Some results on optimal estimation and control for lossy NCS

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Joint work w/ Bruno Sinopoli (CMU),
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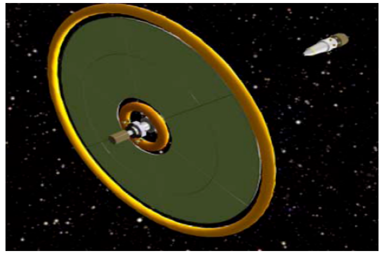
KTH, 14/09/2007

Networked Control Systems

Drive-by-wire systems

Swarm robotics

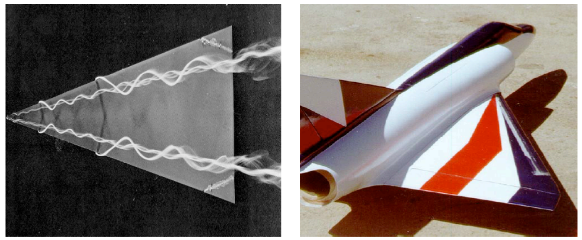
Smart structures: adaptive space telescope



Wireless Sensor Networks

Traffic Control: Internet and transportation

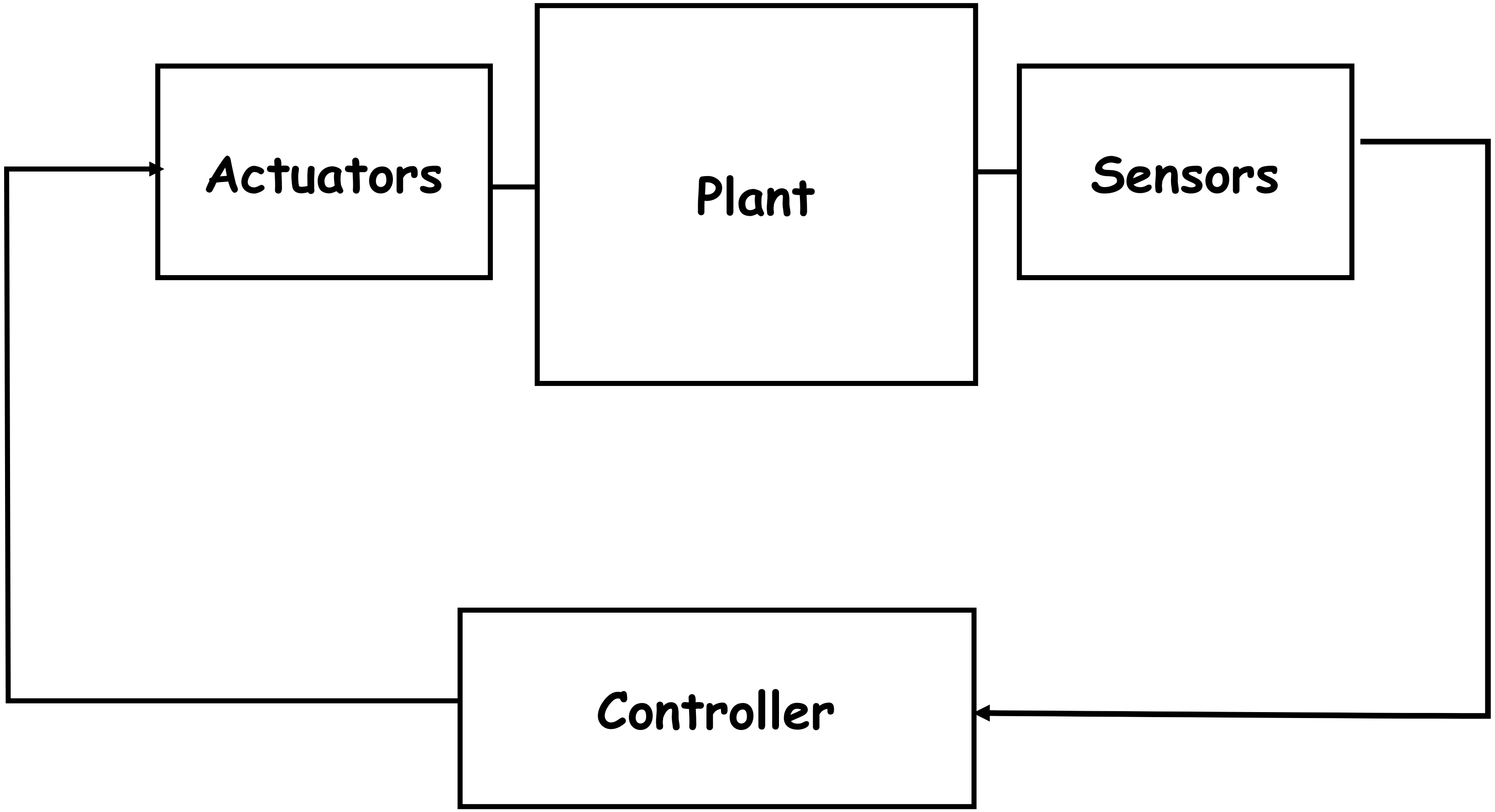
Smart materials: sheets of MEMS sensors and actuators



NCSs: physically distributed dynamical systems interconnected by a communication network

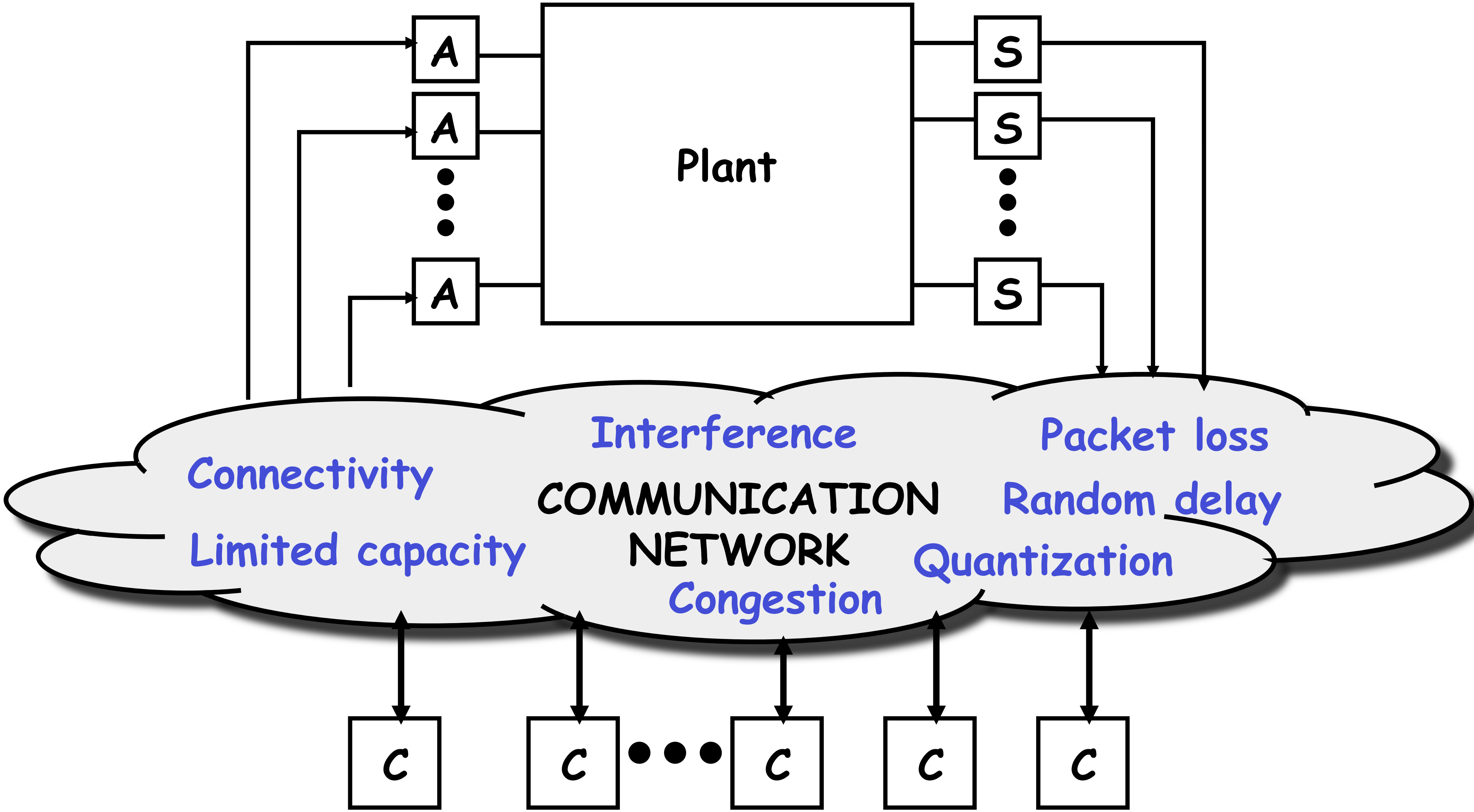
NCSs: what's new for control?

Classical architecture: Centralized structure

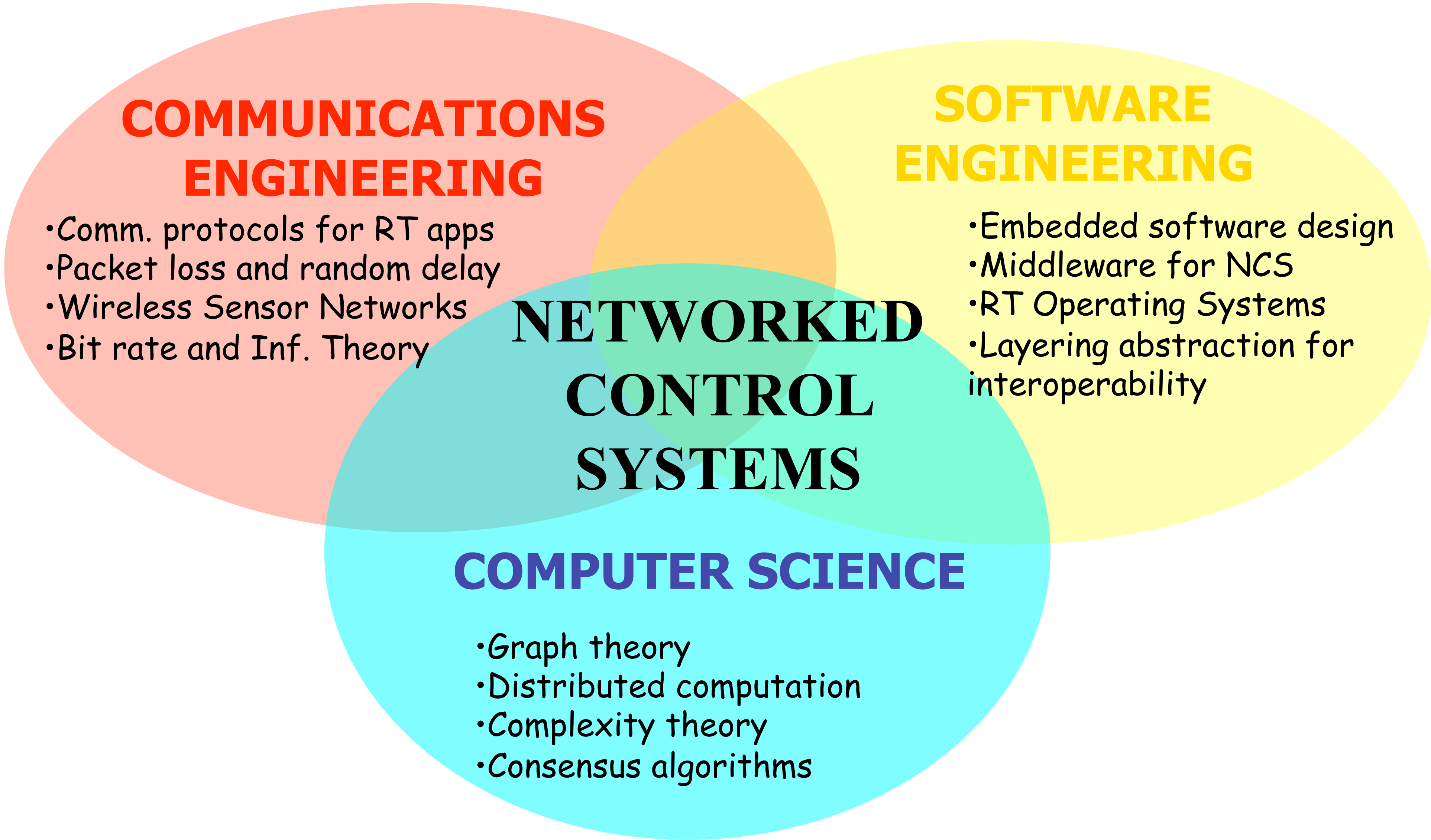


NCSs: what's new for control?

NCSs: Large scale distributed structure



Interdisciplinary research needed



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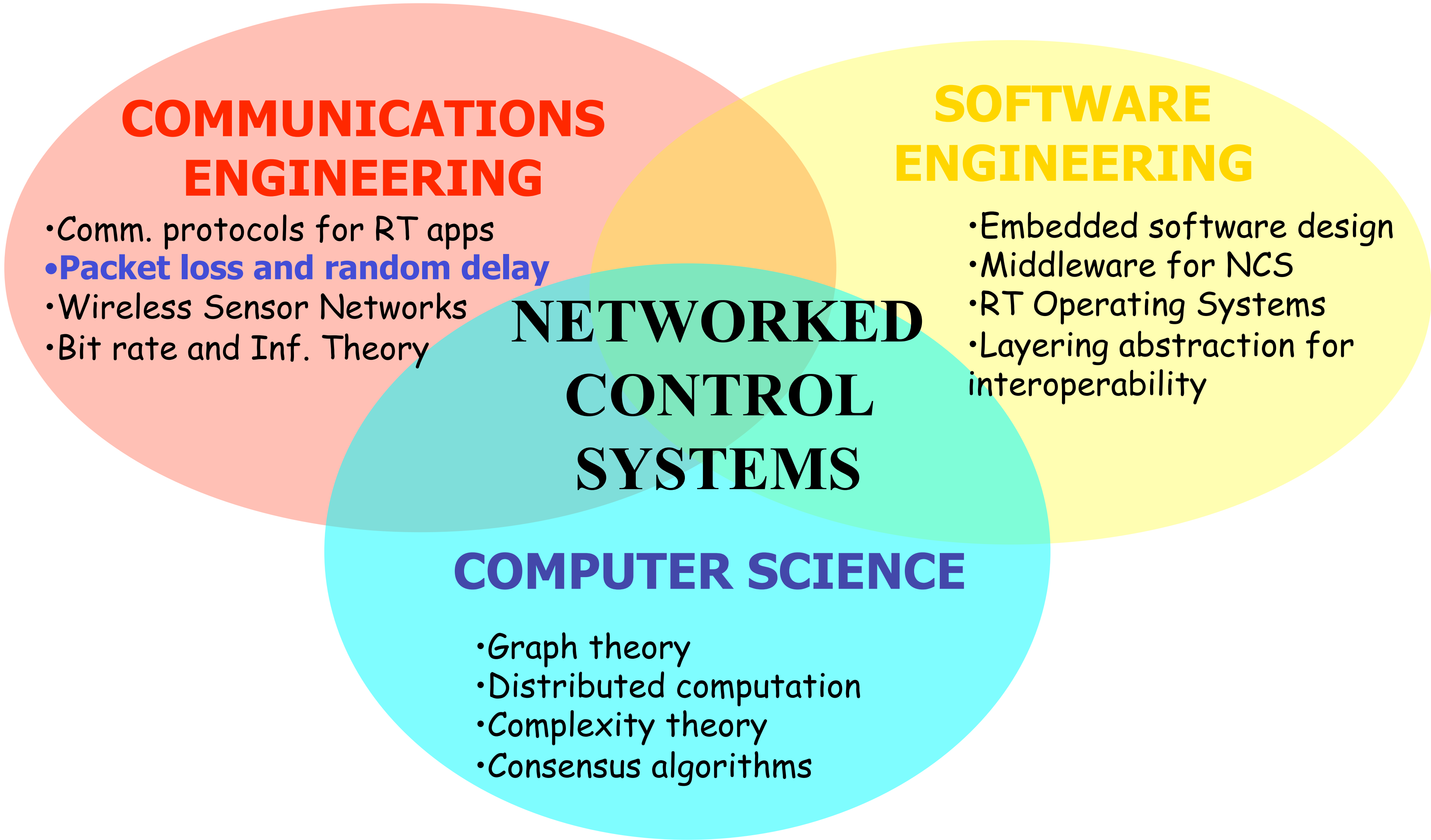
Interdisciplinary research needed



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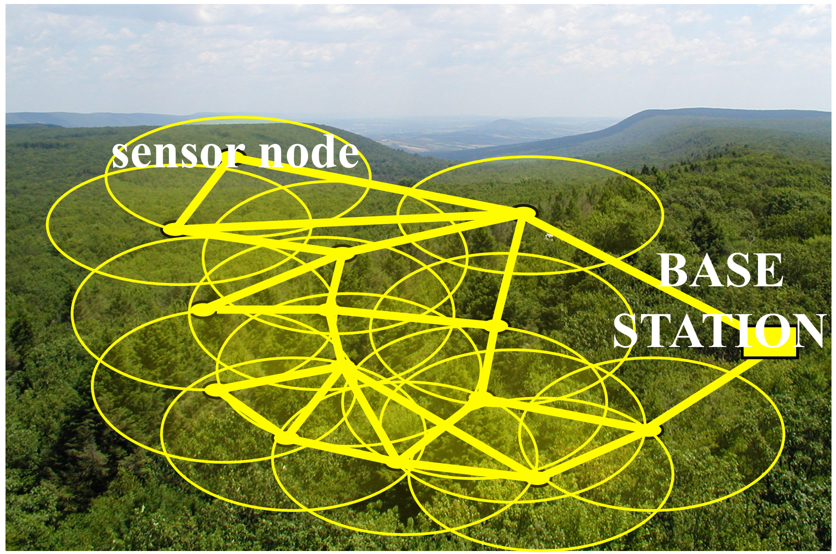
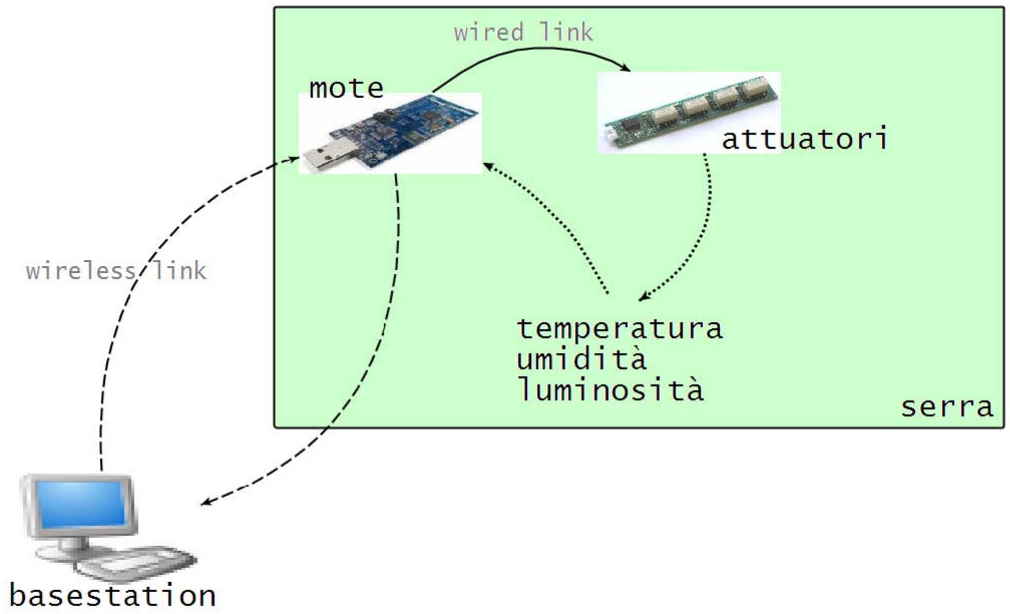
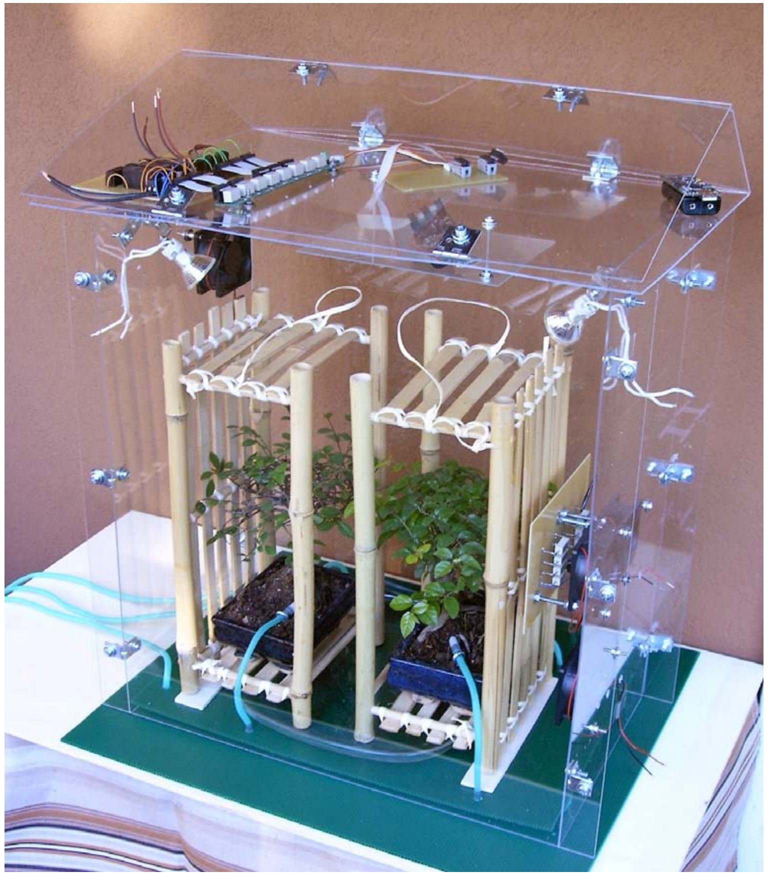
Wireless Sensor Actuator Networks (WSANs)

- Small devices
  - μController, Memory
  - Wireless radio
  - Sensors & Actuators
  - Batteries
- Inexpensive
- Multi-hop communcation
- Programmable (micro-PC)



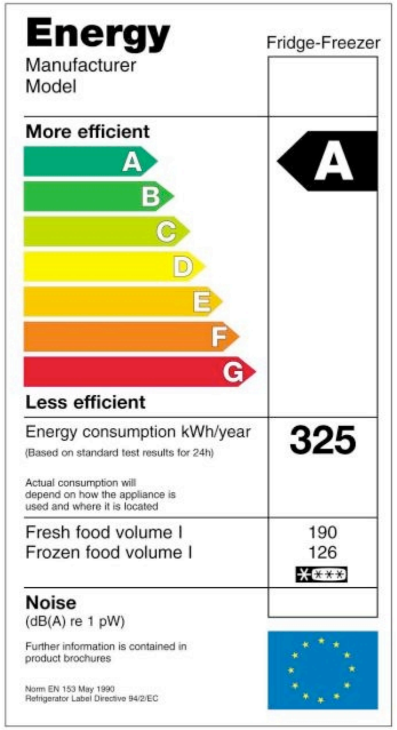
NCS example: Smart Greenhouse





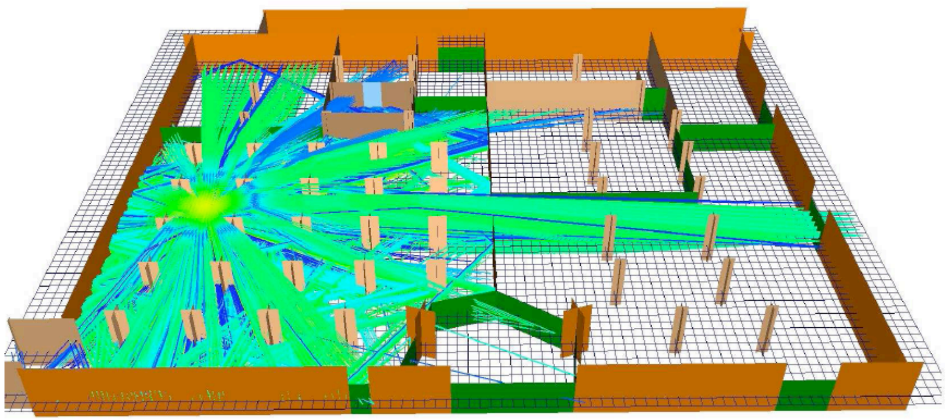
- Distributed estimation
- Distributed control
- Control under packet loss & random delay
- Sensor fusion
- Distributed time synchronization

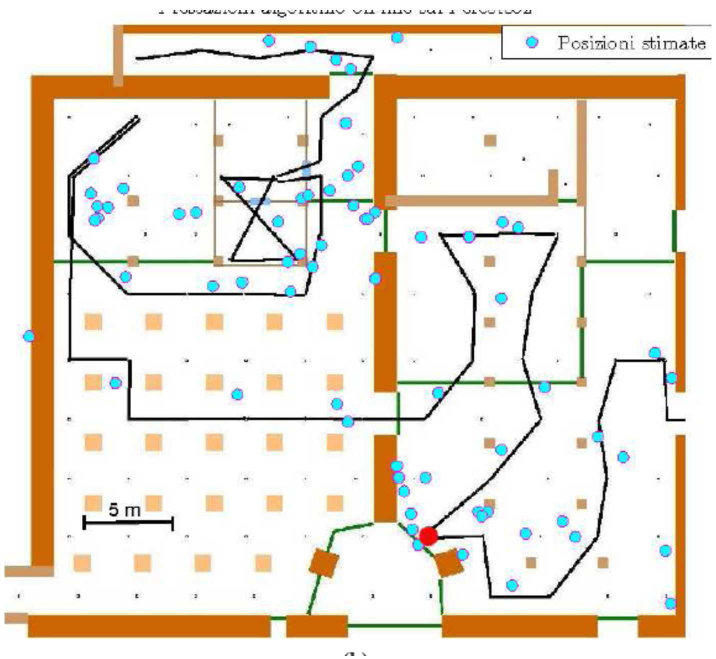
NCS example: ThermoEfficiency Certification



- Building thermodynamics model identification
- Sensor selection for identification
- Optimal sensor placement

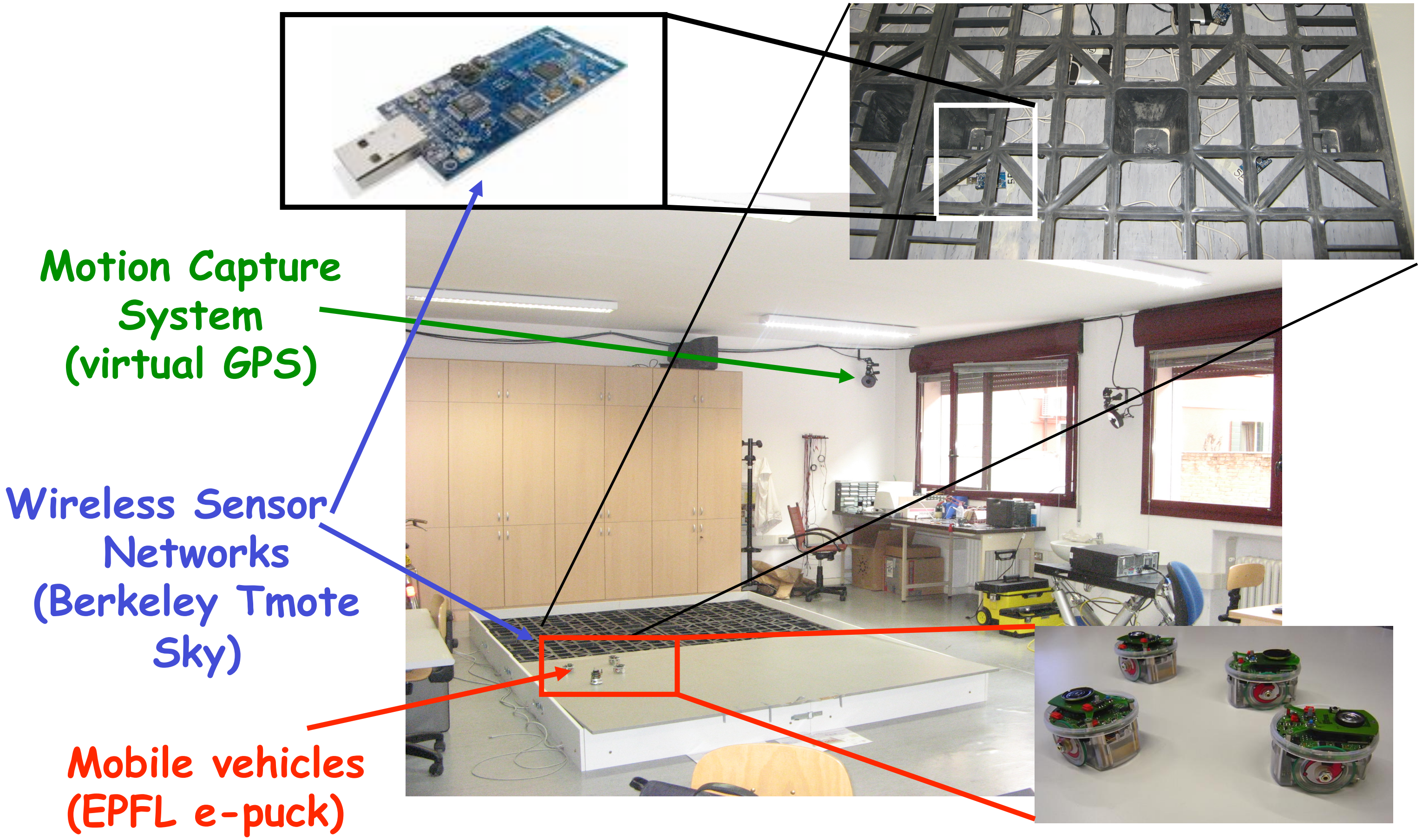
NCS example: Distributed Localization&Tracking





- Indoor radio signal modeling
- Real-time localization
- Distributed tracking
- Coordination

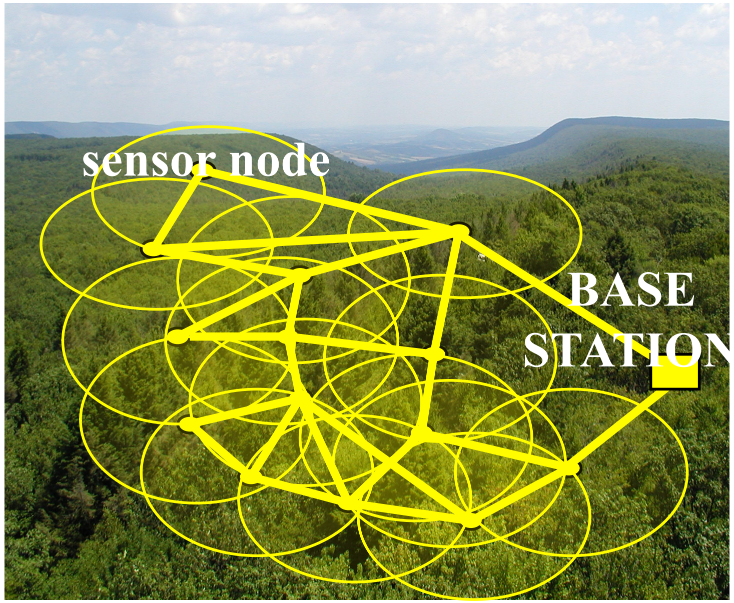
NCS example:
Coordinated Robotics & WSNs



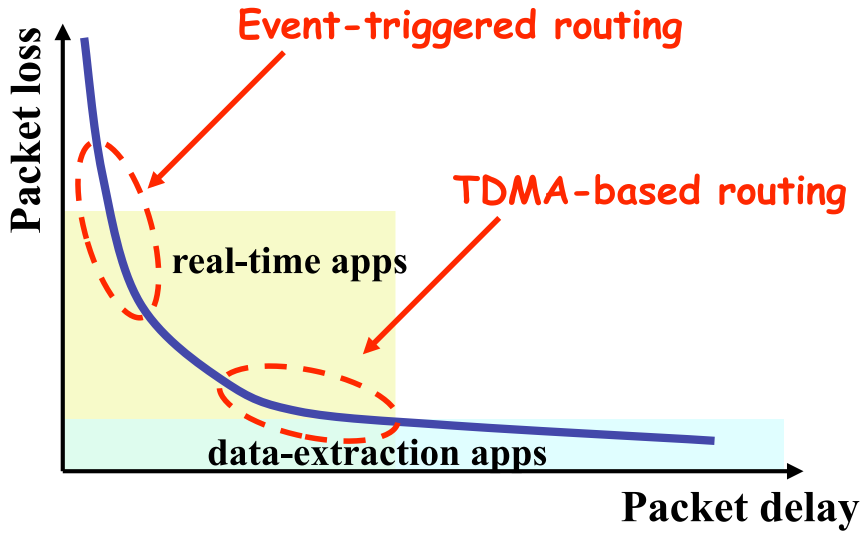
- Coordination & consensus algorithms
- Integration mobile nodes w/ static nodes
- WSN-based localization & navigation

Motivating example: wireless sensor networks

Forest Temperature Monitoring
(data-extraction application)

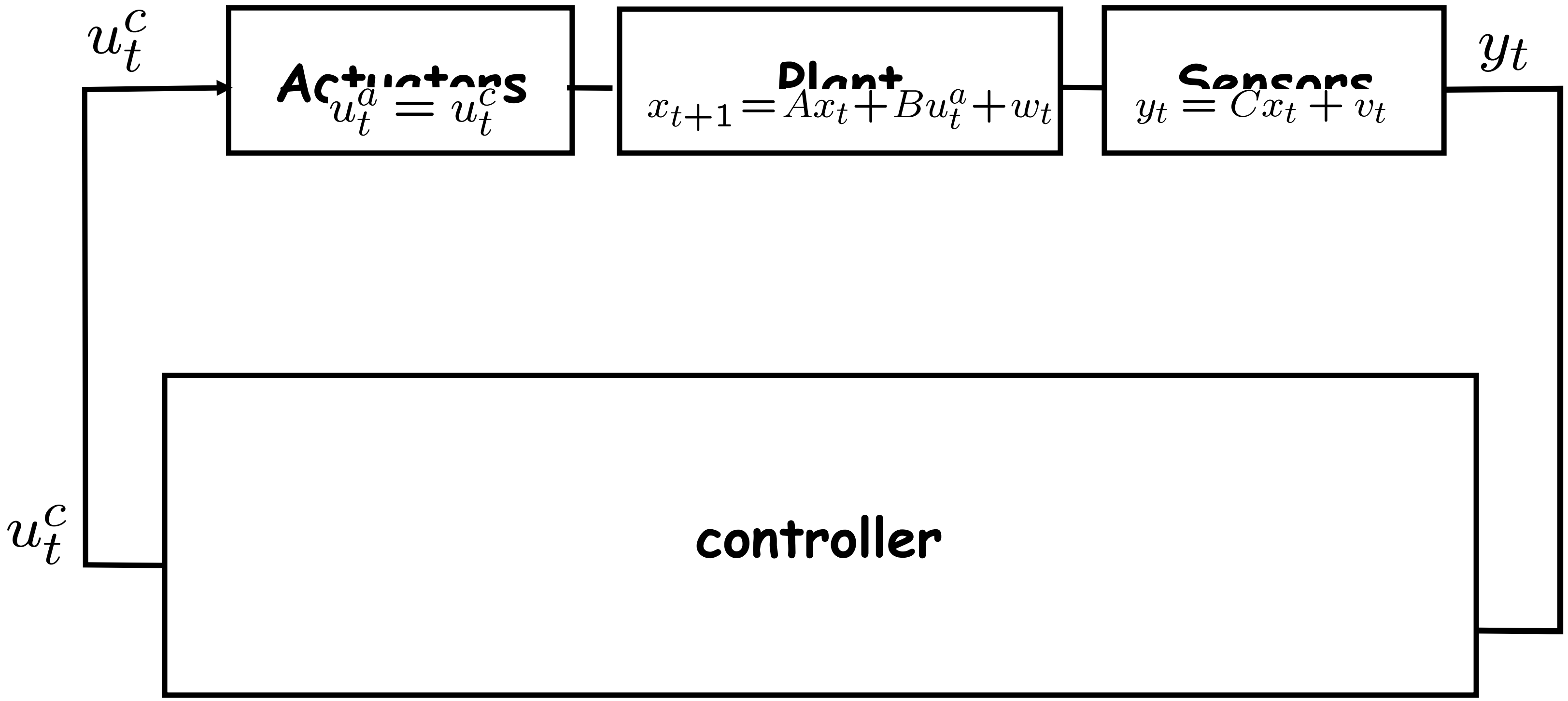


Wildfire detection & tracking
(real-time application)



- Can we design optimal estimators that compensate for random delay and packet loss ?
- What is the performance if we have packet arrival statistics ?
- How can we compare different communication/routing protocols in terms of estimation performance ?

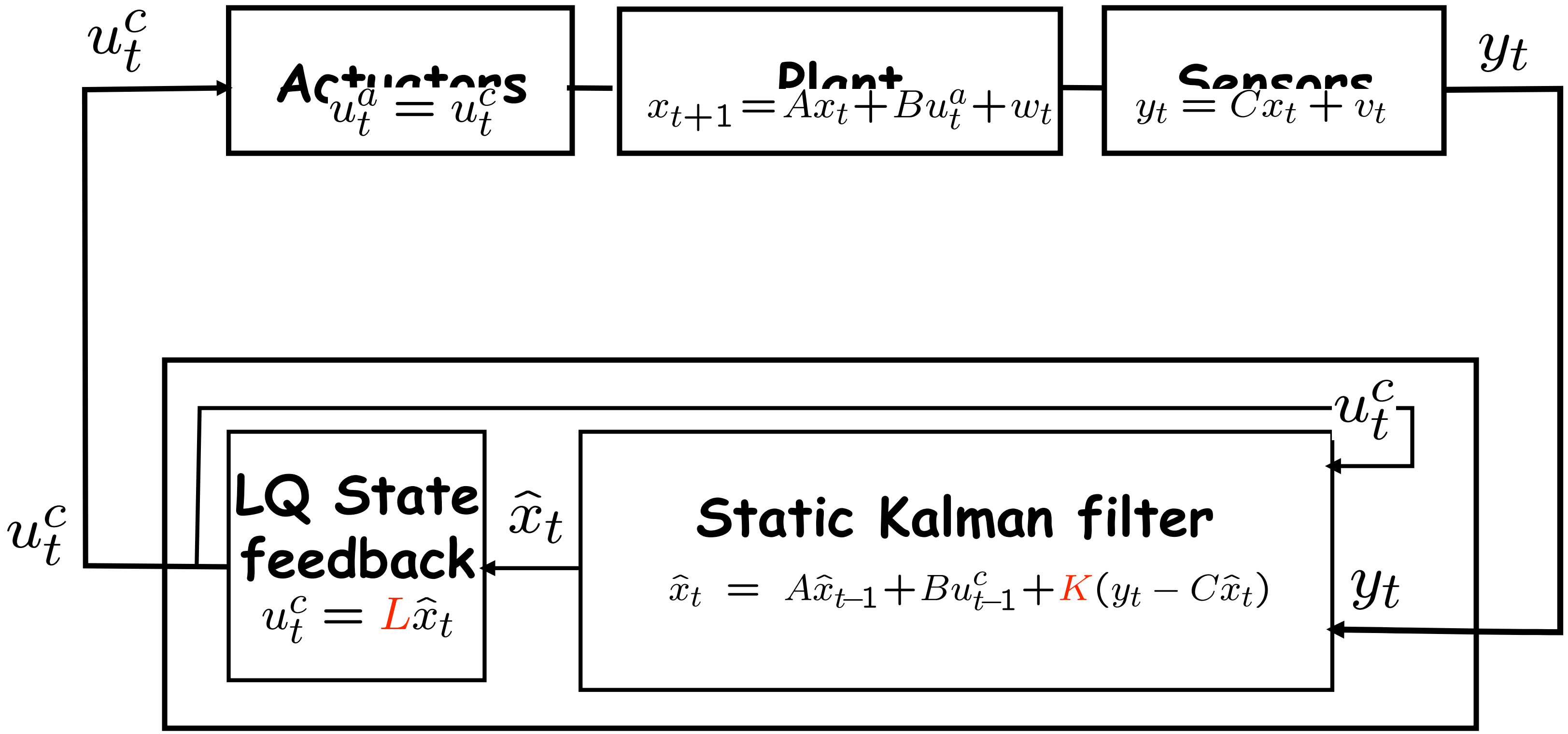
Optimal LQG



$$\min_{u_1^c, \ldots u_T^c} J = \sum_{t=1}^T E[x_t^T W x_t + u_t^T U u_t], \quad T \to \infty$$

Sensors and actuators are co-located, i.e. no delay nor loss

Optimal LQG



1. Separation principle holds: Optimal controller = Optimal estimator design + Optimal state feedback design
2. Closed Loop system always stable (under standard reach./det. hypotheses)
3. Gains K,L are constant solution of Algebraic Riccati Equations

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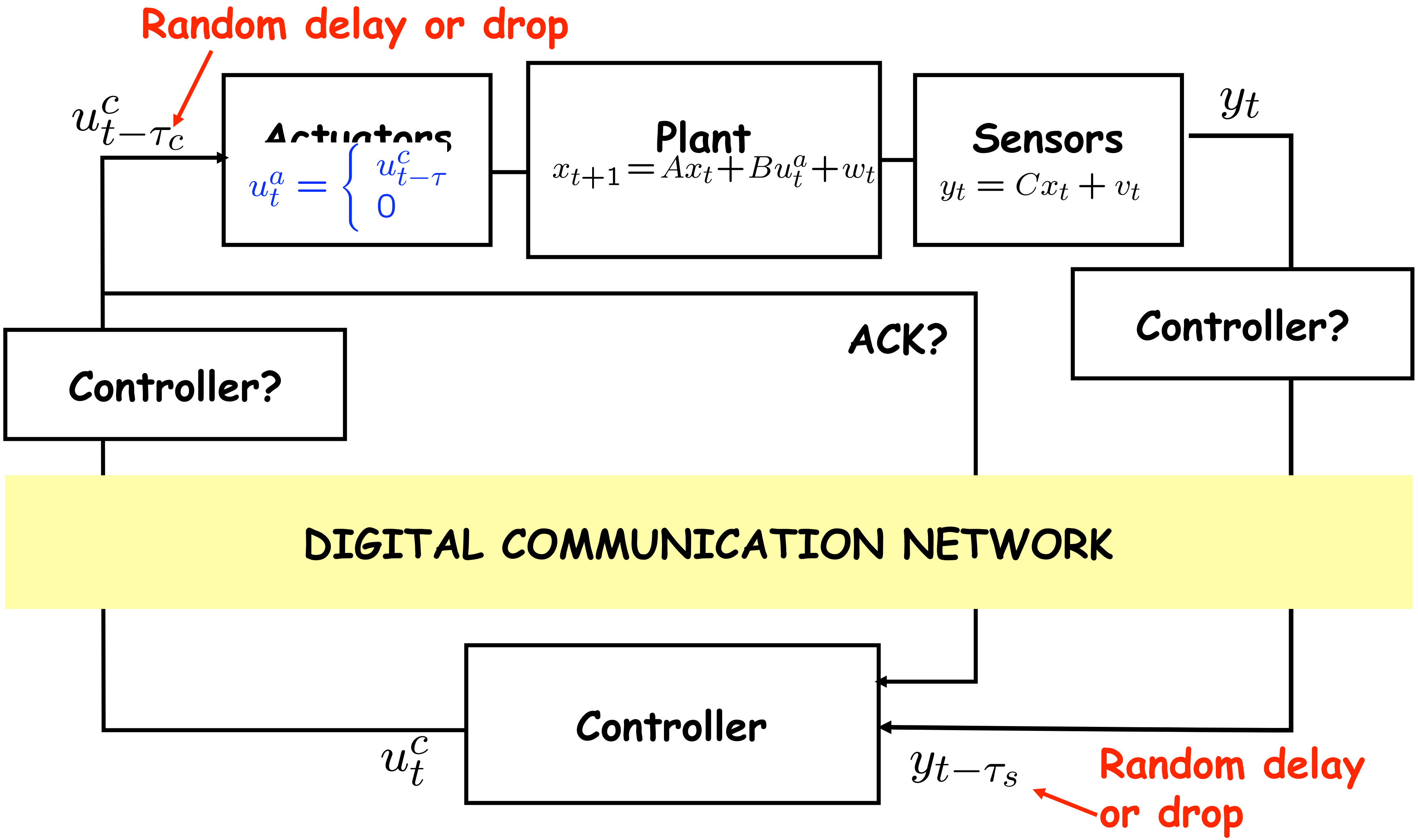
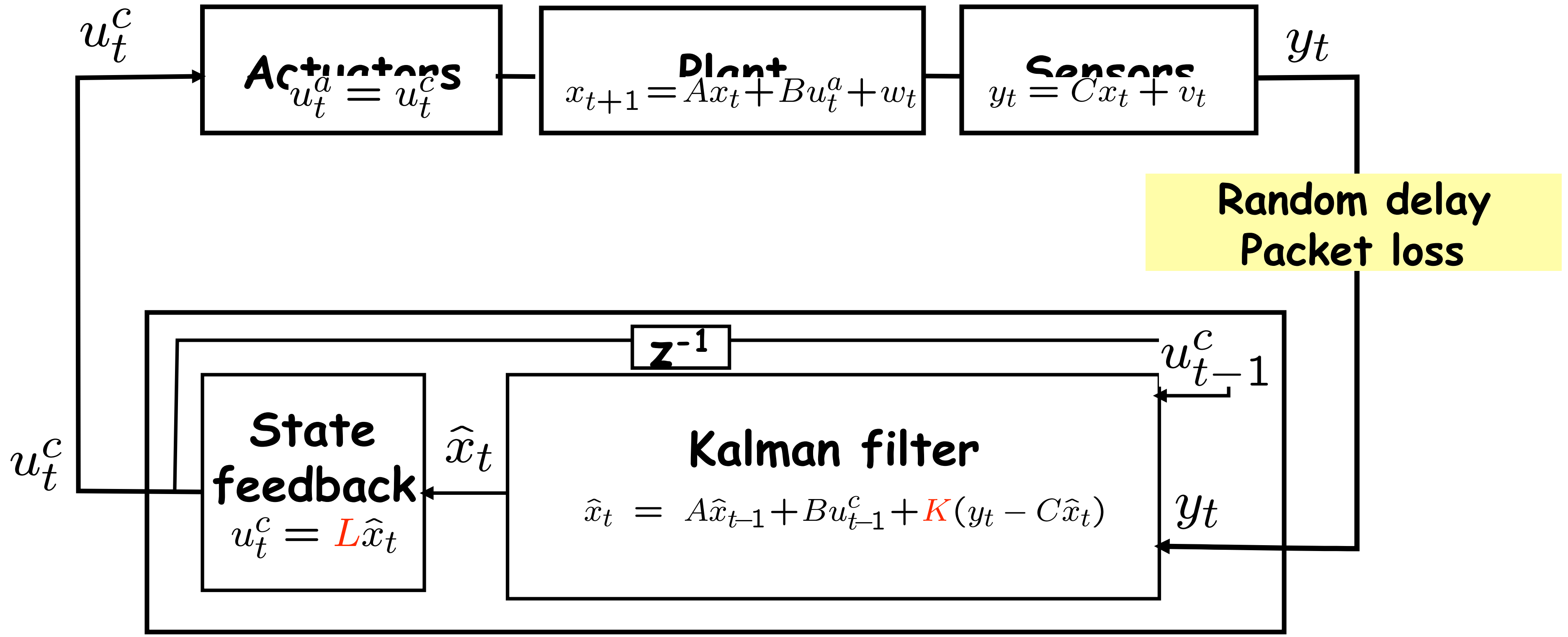
Optimal LQG control over DCN



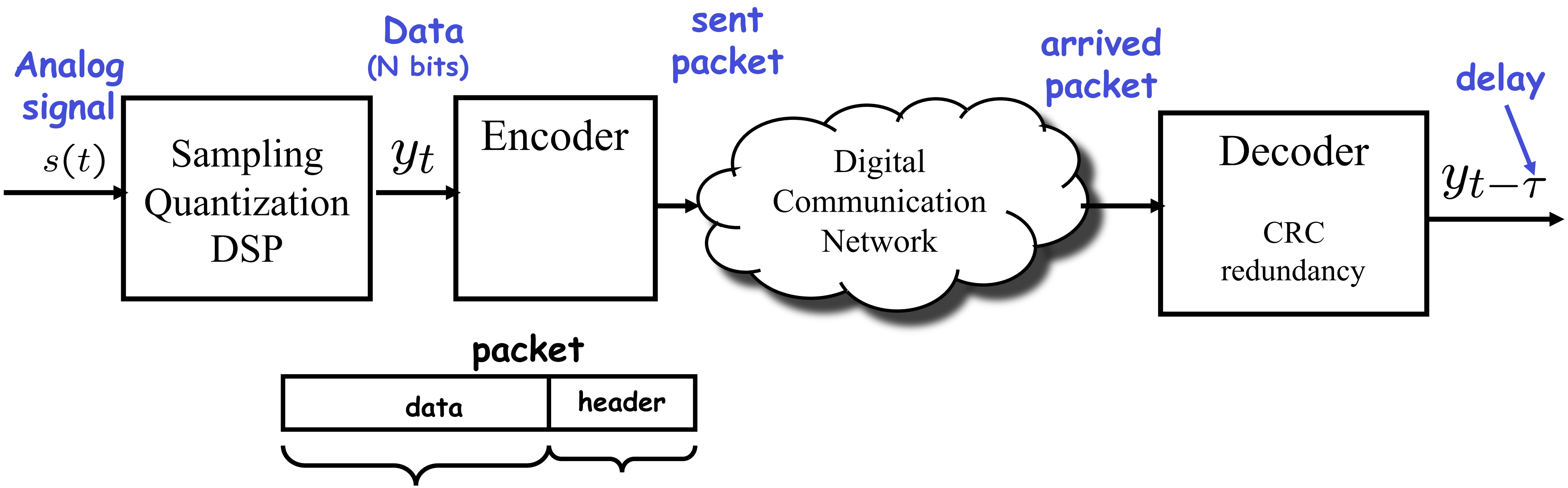
Some consideration on the separation principle



$$\hat{x}_t = E[x_t | y_t, y_{t-1}, .., y_0, u_{t-1}^a, .., u_1^a]$$

if $(u_{t-1}^a, \dots u_1^a)$ known $\Longrightarrow e_t = x_t - \hat{x}_t = f(y_t, y_{t-1}, .., y_1, y_0)$

Modeling of Digital Communication Network (DCN)



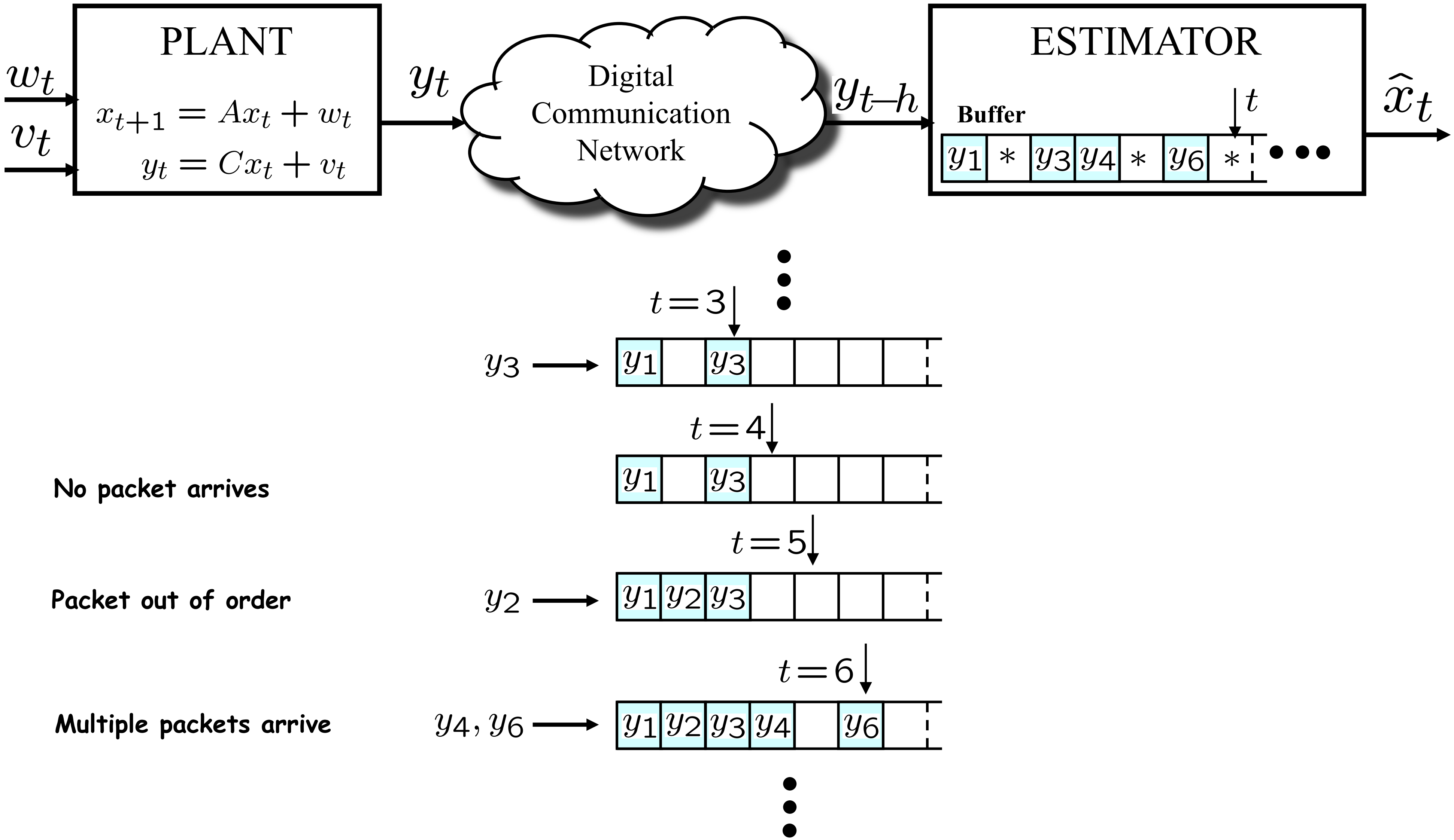
	data	header
ATM	384 bits	40 bits
Ethernet	>368 bits	112 bits
Bluetooth	>499 bits	~100 bits
Zigbee	<1000 bits	128 bits

Assumptions:

(1) Quantization noise<<sensor noise
(2) Packet-rate limited (≠ bit-rate)
(3) No transmission noise (data corrupted=dropped packet)
(4) Packets are time-stamped

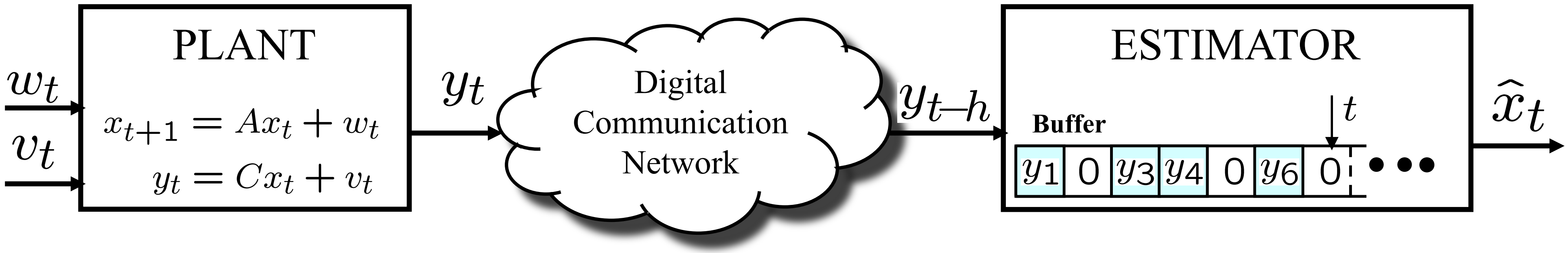
→ **Random delay & Packet loss at receiver**

Estimation modeling



Minimum variance estimation

$$\hat{x}_t = \mathbb{E}[x_t \mid \{y_k\} \text{ available at estimator at time } t]$$



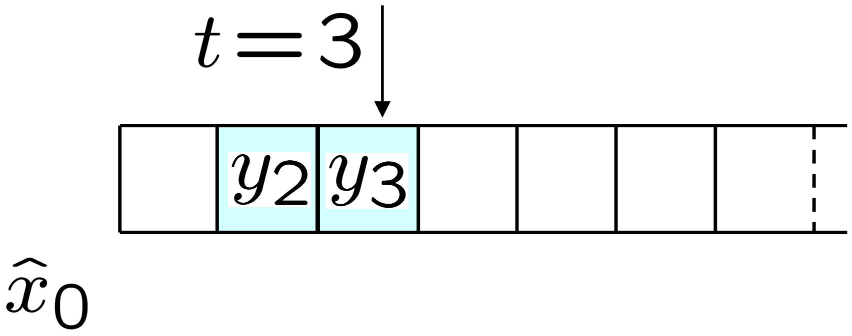
$$\gamma_k^t = \begin{cases} 1 & \textit{if } y_k \text{ arrived before or at time } t, \; t \geq k \\ 0 & \textit{otherwise} \end{cases}$$

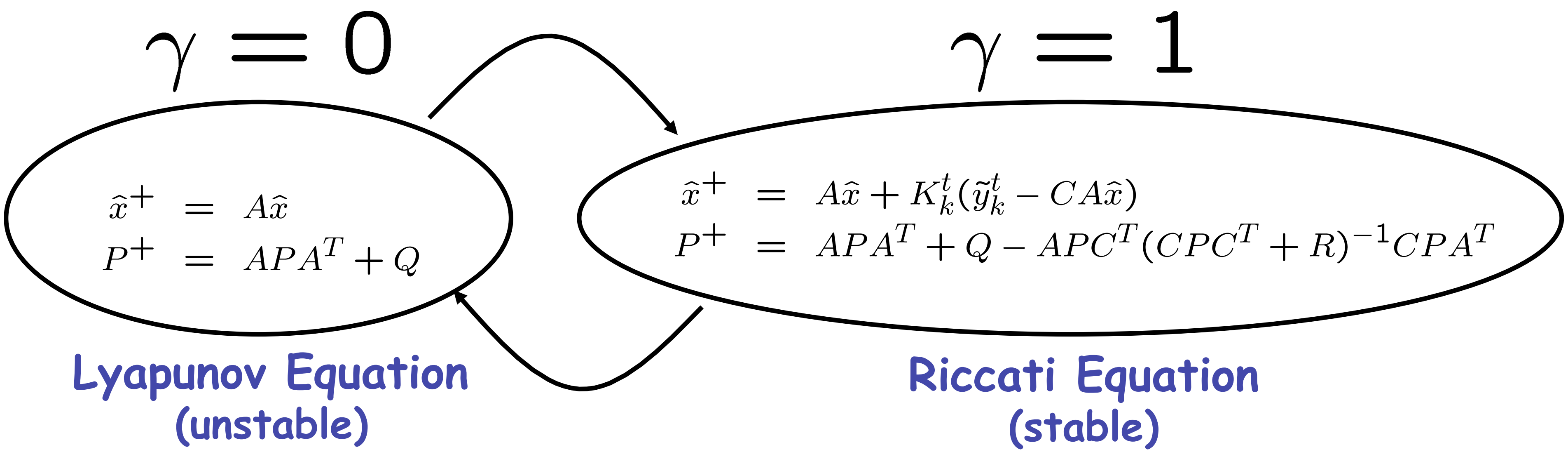
$$\tilde{y}_k = \gamma_k^t (Cx_k + v_k) = C_k^t x_k + u^t$$

Kalman time-varying linear system

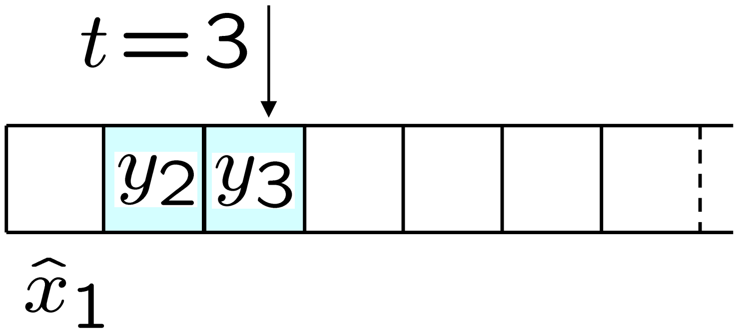
$$\hat{x}_t = \mathbb{E}[x_t \mid \tilde{y}_1, \ldots, \tilde{y}_t, \gamma_1^t, \ldots, \gamma_t^t]$$

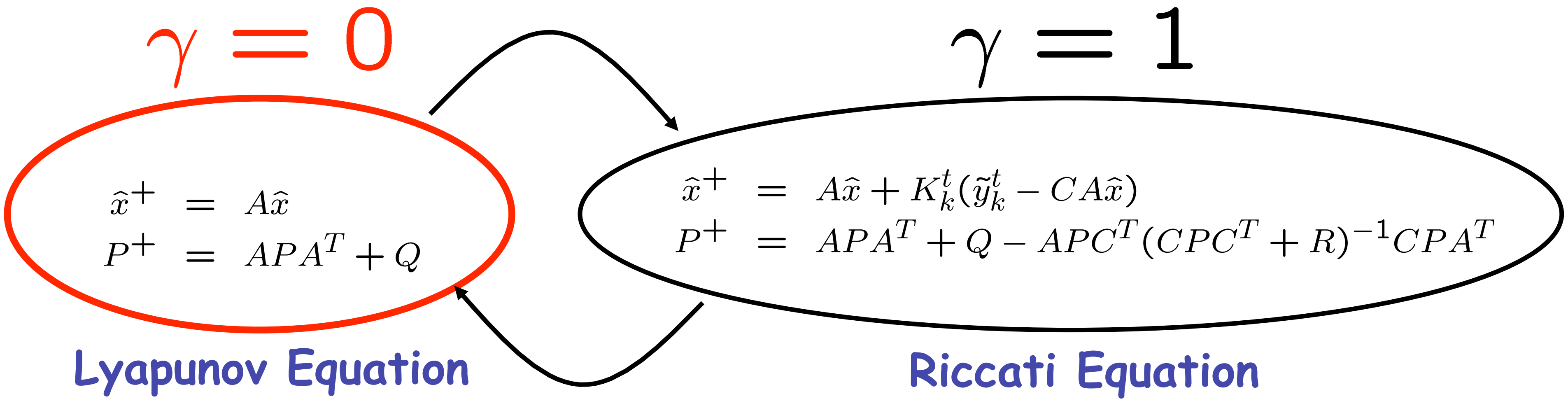
Minimum variance estimation





Minimum variance estimation

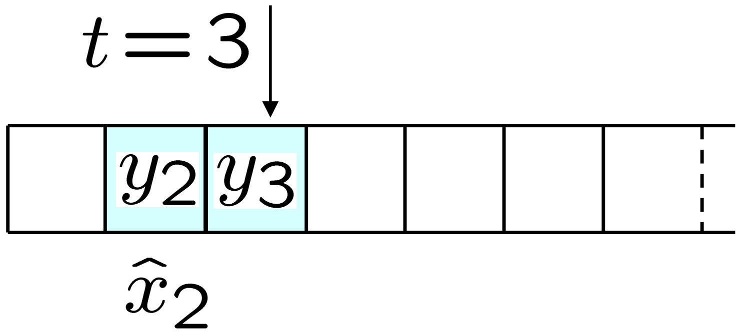


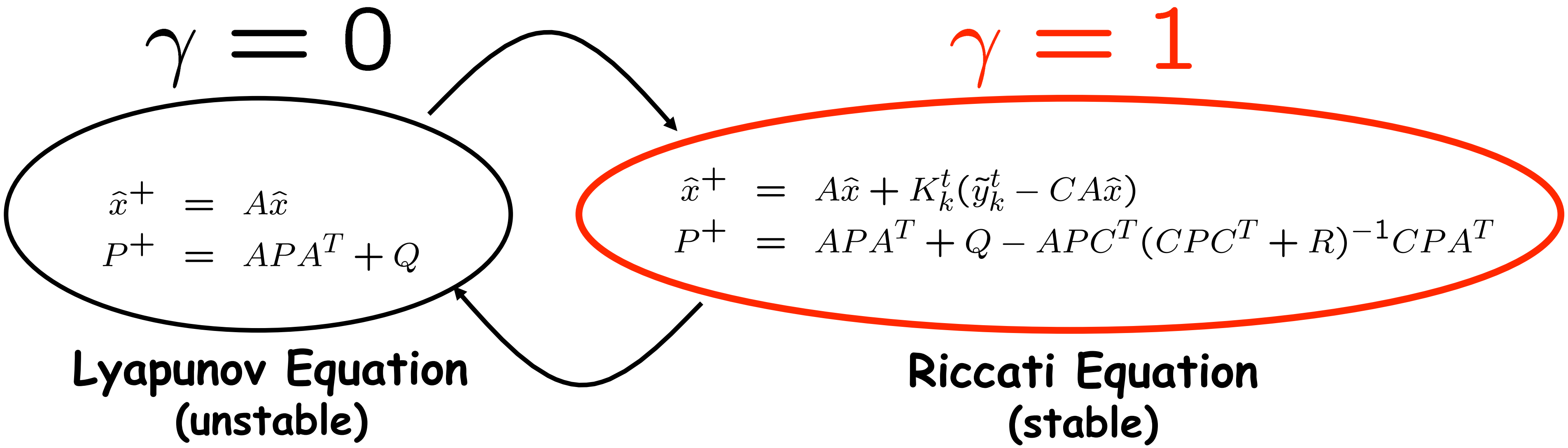


Lyapunov Equation (unstable)

Riccati Equation (stable)

Minimum variance estimation





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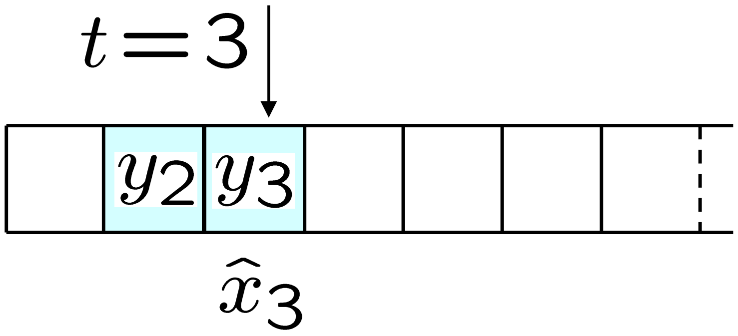

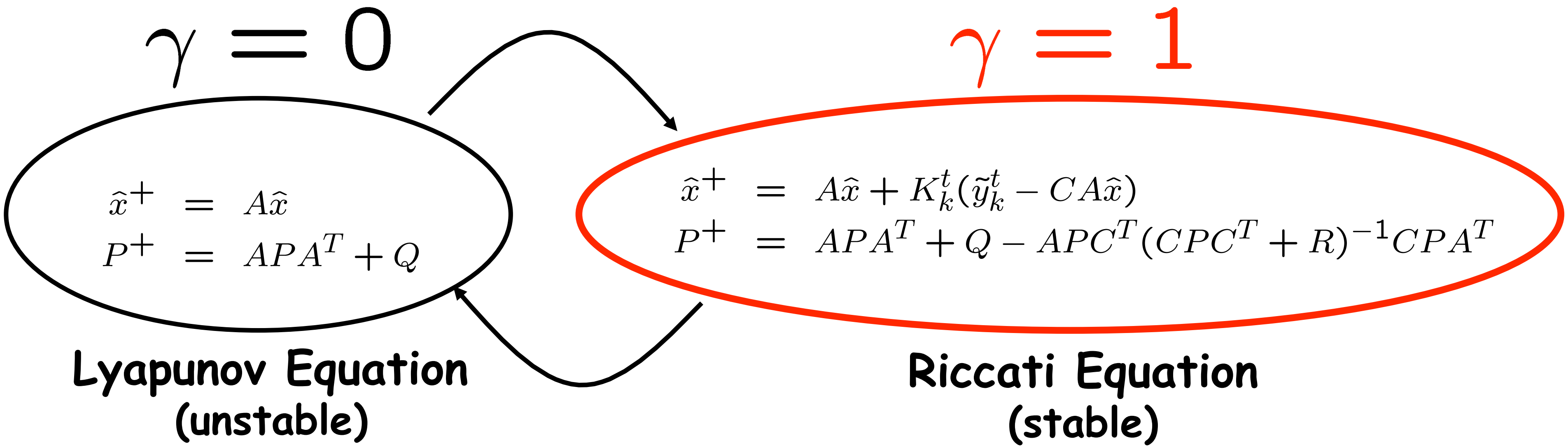

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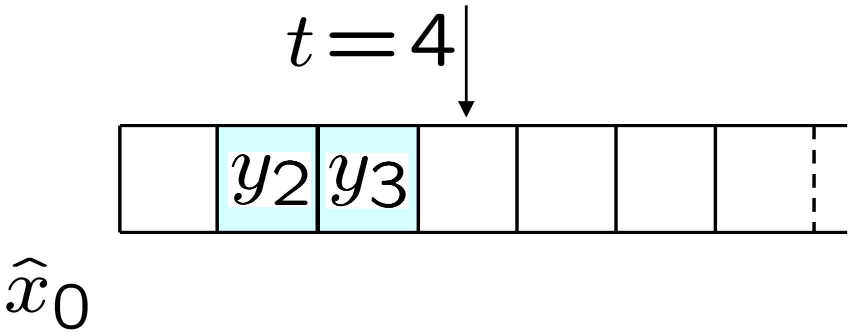

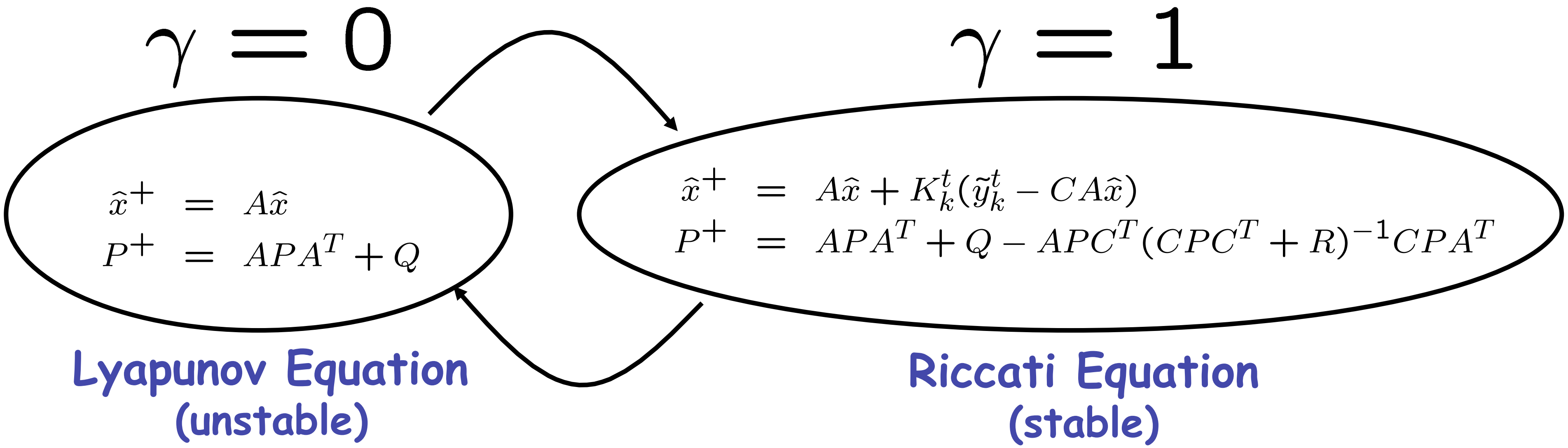

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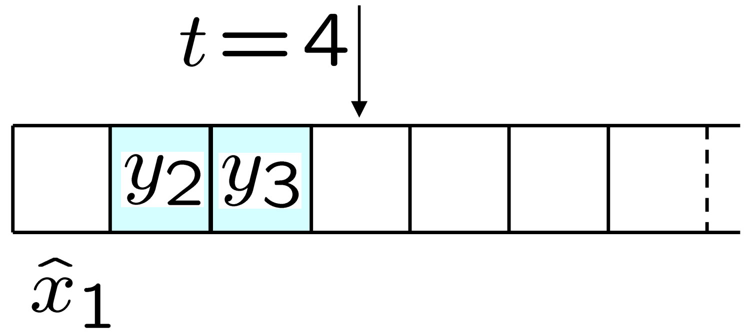

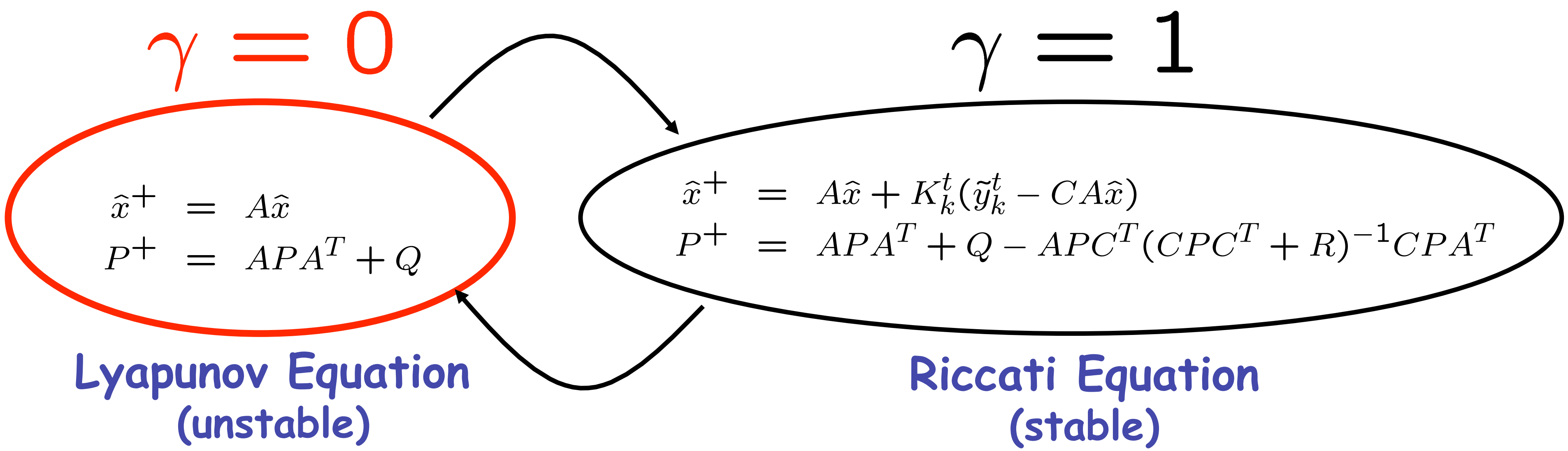

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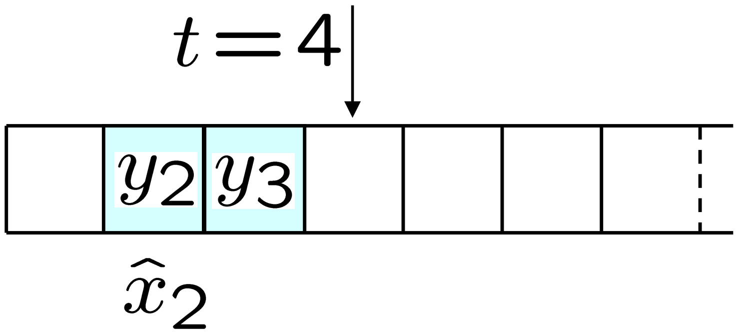

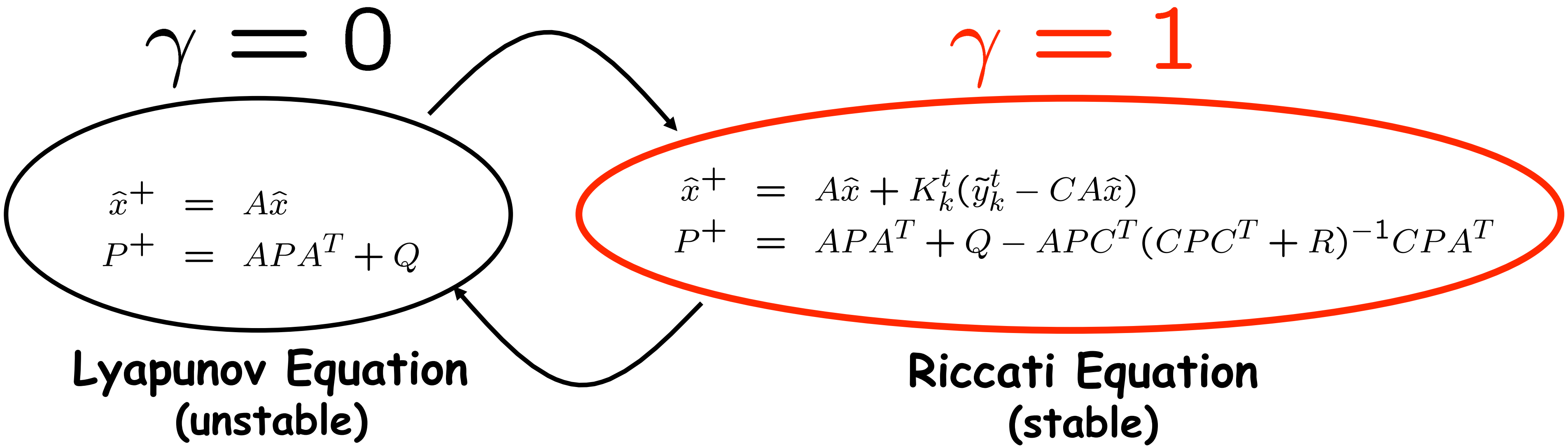

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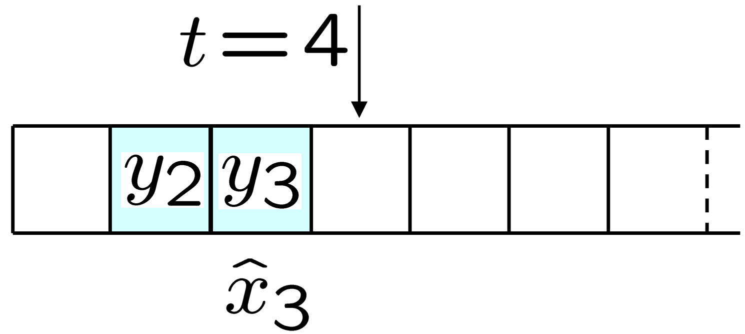

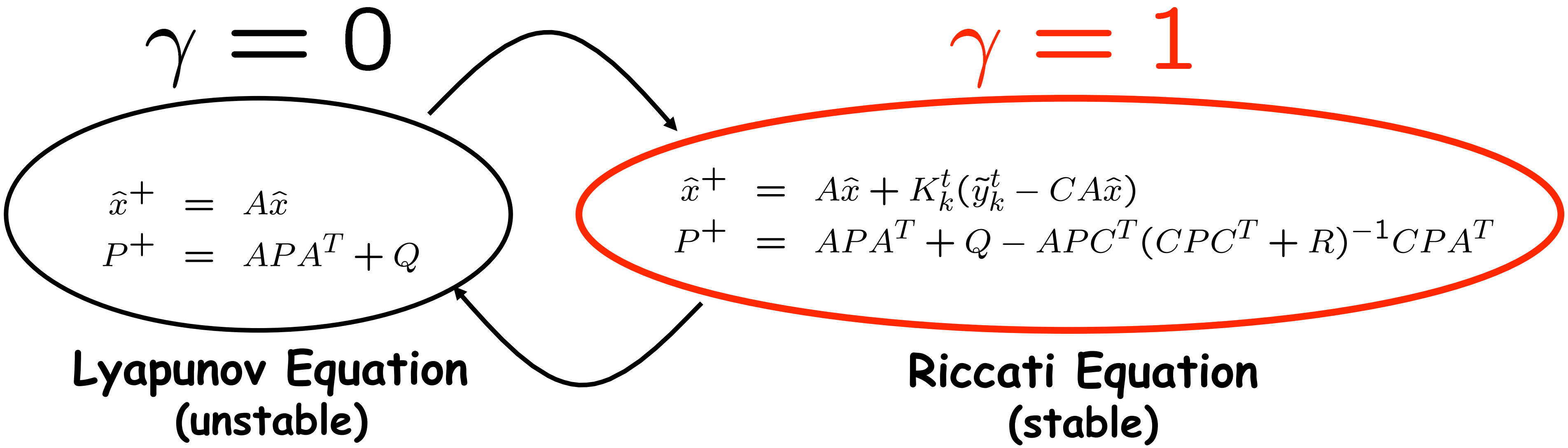

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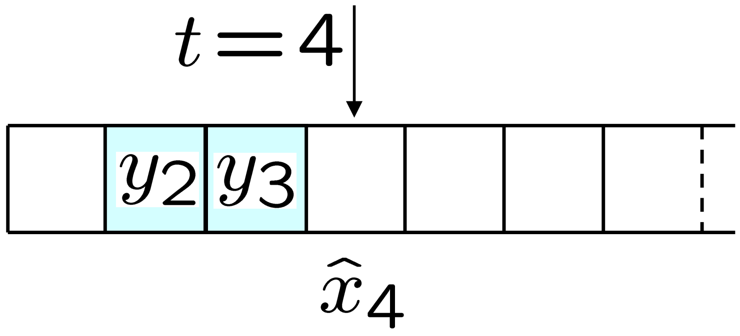

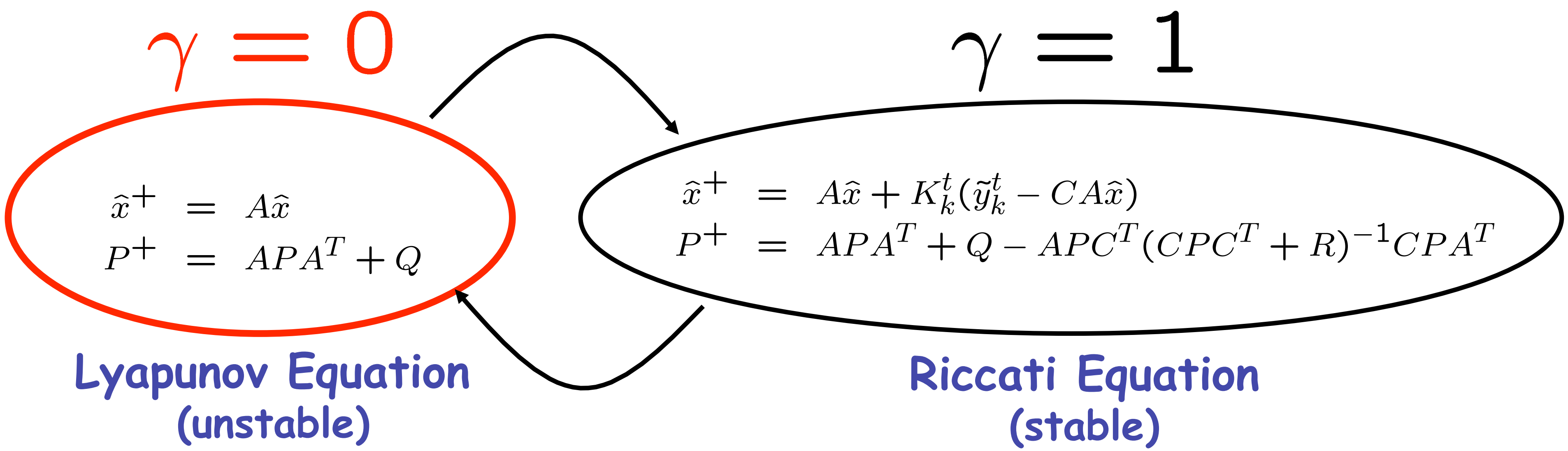

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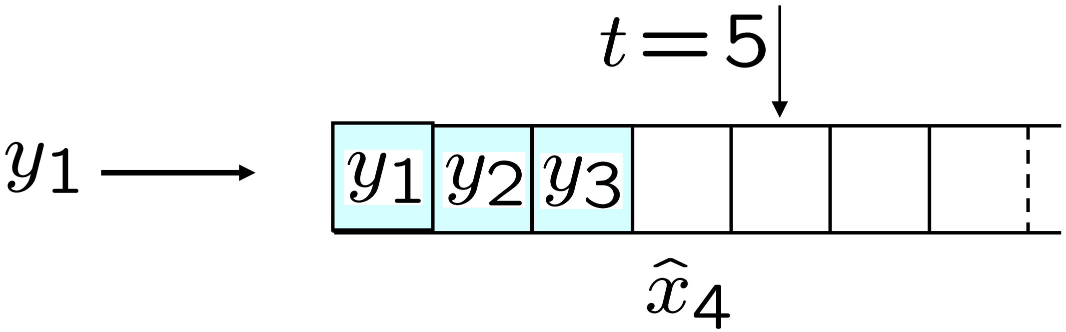
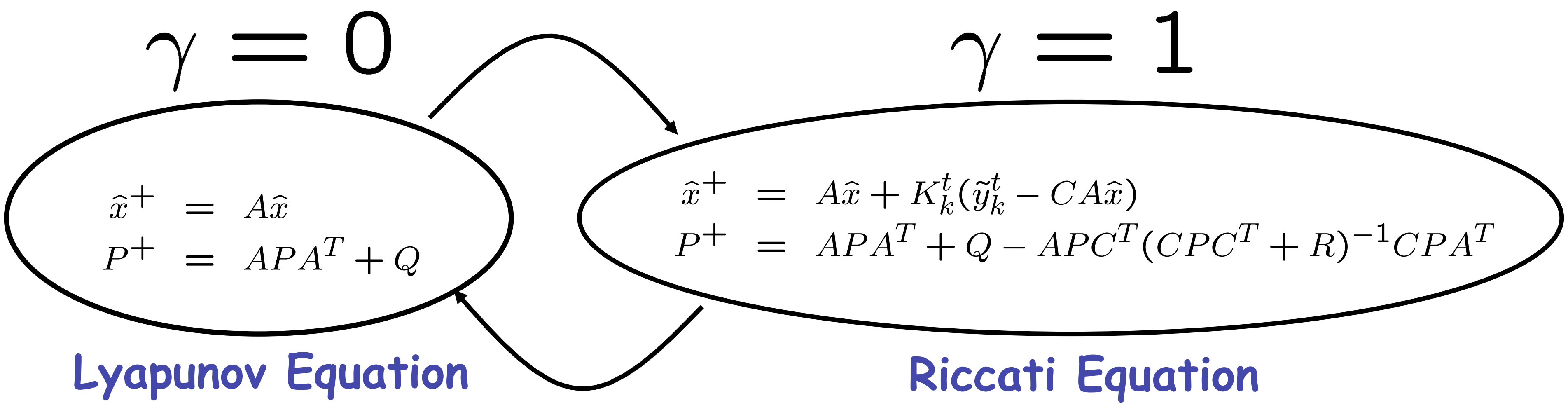
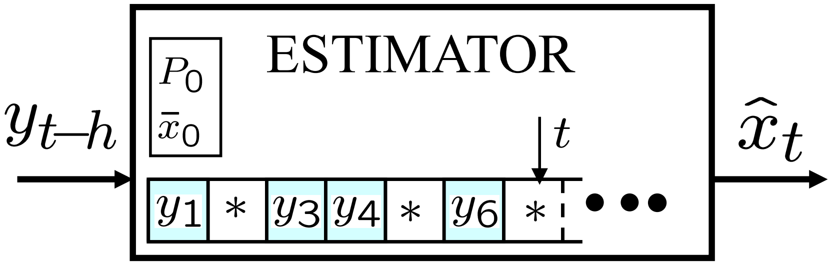




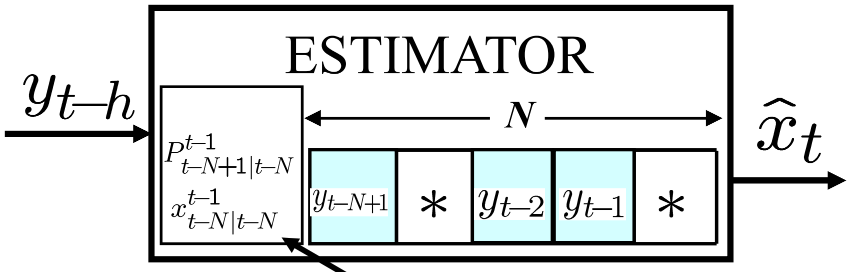
Lyapunov Equation (unstable)

Riccati Equation (stable)

Properties of Optimal Estimator



- Optimal for any arrival process
- Stochastic time-varying gain $K_t=K(\gamma_1,..,\gamma_t)$
- Stochastic error covariance $P_t=P(\gamma_1,..,\gamma_t)$
- Possibly infinite memory buffer
- Inversion of up to *t* matrices at any time *t*

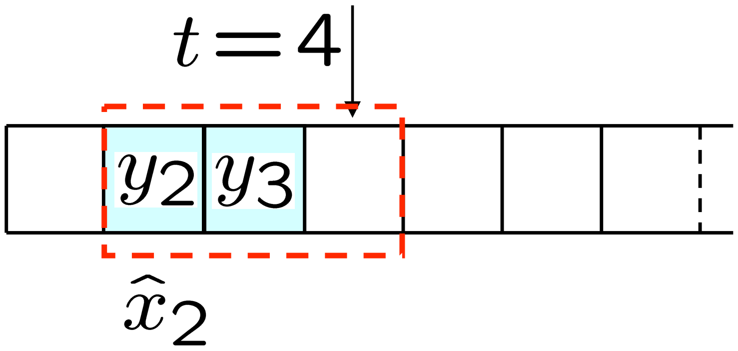


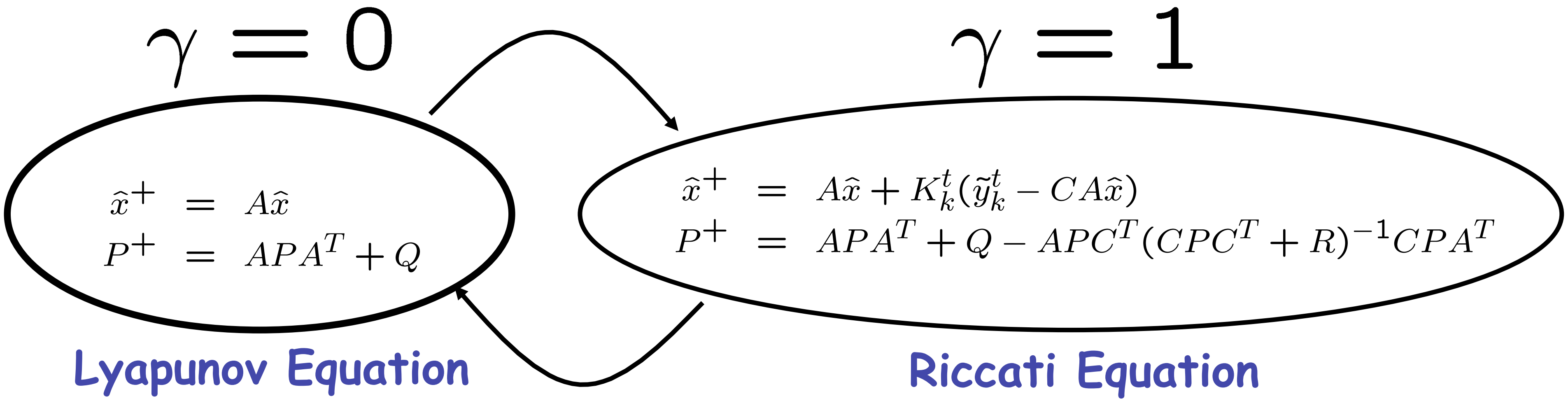
$$\gamma_k^t = \text{cost}, \quad t \geq k + \tau_{max}$$
$$\tau_{max} = N, \quad \text{delay}$$

$$\hat{x}^{t-1}_{t-N|t-N} \triangleq \hat{x}$$
$$P^{t-1}_{t-N+1|t-N} \triangleq P$$

$$\hat{x}^+ = A\hat{x} + \gamma^t_{t-N} PC^T(CPC^T+R)^{-1}(\tilde{y}^t_{t-N} - CA\hat{x}),$$
$$P^+ = APA^T + Q - \gamma^t_{t-N} APC^T(CPC^T+R)^{-1}CPA^T$$

Minimum variance estimation

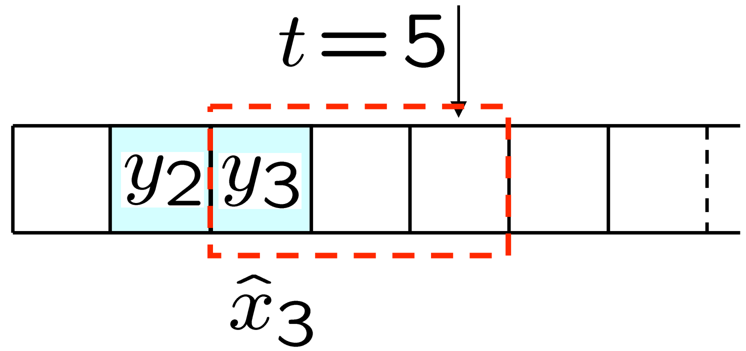


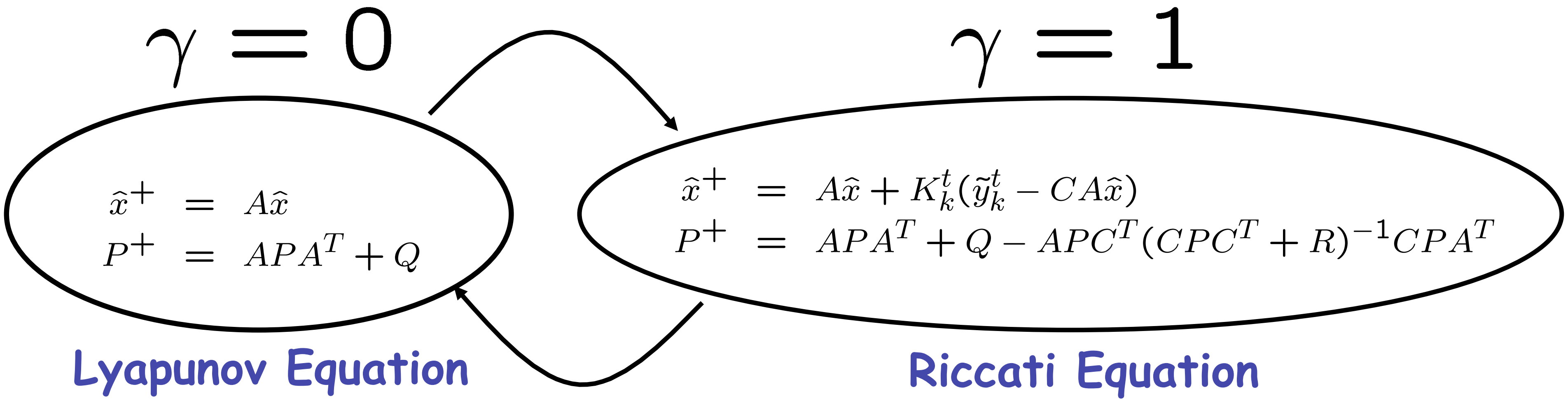


Lyapunov Equation
(unstable)

Riccati Equation
(stable)

Minimum variance estimation





$$\hat{x}^+ = A\hat{x}$$
$$P^+ = APA^T + Q$$

$$\hat{x}^+ = A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x})$$
$$P^+ = APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T$$

Lyapunov Equation (unstable)

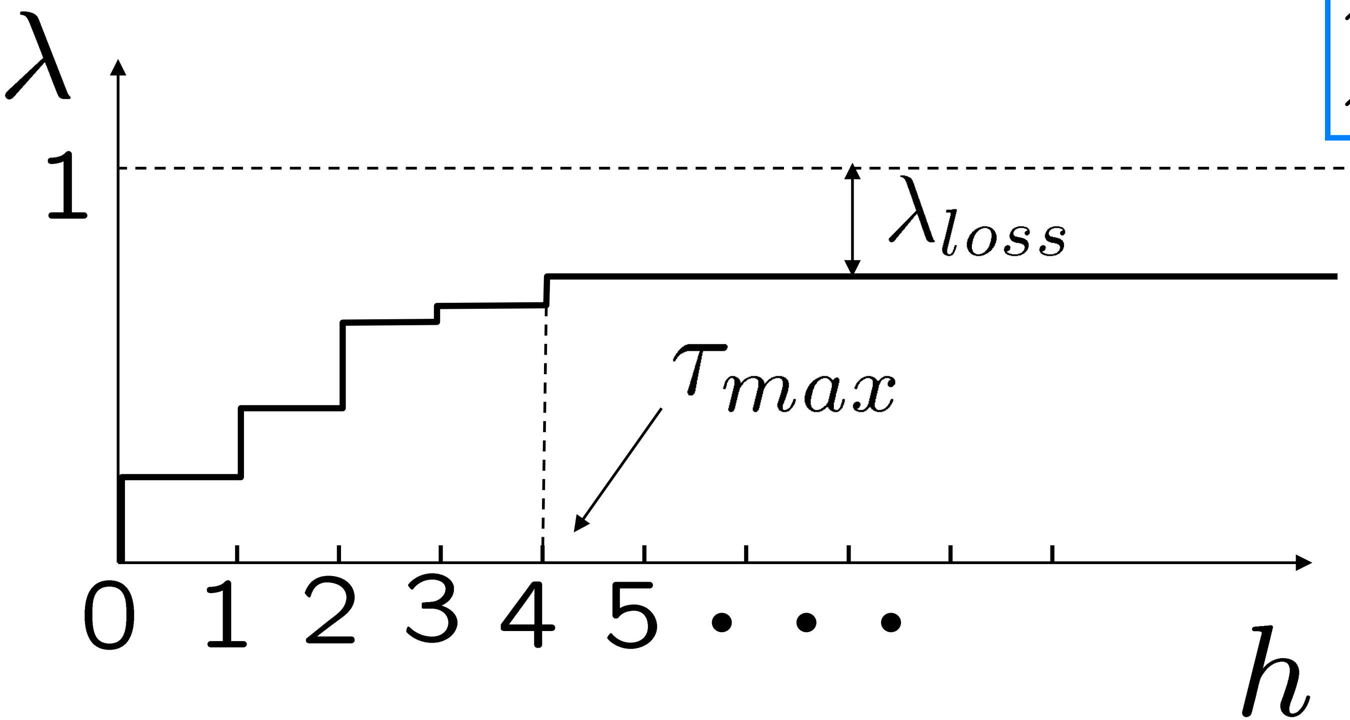
Riccati Equation (stable)

What about stability and performance?

Additional assumption on arrival sequence necessary:
i.i.d. arrival with stationary distribution

τ_k : delay of packet y_k, $\tau_k = \infty$ if y_k never arrives

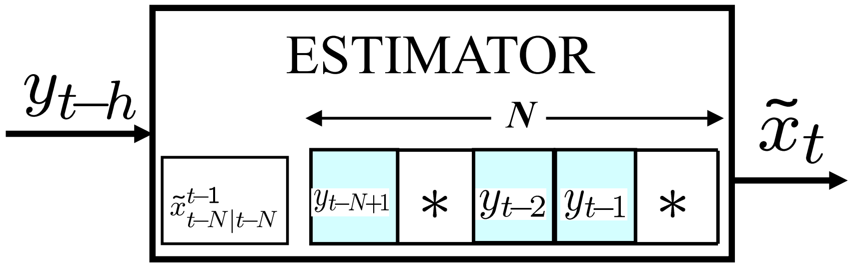
$$\lambda_h \triangleq \mathbb{P}[\tau_k \leq h], \quad \lambda_{loss} \triangleq \mathbb{P}[\tau_k = \infty]$$



Optimal estimation with constant gains and buffer finite memory

$\{K_h\}_{h=0}^{N-1}$, $\quad N$ static gains

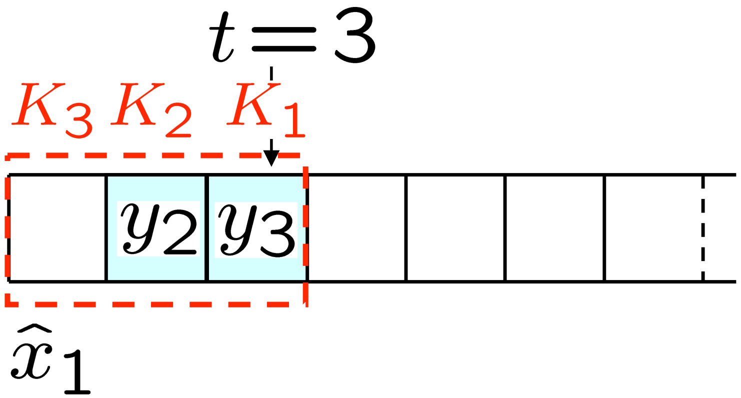
$$\tilde{x}^+ \;=\; A\tilde{x} + \gamma_{t-h}^{t} K_h(\tilde{y}_{t-h}^{t} - CA\tilde{x}), \quad h = N-1,\ldots,0$$

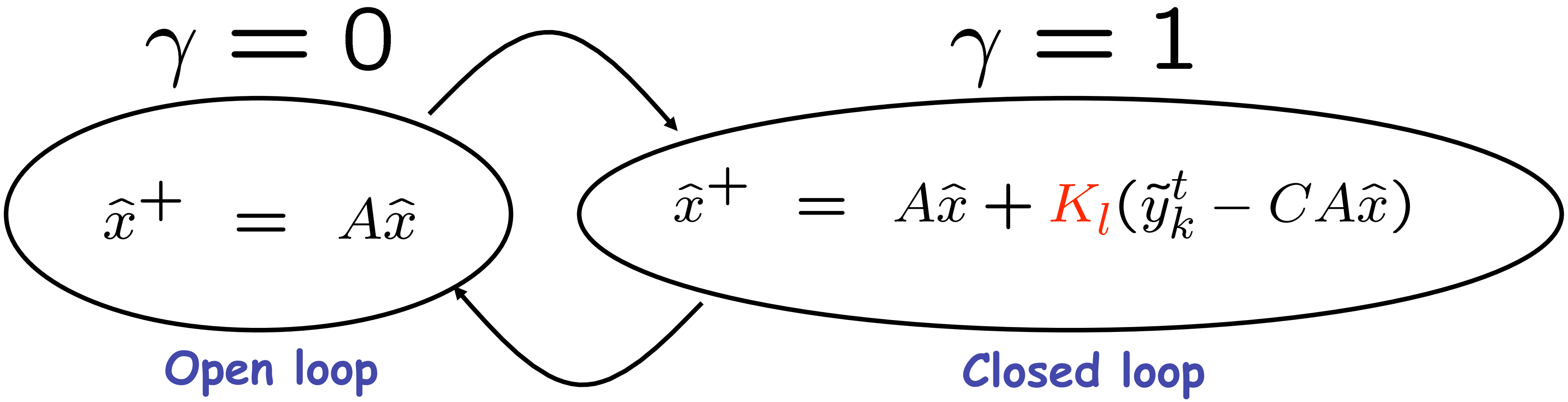


- Does not require any matrix inversion
- Simple to implement
- Upper bound for optimal estimator: $\quad P_t \leq \tilde{P}_{t|t} \Longrightarrow \mathbb{E}_\gamma[P_{t|t}] \leq \mathbb{E}_\gamma[\tilde{P}_{t|t}] = \overline{P}_{t|t}$
- N is design parameter

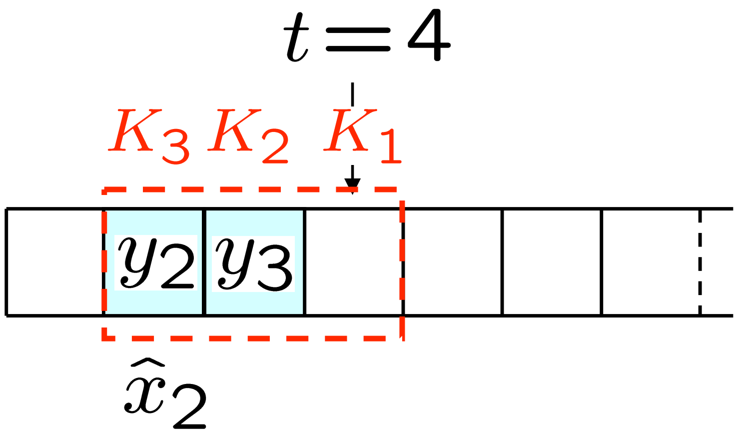
GOAL: compute $\quad \overline{P}_{t|t}$

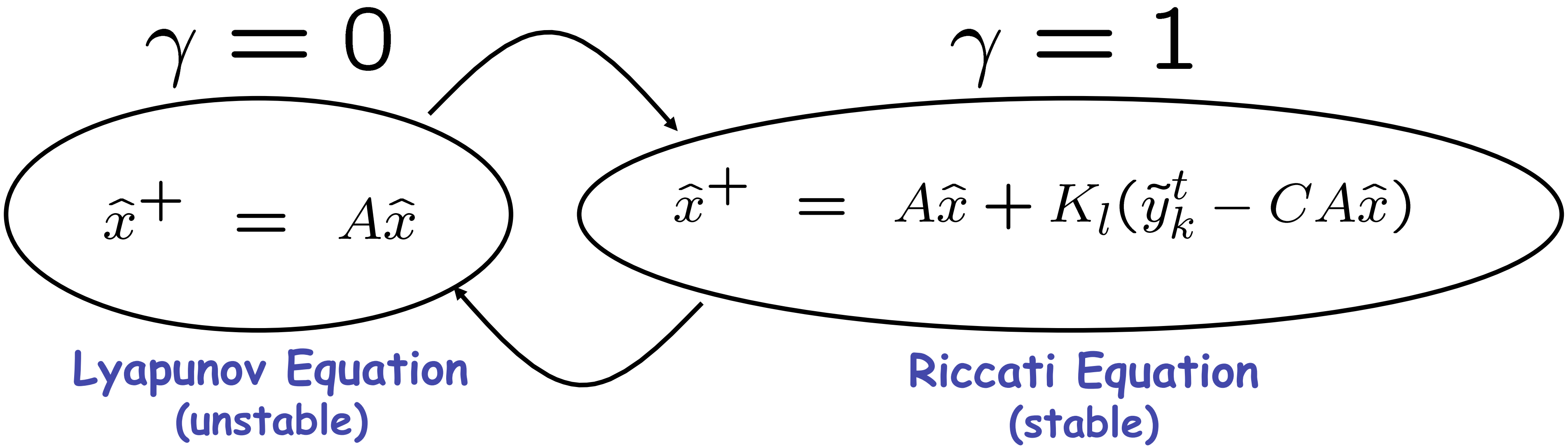
Suboptimal minimum variance estimation





Suboptimal minimum variance estimation





Steady state estimation error

Fixed gains:

$$\mathcal{L}_\lambda(K,P) = \lambda A(I-KC)P(I-KC)^T A^T + (1-\lambda)APA^T + Q + \lambda AKRK^T A^T$$

$$\begin{aligned} \overline{P} &= \mathcal{L}_{\lambda_{N-1}}(K_{N-1},\overline{P}) \\ \overline{P}^+ &= \mathcal{L}_{\lambda_k}(K_k,\overline{P}), \qquad k = N-2,\dots,0 \end{aligned}$$

$$\lim_{t\to\infty} \overline{P}_{t|t} = \overline{P}$$

Optimal fixed gains:

$$\Phi_\lambda(P) = APA^T + Q - \lambda\, APC^T(CPC^T+R)^{-1}CPA^T$$

Modified Algebraic Riccati Equation (MARE) (Φ_1(P)=ARE)

$$\min_{K_0,\dots,K_{N-1}} \overline{P} \quad \Longrightarrow \quad \begin{aligned} \overline{P}_{N-1} &= \Phi_{\lambda_{N-1}}(\overline{P}_{N-1}) \\ \overline{P}_k &= \Phi_{\lambda_k}(\overline{P}_{k+1}), \qquad k = N-2,\dots,0 \\ K_k &= \overline{P}_k C^T (C\overline{P}_k C^T + R)^{-1} \end{aligned}$$

(off-line computation)

Stability issues

Static estimator is stable iff there exists $P \geq 0$ such that:

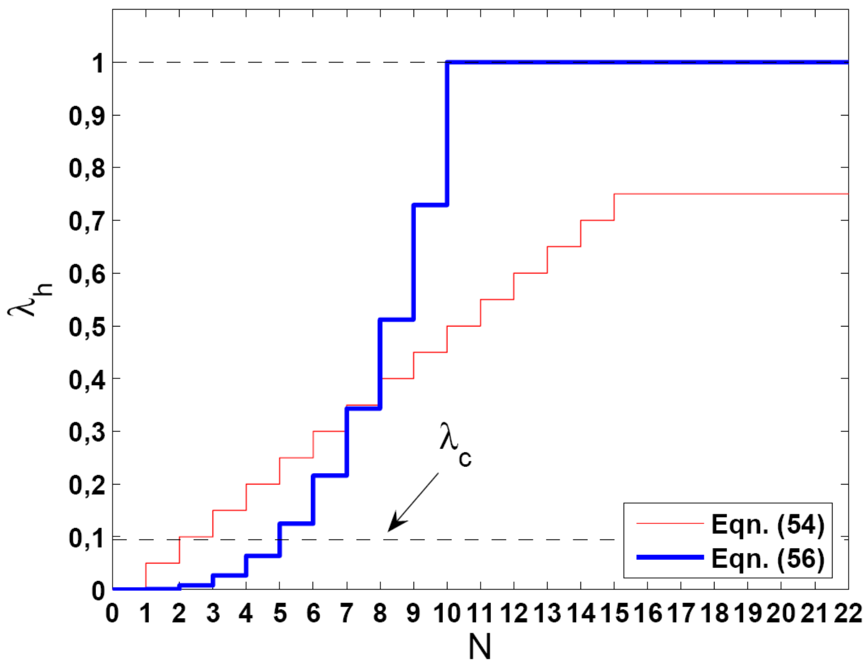
$$P = APA^T + Q - (1-\lambda)\,APC^T(CPC^T + R)^{-1}CPA^T$$

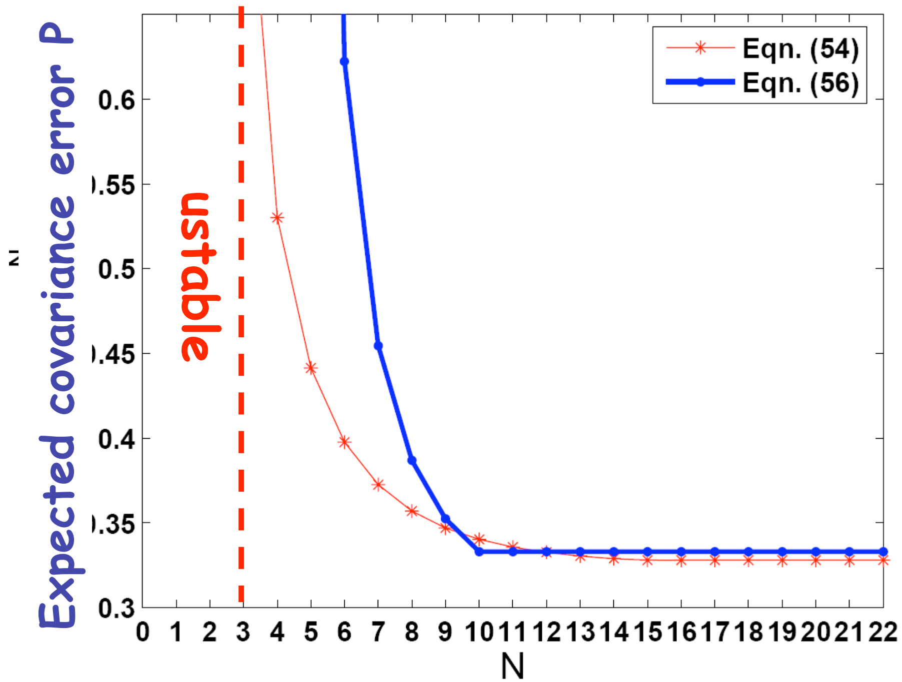
- If $\lambda = 0$ then standard ARE
- Modified Riccati Algebraic Equation known since [Nahi TIF'69]
- If A is unstable then there exist critical probability: if $\lambda < \lambda_c$ stable, if $\lambda > \lambda_c$ unstable
- Upper bound $\lambda_c \leq \frac{1}{\max|\text{eig}(A)|^2}$. Equality if C invertible [Katayama TAC"76]
- Lower bounf $\lambda_c \geq \frac{1}{\prod_{unstable} |\text{eig}(A)|^2}$. Equality if rank$(C) = 1$ [Elia TAC'01, SCL'05]
- Closed form expression for λ_c not known for general (A, C)

Numerical example (I)

Discrete time linearized inverted pendulum:

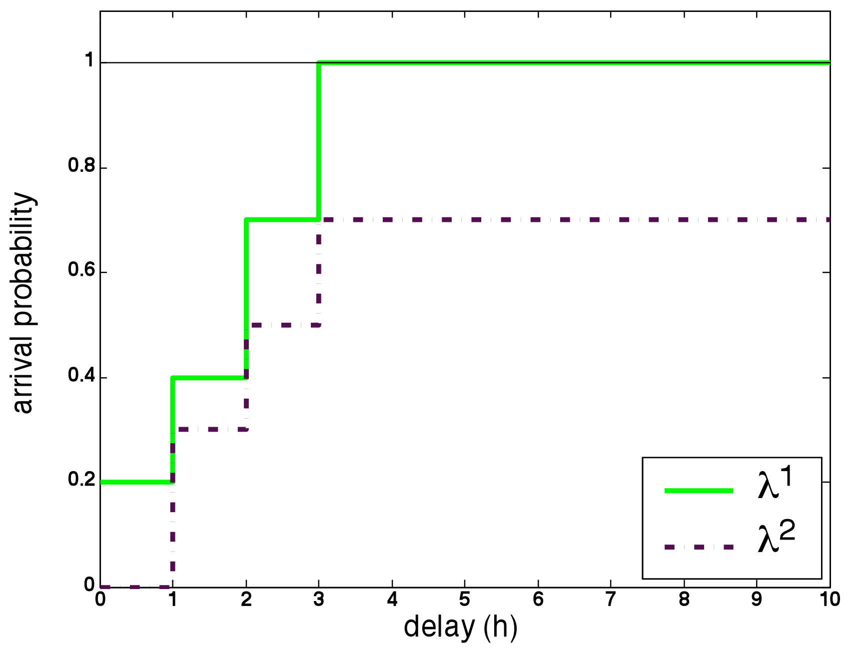
$$A = \begin{bmatrix} 1.01 & 0.05 \\ 0.05 & 1.01 \end{bmatrix}, \quad C = [\, 1 \quad 0 \,], \quad R = 1, \quad Q = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 1 \end{bmatrix}$$





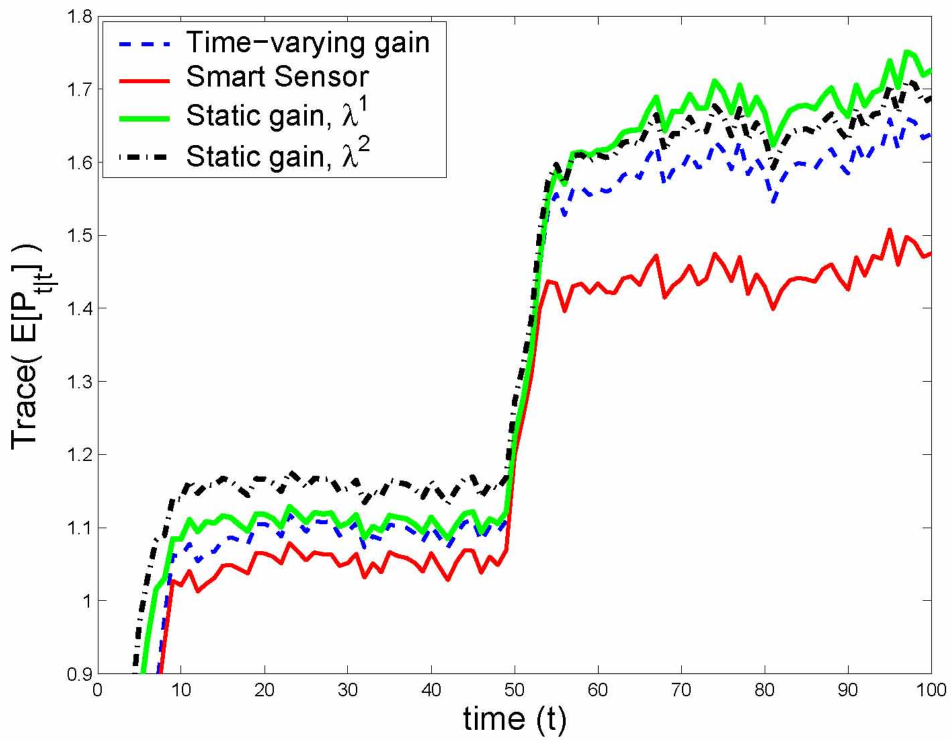
Numerical example (II)

Time-varying arrival probability distribution

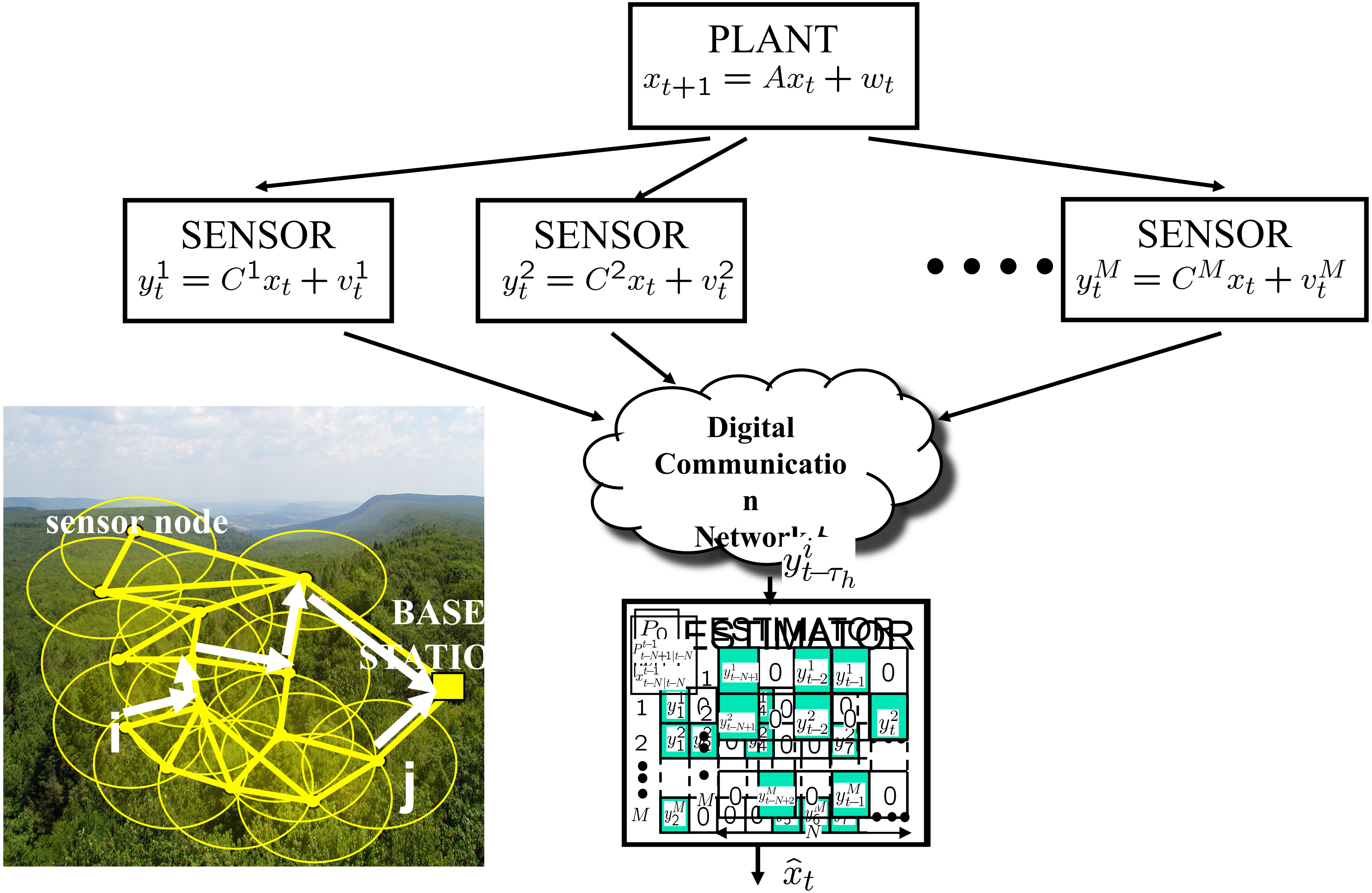


$$\lambda^1 \qquad 0 \leq t \leq 50$$

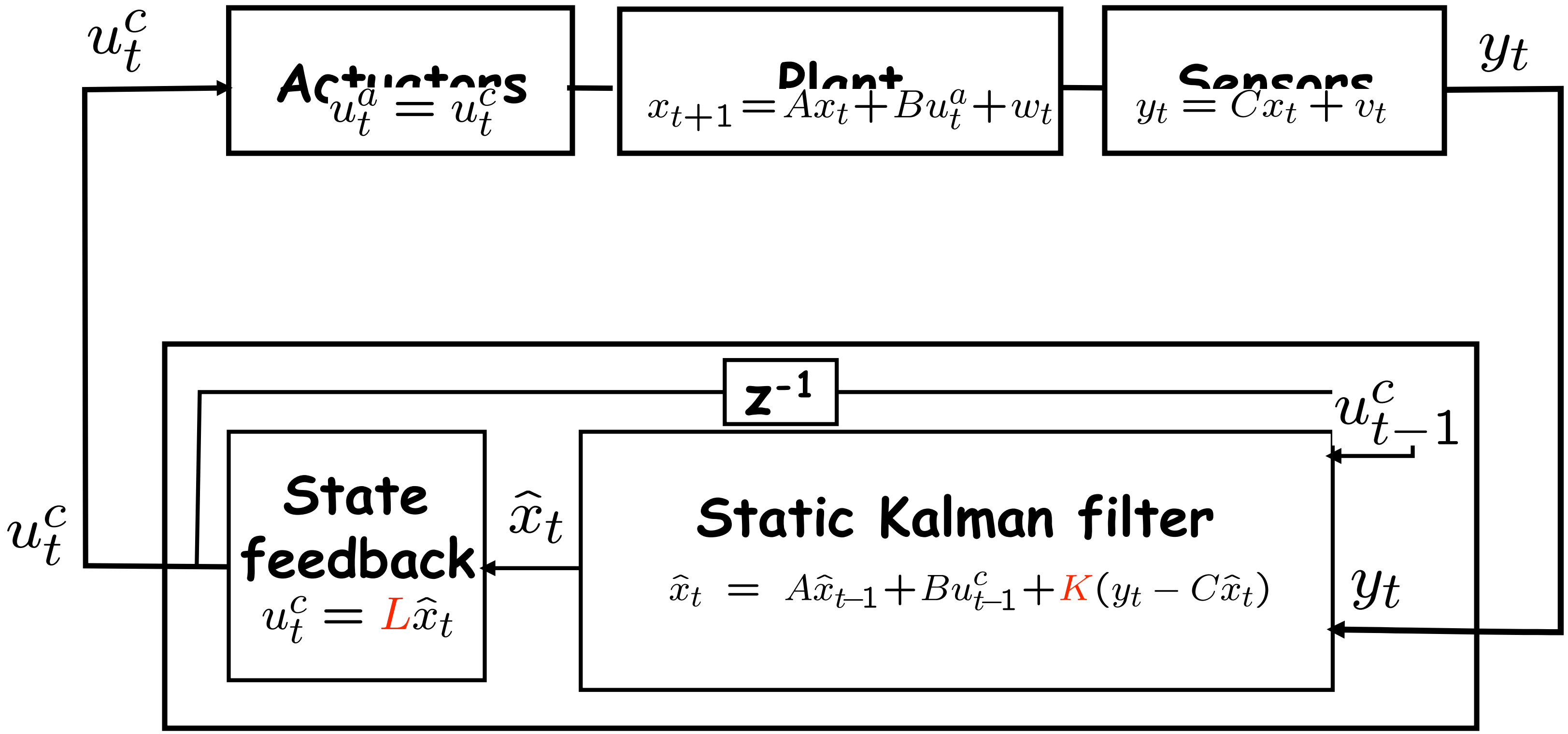
$$\lambda^2 \qquad t > 50$$



Multiple sensors



Back to the control problem



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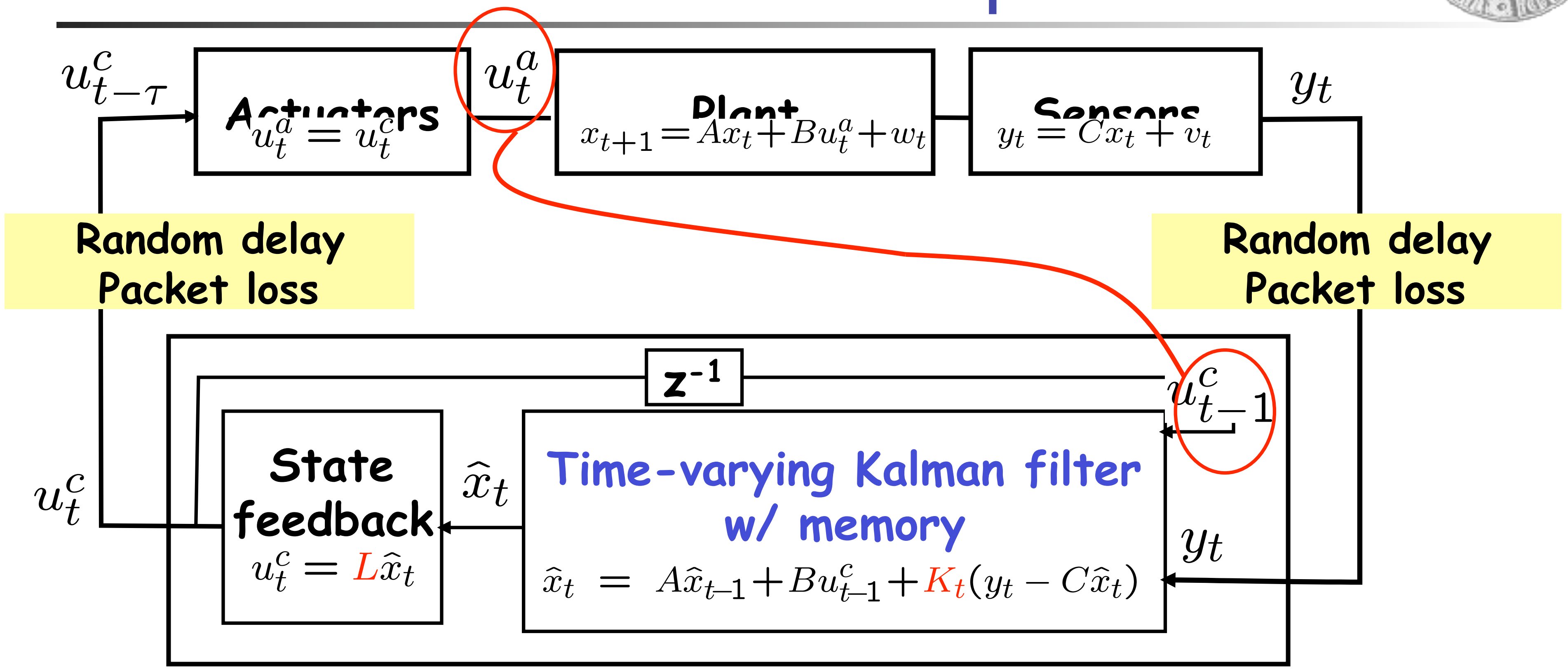
Back to the control problem



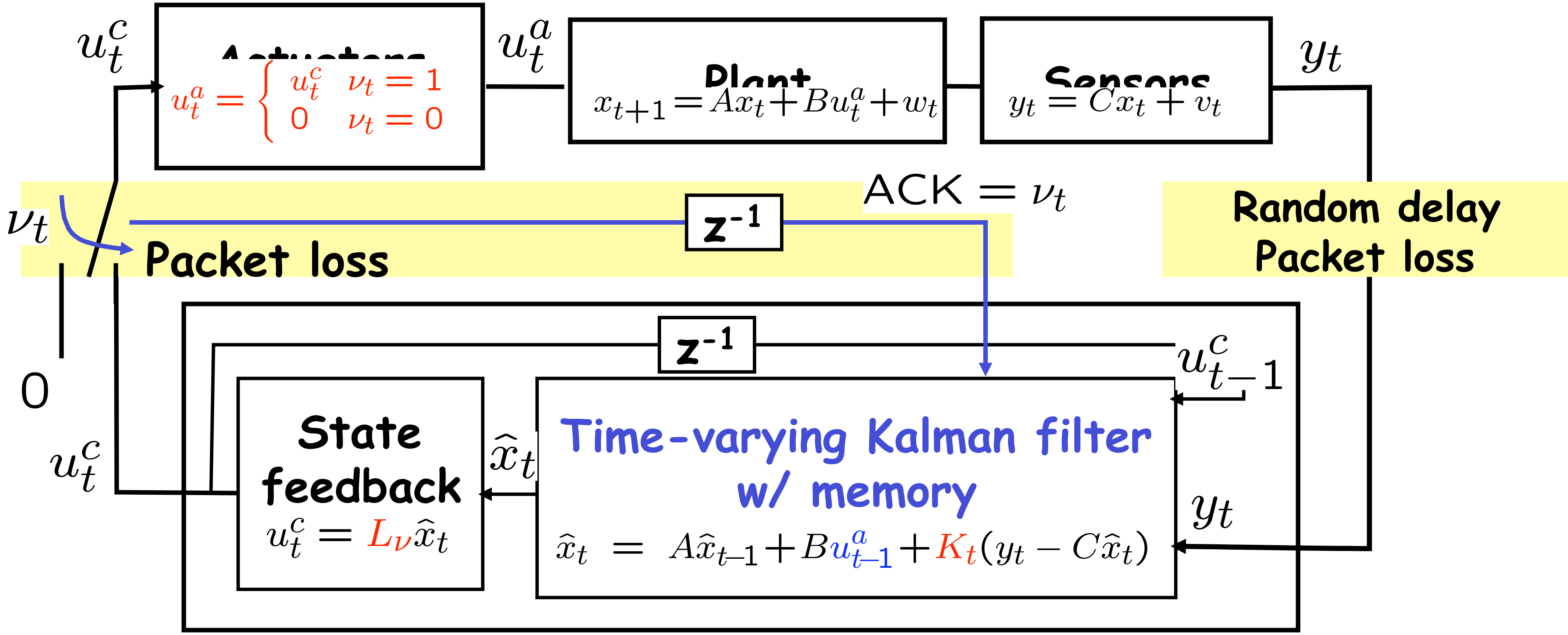
$$\hat{x}_t = E[x_t | y_t, y_{t-1}, .., y_0, u^a_{t-1}, .., u^a_1]$$

if $u^c_{t-1} \neq u^a_{t-1} \Longrightarrow e_t = x_t - \hat{x}_t = f(y_t, .., y_0, u^c_t, .., u^c_0, u^a_t, .., u^a_0)$

$$P_{t|t-1} = AP_{t-1|t-1}A^T + Q + B(u^a_{t-1} - u^c_{t-1})(u^a_{t-1} - u^c_{t-1})^T B^T$$

Estimation error coupled with control action ➔ no separation principle

LQG over TCP-like (ACK-based) protocols



- Separation principle hold (I know exactly u^a_{t-1})
- ν_t Bernoulli rand. var and independent of observation arrival process
- Static state feedback, L_ν solution of dual MARE

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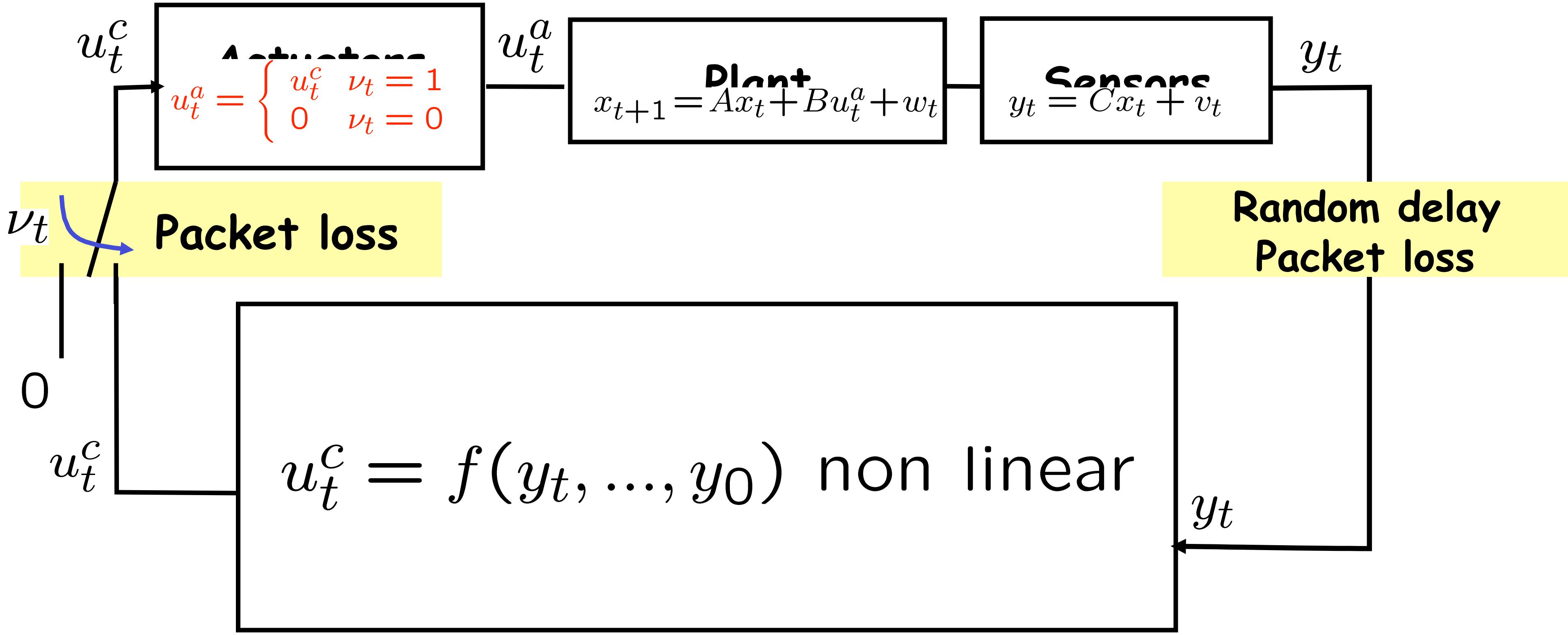
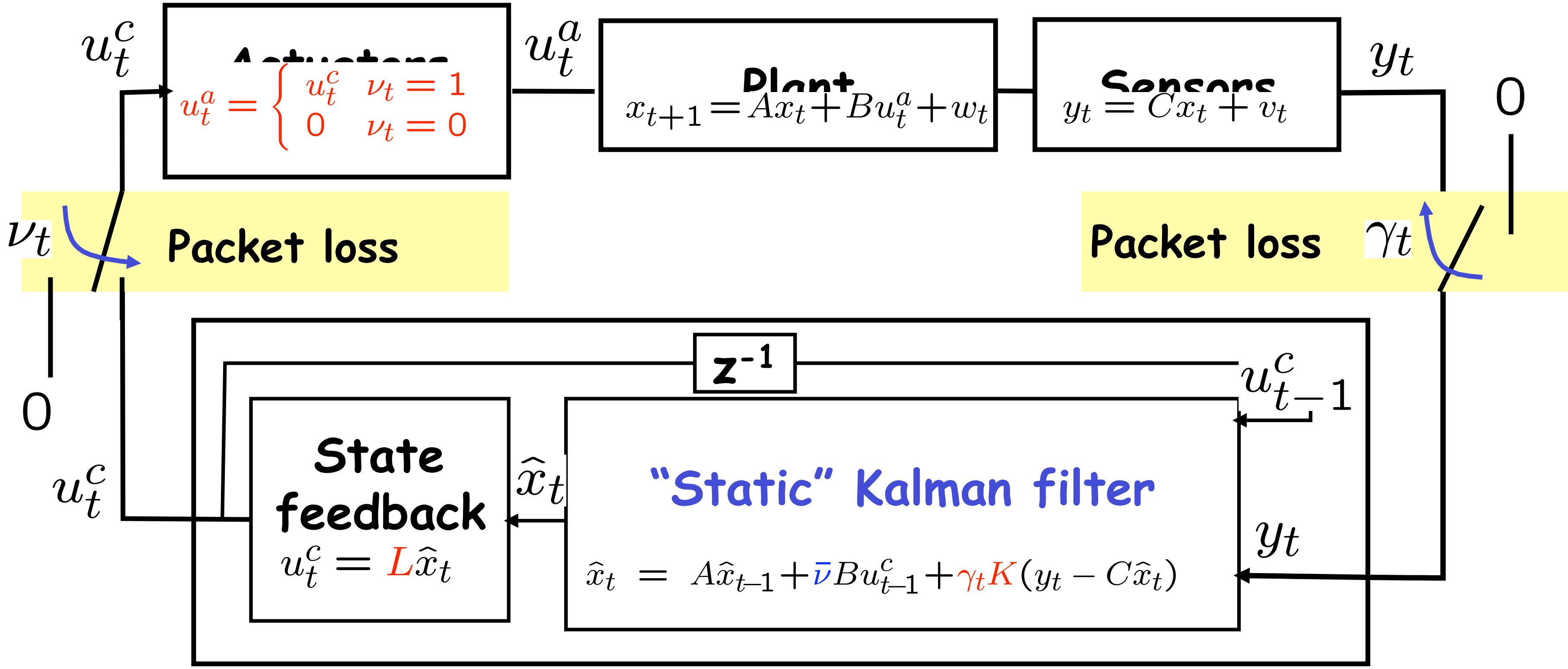
LQG over UDP-like (no-ACK) protocols



- LQG problem still well defined: $\min_{u_t^c,\ldots,u_1^c} E[\sum_{h=1}^{t} x_t^T W x_t + (u_t^a)^T U u_t^a]$
- No separation principle hold (u^a_{t-1} NOT known exactly)

- … but still have some statistical information about u^a_{t-1}

LQG over UDP-like (no-ACK) protocols



- Bernoulli arrival process $P[\nu_t = 1] = \bar{\nu}, P[\gamma_t = 1] = \bar{\gamma}$
- $\bar{\nu}u_{t-1}^c = E[u_{t-1}^a]$
- Sub-optimal controller forced to be state estimator+state feedback
- Optimal choice of K,L is unique solution of 4 coupled Riccati-like equations

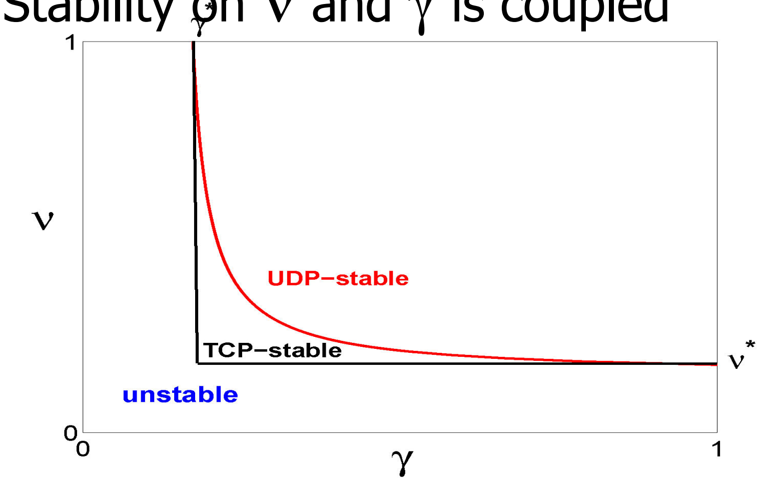
"Compensability and Optimal Compensation of systems with white parameters", De Koning, TAC'92

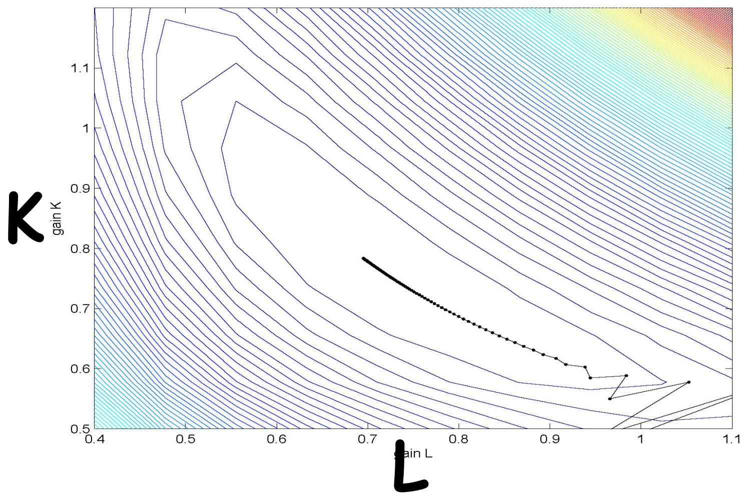
LQG as optimization problem

$$P \triangleq \mathbb{E}\left[\begin{bmatrix} x \\ \hat{x} \end{bmatrix}\begin{bmatrix} x^T & \hat{x}^T \end{bmatrix}\right] = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}{}^T & P_{22} \end{bmatrix}$$

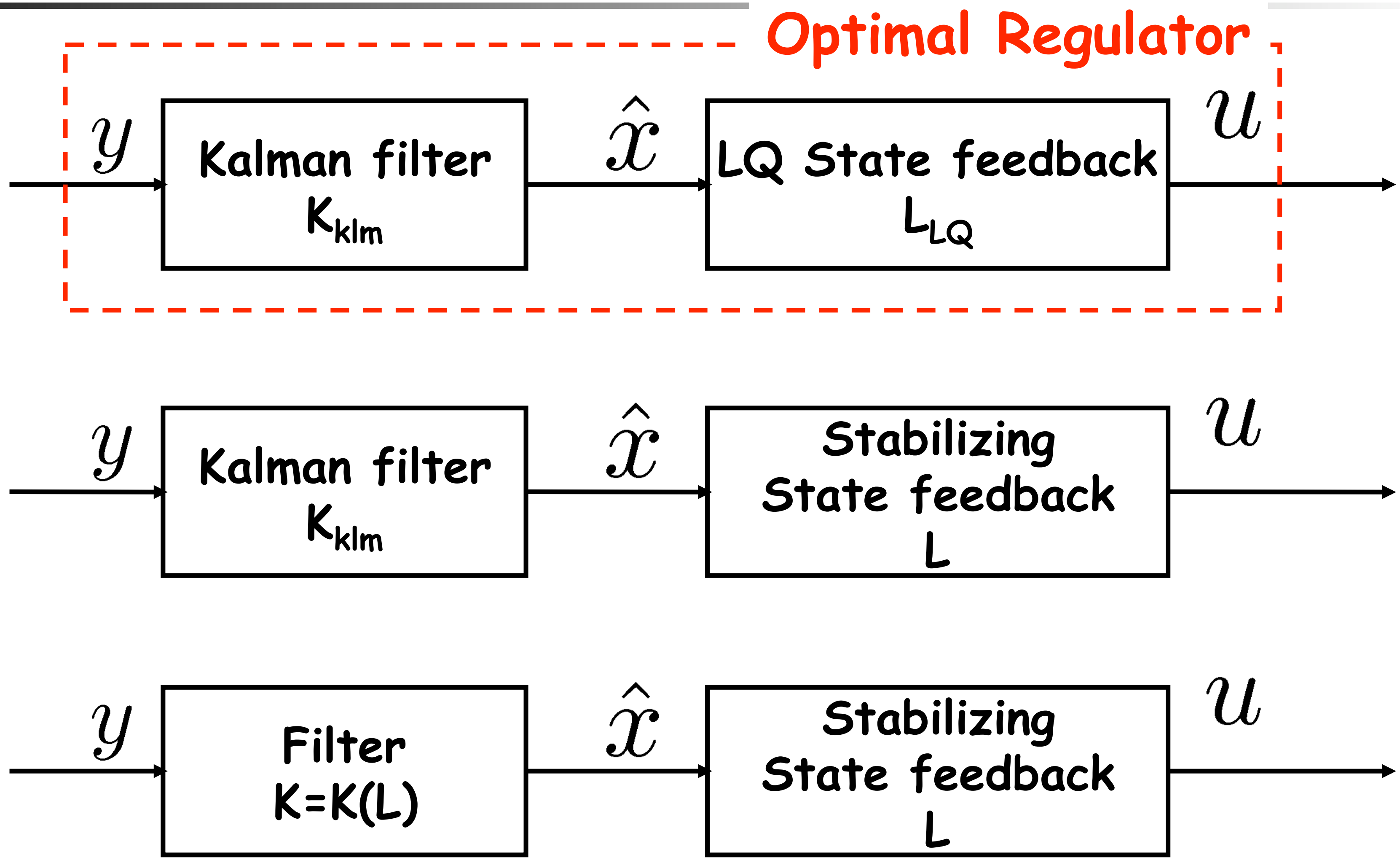
$$\begin{aligned} \mathrm{Min}_{K,L} \quad & \mathrm{Trace}\left(\begin{bmatrix} W & 0 \\ 0 & \bar{\nu}L^T U L \end{bmatrix} P\right) \\ s.t. \quad & P = \mathbb{E}\left[\begin{bmatrix} A & -\nu_k BL \\ \gamma_k KC & A - \bar{\nu}BL - \gamma_k KC \end{bmatrix} P \begin{bmatrix} A & -\nu_k BL \\ \gamma_k KC & A - \bar{\nu}BL - \gamma_k KC \end{bmatrix}^T\right] + \begin{bmatrix} Q & 0 \\ 0 & \bar{\gamma}KRK^T \end{bmatrix} \\ & P \geq 0 \end{aligned}$$

- Non convex problem even for $\nu=\gamma=1$, i.e. classic LQG
- Classic and TCP-based LQG become convex when exploiting optimality conditions like uncorralation between estimate and error estimate $\mathbb{E}[x(x-\hat{x})^T] = 0$
- For UDP-like problem non convex but unique solution using Homotopy and Degree Theory (DeKoning,Athans,Bernstain) (maybe using Sum-of-Squares?)
- Stability on ν and γ is coupled





Side note: Kalman filter is not always optimal !



- Kalman filter always gives smallest estimate error **regardless** of controller L
- If controller $L \neq L_{LQ}$, then performance improves if my estimate is "bad" !

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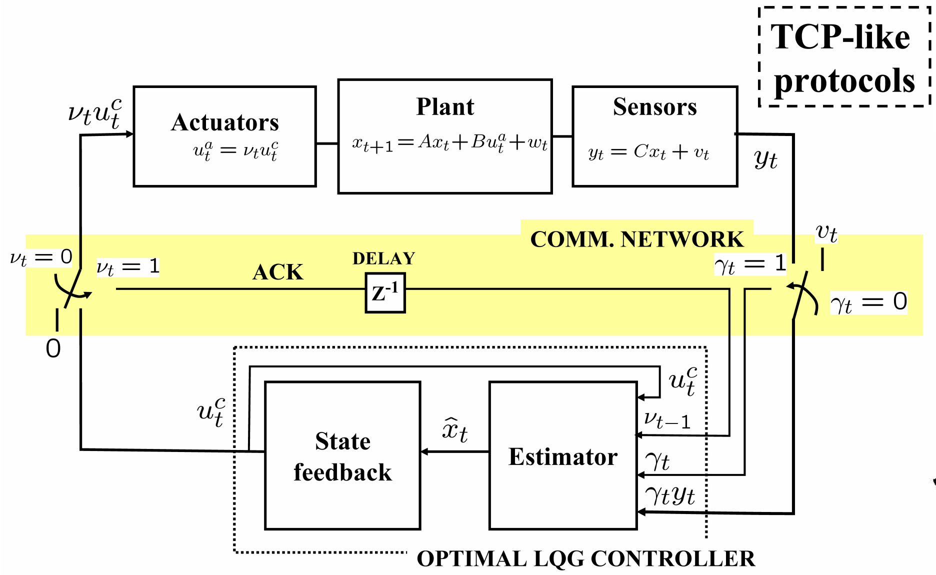
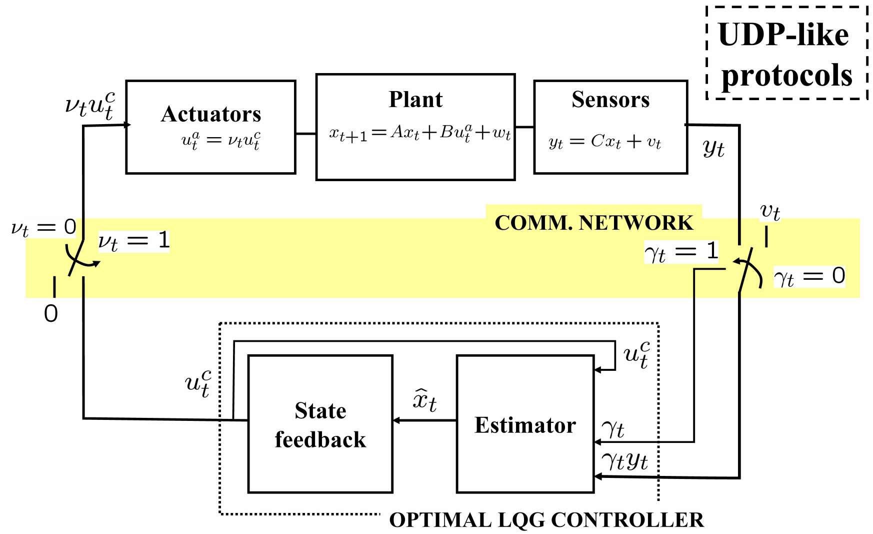
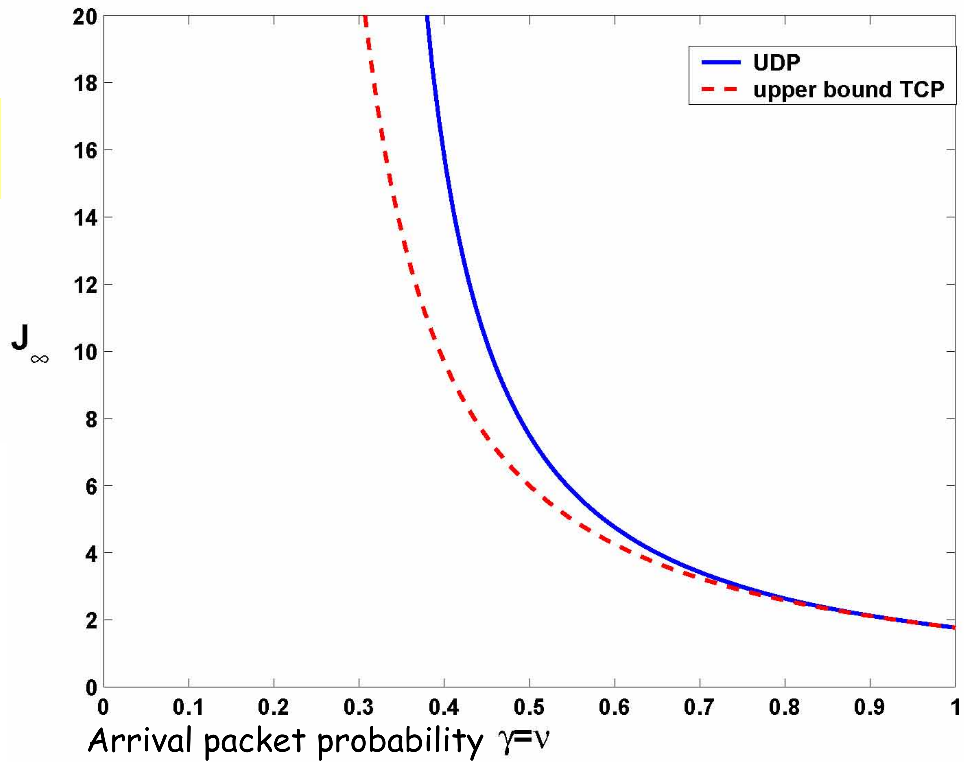
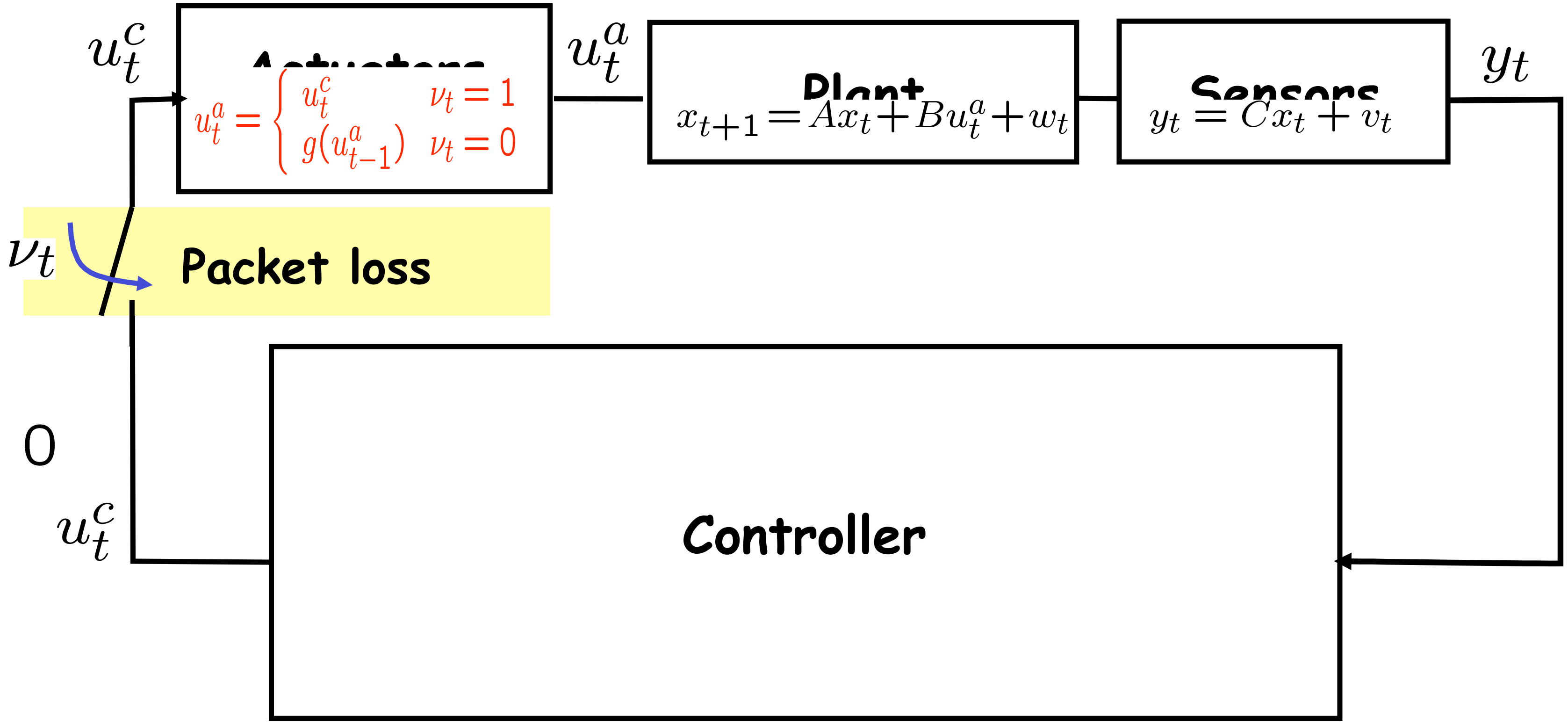
Numerical example: TCP vs UDP







To hold or to zero control input?



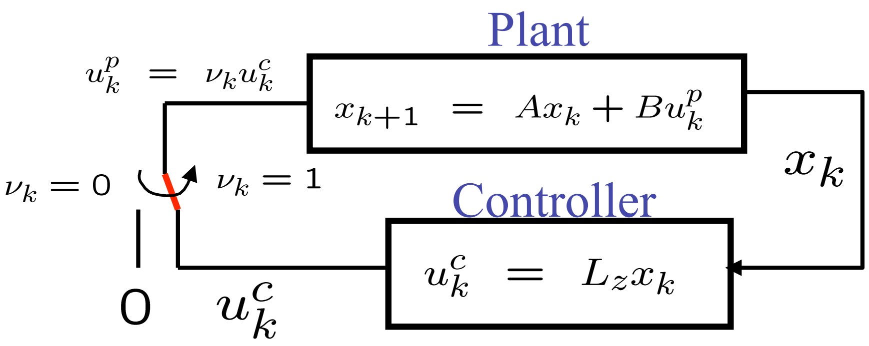
Most common strategy:

$g(u_{t-1}^a) = 0$ zero-input strategy **(mathematically appealing)**

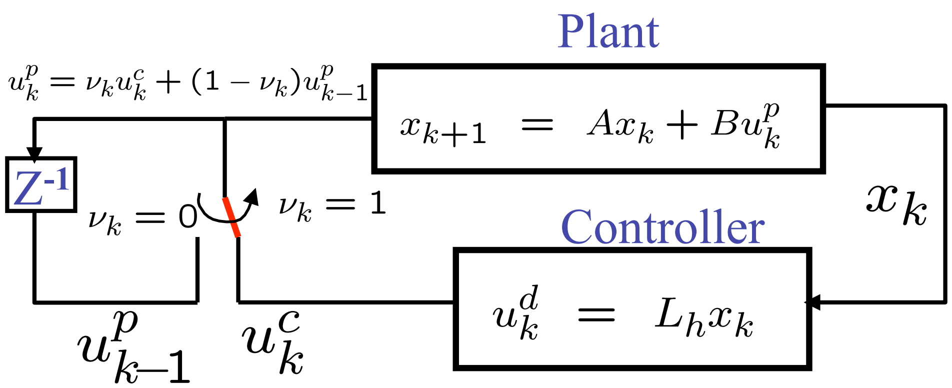
$g(u_{t-1}^a) = u_{t-1}^a$ hold-input strategy **(most natural)**

To hold or to zero control input: no noise (jump linear systems)

Zero-input Strategy



Hold-input Strategy



$$J_z^* = \min_{L_z} E[\sum_{t=1}^{\infty} x_t^T W x_t + (u_t^a)^T U u_t^a]$$

$$J_h^* = \min_{L_h} E[\sum_{t=1}^{\infty} x_t^T W x_t + (u_t^a)^T U u_t^a]$$

Using cost-to-go function (dynamic programming)

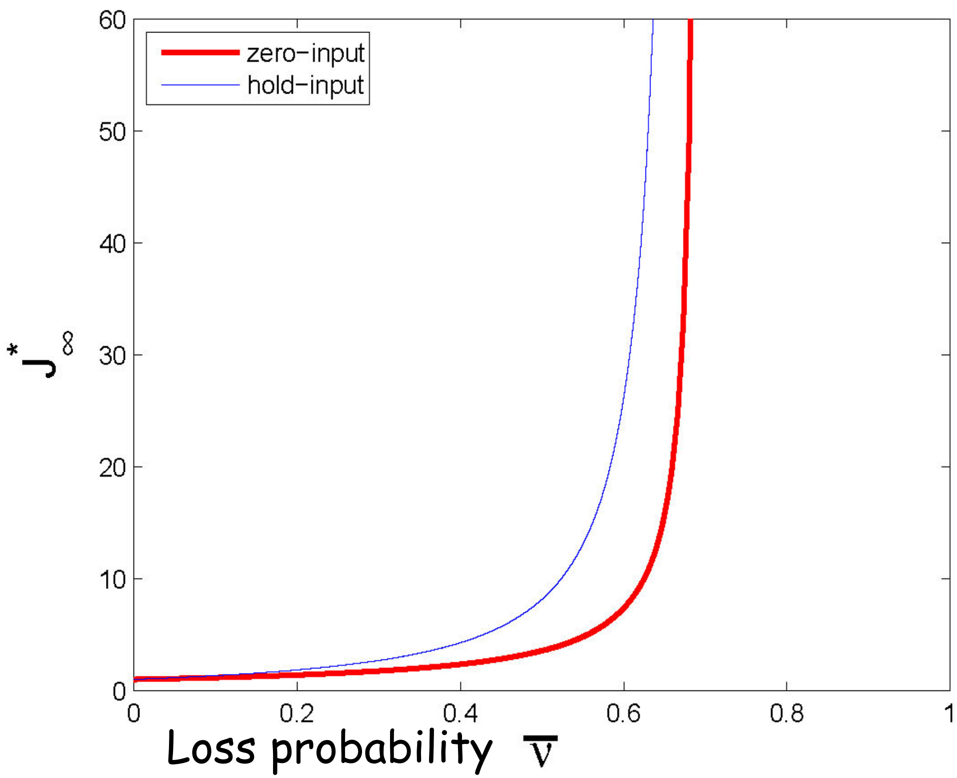
$$J_z^* = E[x_0^T S_z x_0] \qquad J_h^* = E[x_0^T S_h x_0]$$

$S_z = \Phi_z(S_z)$ ← **Riccati-like equation** → $S_h = \Phi_h(S_h)$

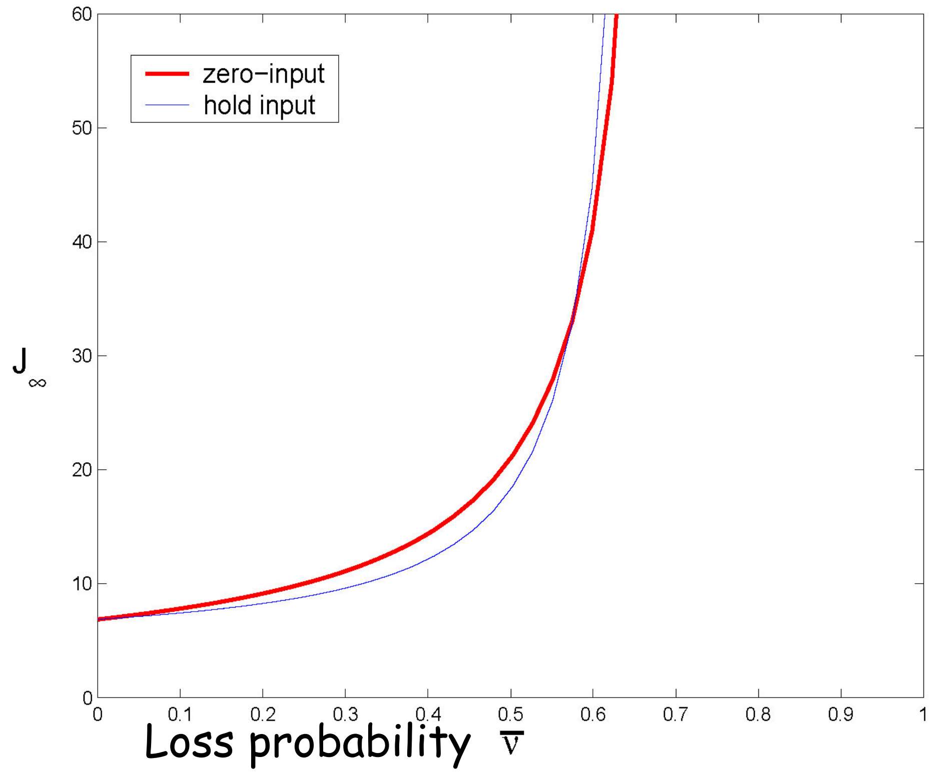
$$L_z^* = f_z(S_z) \qquad L_h^* = f_h(S_h)$$

Example: unstable scalar system

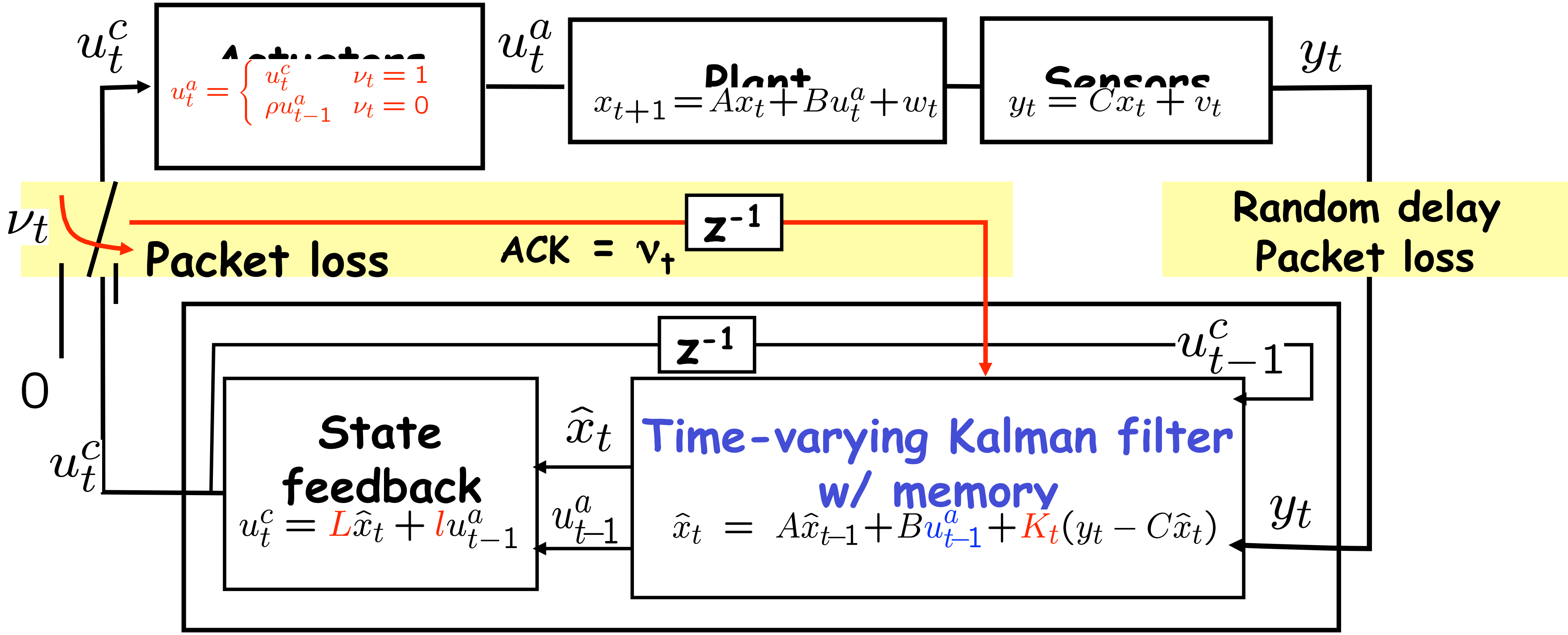
A=1.2, U=0 (fastest convergence)



A=1.2, U=10 (small input)



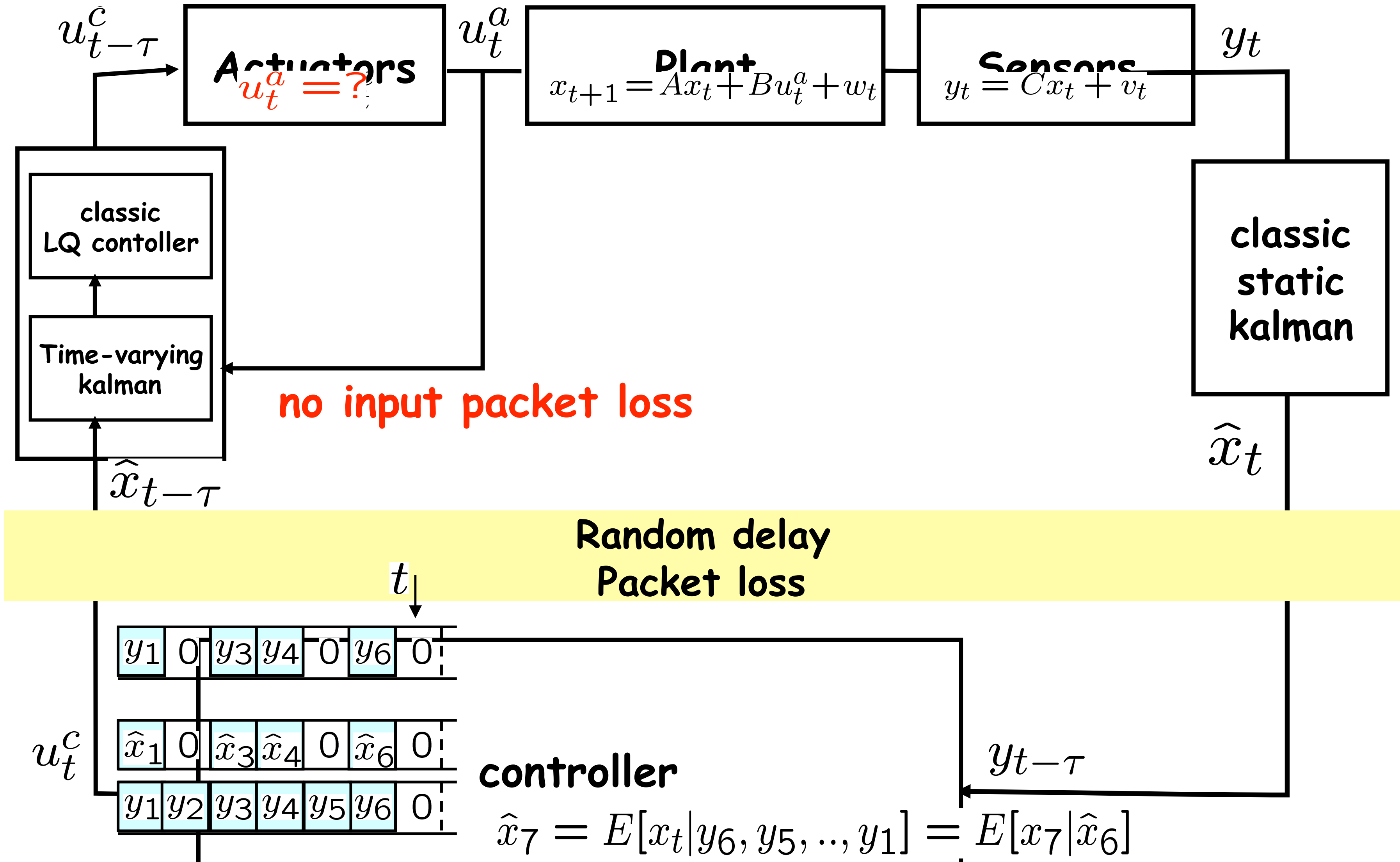
LQG over TCP-like protocols revised



Conjecture:

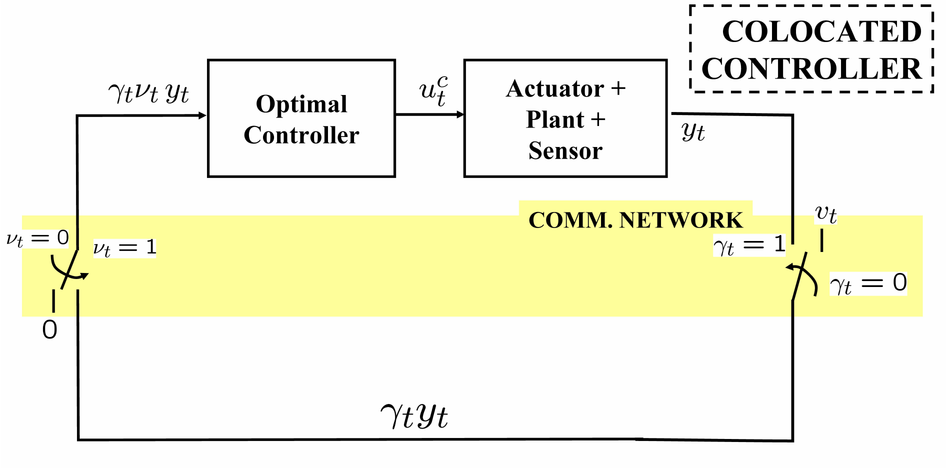
- Separation principle hold
- Optimal function $g(u_{t-1}^a) = \rho u_{t-1}$
- Design parameter L, l, ρ obtained via LQ-like optimal state feedback

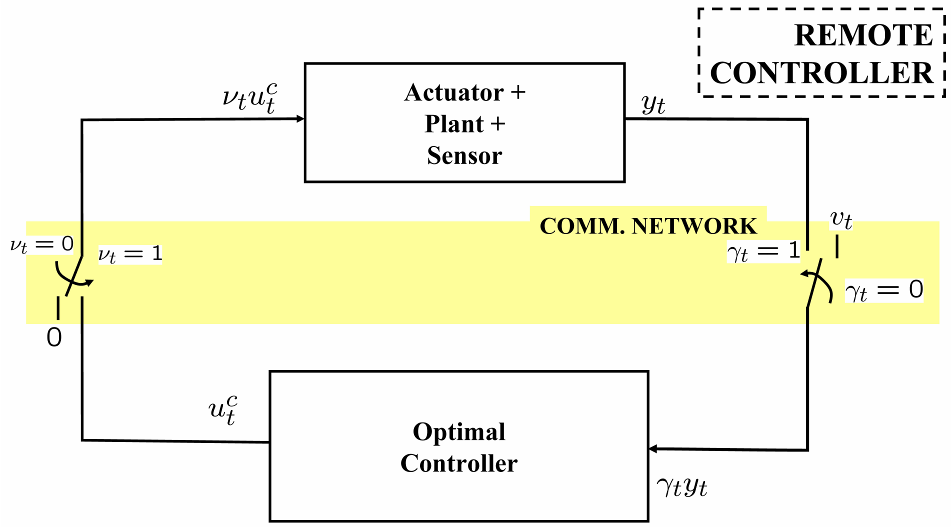
Smart sensors & smart actuators

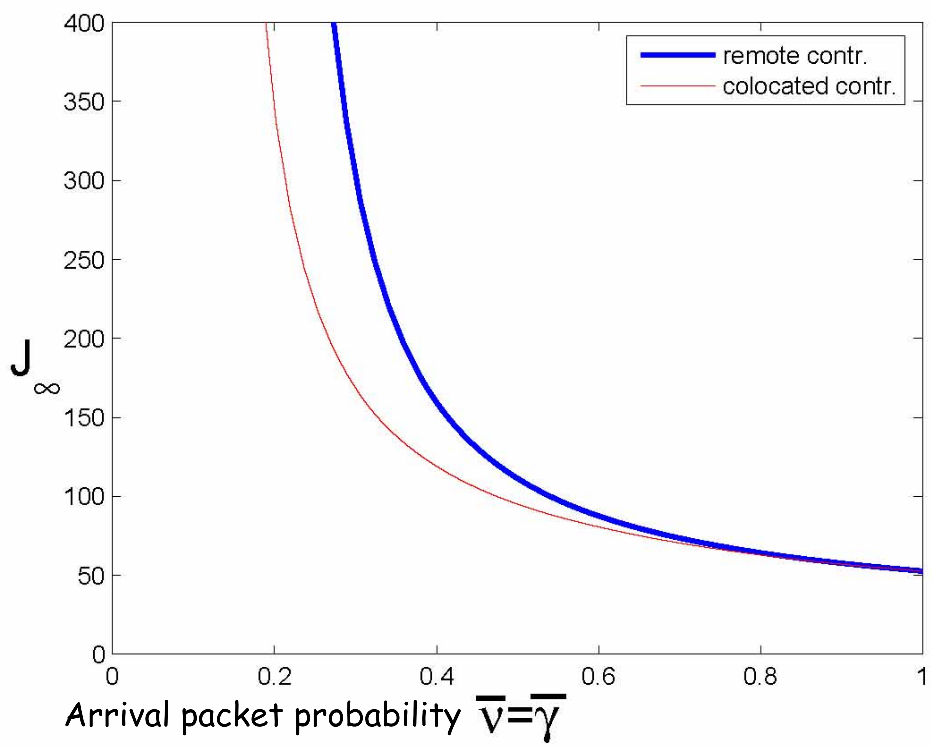


"Optimal LQG control across a packet-dropping link", Gupta, Spanos, Murray, Submitted to Sys.Cont.Lett. 05
"Estimation under controlled and uncontrolled communications in networked control systems", Xu, Hespanha, CDC 05

Numerical example: remote vs co-located controller



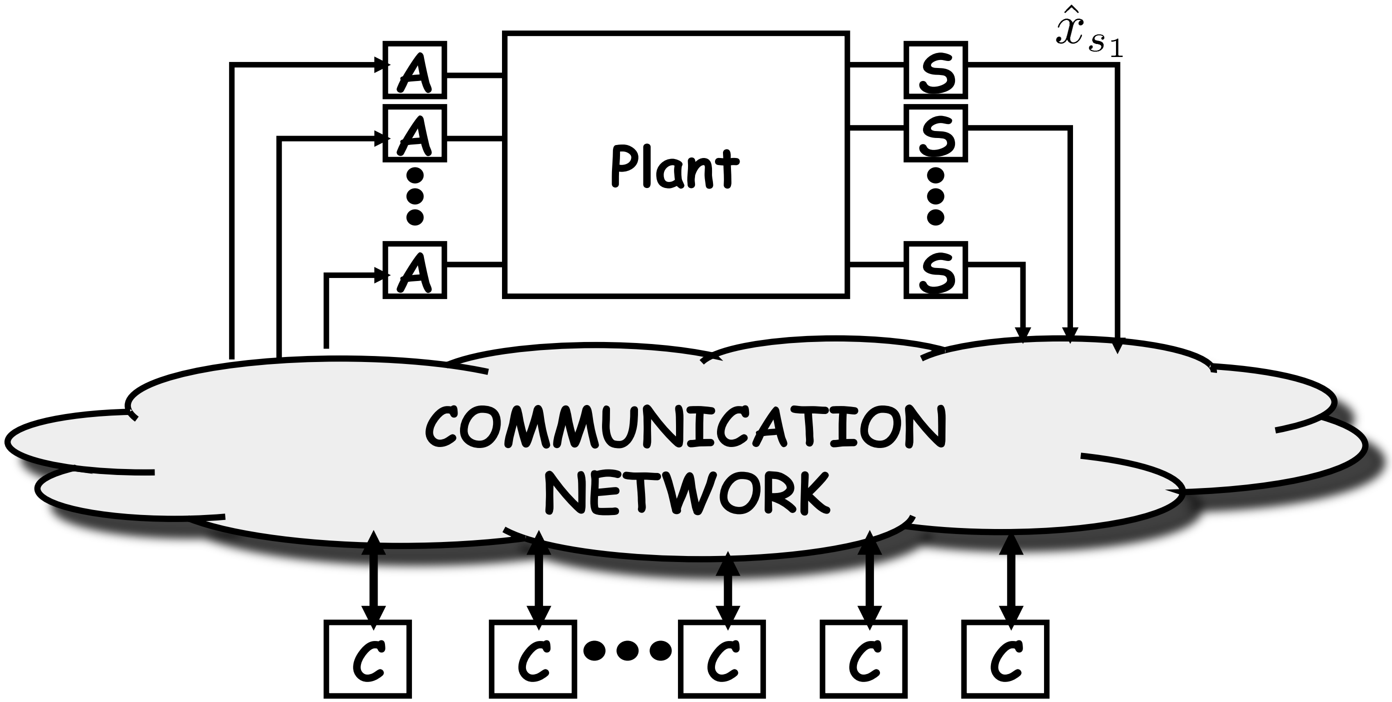




Takeaway points

- Input packet loss more dangerous than measurement packet loss
- TCP-like protocols help controller design as compared to UDP-like (but harder for communication designer)
- If you can, place controller near actuator
- If you can, send estimate rather than raw measurement
- Zero-input control seems to give smaller closed loop state error ($||x_t||$) than hold-input (but higher input)
- Trade-off in terms of performance, buffer length, computational resources (matrix inversion) when random delay
- Can help comparing different communication protocols from a real-time application performance

Future work



- Multiple sensors:
  - data fusion, i.e. $y_1, .., y_m$ arrive at different times
  - distributed estimation & consensus $E[x|y_1, ..., y_N] \stackrel{?}{=} E[x|\hat{x}_{s_1}, \hat{x}_{s_N}]$
- Multiple actuators
  - trade-off between distributed control & centralized coordination

Distributed sensor fusion & Consensus-based estimation