+1)} / \nu_i^{(t+1)}$$

Discrete-time switched system

- families of functions $\mathfrak{F} = \{f_\omega\}_{\omega \in \{\alpha,\beta\}^N}$, $\mathfrak{G} = \{g_\omega\}_{\omega \in \{\alpha,\beta\}^N}$
- closed-loop switched system: switching policy determined by $\theta^{(t)}$
- stabilization of $\widehat{T}^{(t)} \iff \exists \omega^\star \in \{\alpha, \beta\}^N : \widehat{\theta}^{(t)} \in \Theta_{\omega^\star}$ definitively
- candidate limit points for $\widehat{\theta}^{(t)}$

$$\bar{y}_\omega = \frac{\sum_{i \in \mathcal{V}} y_i \omega_i^{-2}}{\sum_{i \in \mathcal{V}} \omega_i^{-2}} \qquad \omega \in \{\alpha, \beta\}^N$$

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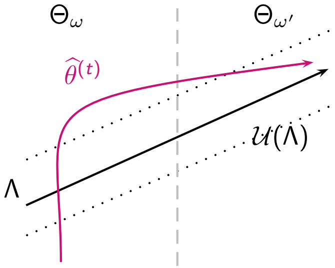
$$\widehat{\theta}_i^{(t+1)} = \mu_i^{(t+1)} / \nu_i^{(t+1)}$$

For large t: $\Lambda = \{\lambda \mathbb{1} | \lambda \in \mathbb{R}\}$

2.1 $\mu^{(t)}$, $\nu^{(t)}$ close to consensus vectors $\bar{\mu}^{(t)}\mathbb{1}$ and $\bar{\nu}^{(t)}\mathbb{1}$
   ($\Rightarrow \widehat{\theta}^{(t)} \in \mathcal{U}(\Lambda)$ definitively)

2.2 motion of $\theta^{(t)}$ thorough contiguous regions



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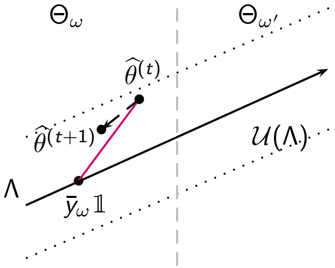
For large t:

$$\widehat{\theta}^{(t+1)} - \theta^{(t)} = c^{(t)} \gamma^{(t)} (\bar{y}_\omega \mathbb{1} - \theta^{(t)}) + O((\gamma^{(t)})^2)$$

$$\bar{y}_\omega = \frac{\sum_{i \in \mathcal{V}} y_i \omega_i^{-2}}{\sum_{i \in \mathcal{V}} \omega_i^{-2}}$$

2.3 If $\bar{y}_\omega \mathbb{1} \in \Theta_\omega \Rightarrow \exists$ an asymptotic invariant set in $\mathcal{U}(\Lambda) \cap \Theta_\omega$



2. Stabilization (a.s.) of $\widehat{T}^{(t)}$

Given observations y and fixed $\omega \in \{\alpha, \beta\}^N$

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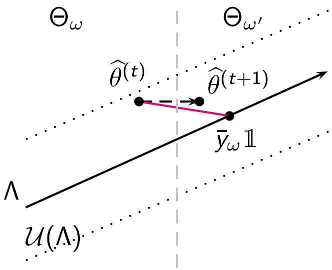
For large t:

$$\widehat{\theta}^{(t+1)} - \theta^{(t)} = c^{(t)} \gamma^{(t)} (\bar{y}_\omega - \theta^{(t)}) + O((\gamma^{(t)})^2)$$

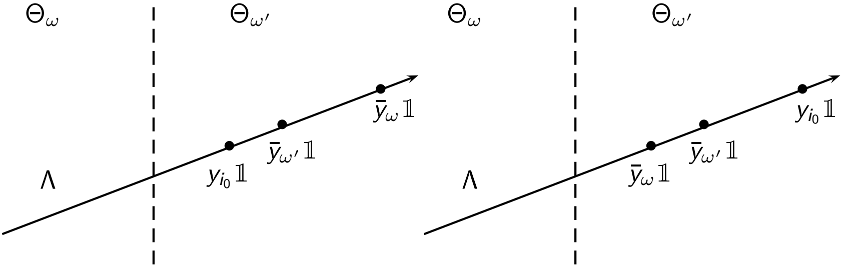
$$\bar{y}_\omega = \frac{\sum_{i \in \mathcal{V}} y_i \omega_i^{-2}}{\sum_{i \in \mathcal{V}} \omega_i^{-2}}$$



2.4 If $\bar{y}_\omega \mathbb{1} \notin \Theta_\omega \Rightarrow \theta^{(t)} \notin \Theta_\omega$ definitevely (oscillations not allowed)

2. Stabilization (a.s.) of $\widehat{T}^{(t)}$

2.4 Oscillations not allowed: $\omega, \omega' \in \{\alpha, \beta\}^N$, $\omega_i = \omega'_i \; \forall i \neq i_0$



$\omega_{i_0} = \beta$ and $\omega'_{i_0} = \alpha$

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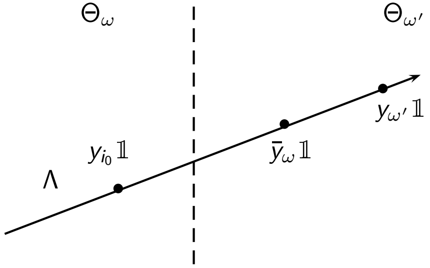

$$\omega_{i_0} = \alpha \text{ and } \omega'_{i_0} = \beta$$

Concluding remarks

Summary

- ML-approach for classification and estimation in sensor networks
- New distributed algorithm: IDCA
  1. convergence to a local maximum of ML function
  2. convergence time $\rightsquigarrow \gamma^{(t)}$
  3. lower bound on relative classification error

Future developments

1. ML-estimation
   - study of relative classification error
   - asymptotic behavior of local and global maxima when $N \to \infty$
2. IDCA
   - study of relative classification error
   - protocol for adaptive search of sequence $\gamma^{(t)}$
   - robustness to outliers
   - resilience to node failures
   - asynchronous versions of IDCA (gossip-type?)