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TRANSMISSION SCHEDULING FOR REMOTE ESTIMATION WITH MULTI-PACKET RECEPTION UNDER MULTI-SENSOR INTERFERENCE

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ABSTRACT

In networked control systems, due to competing demands on various resources such as bandwidth, energy constraints etc., sensor scheduling is an important problem for remote estimation and control tasks. Traditionally, a single sensor is scheduled in each resource block to avoid interference or collision so that the probability of packet loss is reduced. However, receiving multiple packets from different sources under interference is routinely achieved in wireless networks using multi-user detection or multi-packet reception techniques. In this work, we explore the problem of sensor scheduling for remote estimation when the estimator is able to simultaneously receive multiple packets successfully. We use the typical signal-to-interference-and-noise-ratio (SINR) based capture model to compute the packet arrival probabilities and an optimal sensor scheduling policy is determined by minimizing expected estimation error covariance subject to a constraint on the average number of total transmissions from all sensors. For the case of two sensors, a scalar system, and a decoupled two-dimensional system, we show that allowing multiple simultaneous transmissions can improve the quality of the estimation achieving lower energy consumptions. We provide structural results on the optimal scheduling policies. Numerical results are provided to illustrate the benefits of multi-packet reception in remote estimation.

Keywords Estimation under communication constraints; Multi-packet reception; Packet loss.

1 INTRODUCTION

The ubiquity of wireless communication networks has paved the way for novel control technologies involving remote processes and networked devices. In particular, remote estimation has gained considerable attention in the past years in many different areas, such as autonomous vehicles, environment monitoring, power distribution, home and factory automation. With respect to standard sensing architectures, Wireless Sensor Networks (WSNs) achieve important practical advantages due to reduced wiring, easy and modular connections, more accurate knowledge of the process with multiple sensors, and increased agility. On the other hand, estimation performances are not necessarily improved through the use of WSNs since, if the network medium access is not suitable designed, resulting information losses and delays cause poor estimates, especially in the case of state estimation of dynamical systems. At the same time, even when losses are avoided or can be neglected, energy consumption becomes an important aspect and frequent communications ca drain the batteries of the sensors prematurely. Therefore it is fundamental to devise a scheduling algorithm that manages the order and the frequency at which each device accesses the network. The solution is not straightforward and many different algorithms have been proposed in recent works for transmission scheduling in remote estimation of dynamical systems.

When only a single sensor is present, scheduling policies are devised to satisfy energy limitations. In [1], the authors propose an event-based scheduling algorithm where a threshold sets the trade-off between the energy consumption and the quality of the estimate. In [2], the authors consider a smart sensor with two transmission energy levels: the higher guarantees that packets are always successfully delivered, while the lower has a loss probability greater than 0. The corresponding optimal periodic schedule transmits at the higher level as "uniformly" as possible. In [3], the authors do not admit totally reliable communications but, similarly to [2], under the optimal solution among the class of periodic schedules, transmissions are as uniformly spread as possible. The case where a single system is observed by multiple sensors has also been studied in many different works. In [4], the authors consider two sensors with different energy consumptions and a constraint on the total energy. Interestingly, if a set of previous measurements is sent, the optimal scheduling is periodic and does not depend on the parameters of the sensors but only on the energy consumption. For a general number of sensors, the finite-horizon case is considered in [5] while the infinite-horizon case is investigated in [6]. Without packet losses, the latter paper shows that the optimal scheduling can be approximated arbitrarily closely by a periodic schedule. The case with multiple sensors with packet losses is studied in [7]. Under the assumptions that the system is observable from each sensor and that estimates of the state are communicated, optimal scheduling (when only a single sensor can be scheduled at each transmission) is a time-varying threshold policy (time-invariant in the infinite-horizon case) based on the estimation error covariance matrix at the remote estimator. In [8], the authors propose a stochastic schedule that, differently from a deterministic schedule, preserves the Gaussian property in such a way the optimal estimator is simply a time-varying Kalman filter. However, losses due to simultaneous transmissions are not taken into account. A stochastic scheduling scheme for multi-hop networks is presented by [9]. The most general case with multiple systems (and multiple sensors) has recently gained attention. The simple case with two Gauss-Markov system has been studied in [10]. With unstable systems, the authors of [11] show that the optimal schedule must have bounded periods among two consecutive transmissions from the same sensor and it must be periodic.

In almost all the above works, it is assumed that either a single sensor can be scheduled at each transmission, or there is no interference caused even with simultaneous transmissions. There are two notable exceptions where interference is taken into account: in [12], the authors propose a channel-adaptive optimal random access scheme for remote control of multiple systems, and in [13], the authors study the optimal power allocation for remote estimation. In the particular case of this work, we explore how to exploit multi-packet reception in remote estimation of dynamical systems. Multi-user decoding/detection in the presence of interference in wireless communications is very common, where the receiver receives a superposition of all signals transmitted simultaneously from different users in noise, and it can employ a number of clever signal processing/decoding techniques to decode the users' data. While the optimal scheme of joint maximum likelihood detection is computationally highly complex, there are several suboptimal schemes that achieve comparable performance, if designed suitably (see [14]). Multi-packet reception is a particular type of multiuser detection technique where the receiver is equipped to decode multiple simultaneous transmissions. This can be achieved in many ways, such as at the signal modulation level (CDMA), by multiple antennas at the transmitter and receiver (MIMO), or by using signal processing based collision resolution methods as discussed in [15]. Multi-packet reception based systems can yield a great improvement for WSNs because, for example, a central node equipped with multiple antennas may be able to simultaneously receive and decode measurements from different sensors, boosting the performance of the remote estimation algorithms. On the other hand, simultaneous transmissions interfere with each other at the receiver and, even if multi-packet reception is possible, the loss probability of a given packet may be higher. The optimal balance between transmission scheduling and interference mitigation in the context of remote estimation performance is an open problem, and largely an unexplored field.

In this work, we aim to partially answer this question. We formulate an optimization problem where the expected trace of the remote estimation error covariance is minimized subject to average transmission energy constraints, but, differently from existing algorithms, we allow multiple simultaneous transmissions. The packet reception model accounts both for the interference due to other incoming communications and to external noise. We consider two types of multi-packet reception schemes, one based on the capture property of the wireless receiver (see [16]), where any sensor that has a received Signal-to-Interference-and-Noise Ratio (SINR) above a certain threshold at the remote estimator, can be successfully decoded. In this scheme, each sensor sees the interference due to other sensor transmissions as noise. We also investigate a more sophisticated receiver based on *successive interference cancellation* (SIC), where the sensor with the strongest received power is decoded first, and its reconstructed signal is subtracted out from the total received signal, so that the sensor that has the second strongest received power can be decoded next where the strongest user's interference is no longer present. In the context of information theory, it is well known that SIC is an optimal scheme

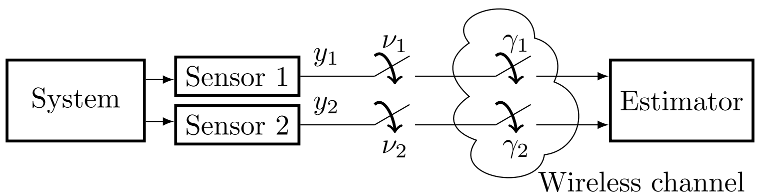

Figure 1: System model.

that achieves the rate region of a multiple access channel while minimizing the total transmission power ([17]). The improvement due to SIC in random access with decentralized power control has also been investigated by [18].

The main contributions of this work can be summarized as follows: (i) in contrast with existing works, we consider multiple simultaneous transmissions, an accurate model of the wireless channel that accounts for mutual interference and the corresponding packet loss probabilities, and two different decoding algorithms, namely with and without SIC, (ii) under the considered framework, an optimal scheduling policy is determined by solving an optimization problem that accounts for the estimation quality and penalizes the total number of sensor transmissions, (iii) we provide the general structure of the optimal scheduling policy for a scalar system and a decoupled two-dimensional system for the two-sensor case, showing their threshold-type behaviour and that they do not depend on the decoding algorithm, and (iv) numerical simulations are used to illustrate the differences between the schemes with and without SIC, and to compare the proposed algorithms with a traditional single-transmission scheme.

2 PROBLEM FORMULATION

In this paper we consider a dynamical system whose state has to be estimated by a remote estimator, as depicted in Fig. 1 for the case of two transmitting sensors. In general, a set of N sensors communicate to the estimator, which plays the role of a fusion centre, through a wireless network. The central node is equipped with a receiver capable of multi-packet reception, thus allowing more than one sensor to transmit simultaneously. To avoid confusion, we denote by *transmission period* the time interval during which each scheduled sensor transmits its measurement to the remote estimator. We assume there is a transmission period in any sampling period. Moreover, all the transmissions scheduled within a transmission period are synchronized, starting at the beginning, and finishing at the end of the period.

2.1 System model

Consider the discrete time state-space linear model

$$x(k+1) = Ax(k) + w(k) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state and $w(k) \in \mathbb{R}^n$ is the process noise modelled as an independent and identically distributed (*i.i.d.*) Gaussian random variables $w(k) \sim \mathcal{N}(0, Q)$ with $Q \geq 0$. A set of N sensors is available. At each sampling instant, the i-th sensor measures the output

$$y_i(k) = C_i x(k) + v_i(k) \tag{2}$$

where $y_i(k) \in \mathbb{R}^{m_i}$ and $v_i(k)$ is the measurement noise modelled as i.i.d. Gaussian random variables (and independent of $\{w(k)\}$) $v_i(k) \sim \mathcal{N}(0, R_i)$ with $R_i > 0$. During the k-th transmission period, a packet containing the sampled output $y_i(k)$ is communicated according to the decision variable $\nu_i(k)$ to a remote estimator: if $\nu_i(k) = 1$, then $y_i(k)$ is transmitted, while it is not transmitted if $\nu_i(k) = 0$. When scheduled, a transmission may not be successfully completed due to interference of other transmissions and channel and receiver noise. We represent this process through the variable $\gamma_i(k)$, which is equal to 1 if the transmission of $y_i(k)$ is successfully completed, 0 otherwise. The information set available at the fusion centre at the time instant k is:

$$\mathcal{I}(k) = \bigcup_{i=1}^{N} \mathcal{I}_i(k) \tag{3}$$

with

$$\mathcal{I}_i(k) = \big\{\nu_i(0)\gamma_i(0)y_i(0),\ \nu_i(1)\gamma_i(1)y_i(1),\ \ldots\ ,\ \nu_i(k-1)\gamma_i(k-1)y_i(k-1)\big\} \tag{4}$$

where, with a little misuse of notation, if $\nu_i(t)\gamma_i(t) = 0$ then $\nu_i(t)\gamma_i(t)y_i(t) = \emptyset$, i.e. $y_i(t)$ is missing. We define the following variables:

$$\widehat{x}(k|k-1) := \mathbb{E}[x(k) \mid \mathcal{I}(k)]$$
$$P(k|k-1) := \mathbb{E}[(x(k)-\widehat{x}(k|k-1))(x(k)-\widehat{x}(k|k-1))' \mid \mathcal{I}(k)].$$

From [19], $\widehat{x}(k|k-1)$ is the optimal estimator given $\mathcal{I}(k)$, and the matrix $P(k|k-1)$ denotes the corresponding estimation error covariance matrix. The optimal estimator with measurements received from multiple sensors can be found in [20]. Due to space limitations, we report only the update of the covariance matrix accounting for packet losses as:

$$P(k|k) = \left(P^{-1}(k|k-1) + \sum_{i=1}^{N} \gamma_i(k)\nu_i(k)C_i'R_i^{-1}C_i\right)^{-1} \tag{5}$$

$$P(k|k-1) = AP(k-1|k-1)A' + Q. \tag{6}$$

The decision variables $\nu_i(k)$, $i = 1, \ldots, N$ are chosen at the central node and are communicated back to the sensors within the time interval $(k-1, k)$ without error.

2.2 Channel model

Denote by P_i^{tx} the transmitted power of the i-th sensor, while g_i denotes the slow fading component of the channel power gain (usually dependent on path loss etc.) from the i-th sensor to the remote estimator, and $h_i(k)$ is the fast fading component of the same channel during the k-th transmission period. We assume that P_i^{tx} and g_i are constant, while $h_i(k)$ is modelled as a temporally independent identically distributed exponential random variable (this corresponds to Rayleigh fading, a common distribution for a wireless environment with large number of scatterers) with unity mean, i.e. $h_i(k) \sim Exp(1)$, with $h_i(k) \perp h_i(t)$ for $t \neq k$ and $h_i(k) \perp h_j(t)$ for $\forall k, t$ and $i \neq j$. It follows that the received power at the remote estimator from the i-th sensor $P_i^{\mathrm{rc}}(k)$ during the k-th transmission interval is

$$P_i^{\mathrm{rc}}(k) = \begin{cases} P_i^{\mathrm{tx}} g_i h_i(k) & \text{if } \nu_i(k) = 1 \\ 0 & \text{if } \nu_i(k) = 0. \end{cases} \tag{7}$$

It is easy to see that, given $\nu_i(k) = 1$, the received power is an exponential random variable with mean $\lambda_i = (g_i P_i^{\mathrm{tx}})^{-1}$, i.e. $P_i^{\mathrm{rc}}(k) \sim Exp(\lambda_i)$. Due to the intrinsic nature of the wireless medium, background channel and/or receiver noise is also present. We model it as an Additive White Gaussian Noise (AWGN) whose average power at the remote estimator is σ^2.

Without SIC, the Signal-to-Interference-and-Noise Ratio (SINR) corresponding to the packet containing $y_i(k)$ is

$$\mathrm{SINR}_i(k) = \frac{\nu_i(k)P_i^{\mathrm{tx}} g_i h_i(k)}{\sum_{j \neq i} \nu_j(k) P_j^{\mathrm{tx}} g_j h_j(k) + \sigma^2} \tag{8}$$

$$= \frac{\nu_i(k)P_i^{\mathrm{rc}}(k)}{\sum_{j \neq i} \nu_j(k) P_j^{\mathrm{rc}}(k) + \sigma^2}. \tag{9}$$

When SIC is employed, we assume without loss of generality that the sensors have been ordered in descending order of receiver power, i.e. $P_1^{\mathrm{rc}}(k) \geq P_2^{\mathrm{rc}}(k) \geq \ldots P_N^{\mathrm{rc}}(k)$. Since the stronger users are decoded before the weaker users, the SINR for the i-th sensor in this case is given by

$$\mathrm{SINR}_i(k) = \frac{\nu_i(k)P_i^{\mathrm{rc}}(k)}{\sum_{j > i} \nu_j(k) P_j^{\mathrm{rc}}(k) + \sigma^2}. \tag{10}$$

A packet from the i-th sensor at the k-the time slot can be decoded without error if $\mathrm{SINR}_i(k) > \alpha$, where $\alpha > 0$ is a threshold that is chosen based on the modulation and coding schemes used. We need to have $\alpha \in (0, 1)$, which is necessary to enable multi-packet reception, and can be achieved e.g. by Code Division Multiple Access (CDMA). It follows that the packet arrival process from the i-th sensor can be expressed as

$$\gamma_i(k) = \begin{cases} 1 & \text{if } \mathrm{SINR}_i(k) > \alpha \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Since the channel gains are independent across time slots, $\gamma_i(k)$ is also an i.i.d Bernoulli process. However $\gamma_i(k), \gamma_j(k)$ for $j \neq i$ may be dependent on each other due to interference within a given time slot.

3 ARRIVAL PROBABILITIES

In this section we provide the probabilities of the arrival process for the case with 2 sensors. When only 1 sensor is scheduled, we have:

$$q_{10} := \boldsymbol{P}(\gamma_1(k) = 1 \,|\, \nu_1(k) = 1,\, \nu_2(k) = 0) = e^{-\alpha\lambda_1\sigma^2}$$
$$q_{01} := \boldsymbol{P}(\gamma_2(k) = 1 \,|\, \nu_1(k) = 0,\, \nu_2(k) = 1) = e^{-\alpha\lambda_2\sigma^2}$$

When both the sensors are scheduled, denote:

$$\begin{aligned}
p_{11} &:= \boldsymbol{P}(\gamma_1(k) = 1,\, \gamma_2(k) = 1 \,|\, \nu_1(k) = 1,\, \nu_2(k) = 1)\\
p_{10} &:= \boldsymbol{P}(\gamma_1(k) = 1,\, \gamma_2(k) = 0 \,|\, \nu_1(k) = 1,\, \nu_2(k) = 1)\\
p_{01} &:= \boldsymbol{P}(\gamma_1(k) = 0,\, \gamma_2(k) = 1 \,|\, \nu_1(k) = 1,\, \nu_2(k) = 1)\\
p_{00} &:= \boldsymbol{P}(\gamma_1(k) = 0,\, \gamma_2(k) = 0 \,|\, \nu_1(k) = 1,\, \nu_2(k) = 1)
\end{aligned}$$

It is easy to compute $p_{11} + p_{10}$ and $p_{11} + p_{01}$:

$$\begin{aligned}
p_{11} + p_{10} &= \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1}\, e^{-\alpha\lambda_1\sigma^2} = \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1}\, q_{10}\\
p_{11} + p_{01} &= \frac{\lambda_1}{\lambda_1 + \alpha\lambda_2}\, e^{-\alpha\lambda_2\sigma^2} = \frac{\lambda_1}{\lambda_1 + \alpha\lambda_2}\, q_{01}.
\end{aligned}$$

Under the assumption that $\alpha \in (0, 1)$, while different expressions exist, it is convenient to write

$$\begin{aligned}
p_{11} &= \left(\frac{\lambda_1}{\lambda_1 + \alpha\lambda_2} + \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1} - 1\right) e^{-(\lambda_1+\lambda_2)\frac{\alpha}{1-\alpha}\sigma^2}\\
p_{00} &= 1 - (p_{11} + p_{10}) - (p_{11} + p_{01}) + p_{11}\\
&= 1 - \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1}\, e^{-\alpha\lambda_1\sigma^2} - \frac{\lambda_1}{\lambda_1 + \alpha\lambda_2}\, e^{-\alpha\lambda_2\sigma^2} + \left(\frac{\lambda_1}{\lambda_1 + \alpha\lambda_2} + \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1} - 1\right) e^{-(\lambda_1+\lambda_2)\frac{\alpha}{1-\alpha}\sigma^2}
\end{aligned}$$

Finally p_{10} and p_{01} can be found by subtracting p_{11} from $p_{11} + p_{10}$ and $p_{11} + p_{01}$.

When SIC is employed, the corresponding probabilities (denoted by the superscript SIC) are given by

$$\begin{aligned}
p_{11}^{SIC} &= \left(\frac{\lambda_2}{\lambda_2+\alpha\lambda_1} e^{-\lambda_1\alpha\sigma^2} + \frac{\lambda_1}{\lambda_1+\alpha\lambda_2} e^{-\lambda_2\alpha\sigma^2}\right) e^{-(\lambda_1+\lambda_2)\alpha\sigma^2} + \left(1 - \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1} - \frac{\lambda_1}{\lambda_1 + \alpha\lambda_2}\right) e^{-(\lambda_1+\lambda_2)\frac{\alpha}{1-\alpha}\sigma^2}\\
p_{10}^{SIC} &= \frac{\lambda_2}{\lambda_2 + \alpha\lambda_1} e^{-\alpha\sigma^2\lambda_1} \left(1 - e^{-\alpha\sigma^2(\lambda_2+\alpha\lambda_1)}\right)\\
p_{01}^{SIC} &= \frac{\lambda_1}{\lambda_1 + \alpha\lambda_2} e^{-\alpha\sigma^2\lambda_2} \left(1 - e^{-\alpha\sigma^2(\lambda_1+\alpha\lambda_2)}\right)\\
p_{00}^{SIC} &= p_{00}.
\end{aligned}$$

Lemma 1. *The following results hold for the arrival probabilities:*

1. $p_{11} + p_{10} < q_{10}$ *and* $p_{11} + p_{01} < q_{01}$
2. $p_{00} + p_{10} > 1 - q_{01}$ *and* $p_{00} + p_{01} > 1 - q_{10}$
3. $p_{00} < \max\{1 - q_{10},\, 1 - q_{01}\}$
4. $p_{00} < \min\{1 - q_{10},\, 1 - q_{01}\}$ *if and only if*

$$e^{-(\lambda_1+\lambda_2)\frac{\alpha}{\alpha-1}\sigma^2} \left(\frac{\lambda_1}{\lambda_1 + \alpha\lambda_2} - \frac{\alpha\lambda_1}{\lambda_2 + \alpha\lambda_1}\right) \quad < \quad \frac{\lambda_1}{\lambda_1 + \alpha\lambda_2}\, e^{-\alpha\lambda_2\sigma^2} \quad - \quad \frac{\alpha\lambda_1}{\lambda_2 + \alpha\lambda_1}\, e^{-\alpha\lambda_1\sigma^2} \tag{12}$$

As an immediate consequence of previous Lemma, according to point (3), if channels are identical, i.e. $q_{10} = q_{01} = q$, then $p_{00} < 1 - q$. The same results hold when the arrival probabilities are replaced by their counterparts with SIC.

4 OPTIMAL SENSOR SCHEDULING

In this section, we are interested in finding the optimal sensor scheduling scheme over the finite horizon $[0, K-1]$, where the sum of the trace of expected prediction error covariance matrices $P(k|k-1)$ over the finite horizon is minimized, along with a penalty on the expected number of total sensor transmissions. In particular, the sensor scheduling problem can be formulated as

$$J\left(U_{[0,K-1]}, P_0\right) = \sum_{k=0}^{K-1} \mathbb{E}\left[\mathrm{Tr}\left(P(k|k-1)\right)\right] + \mu \sum_{k=0}^{K-1}\sum_{i=1}^{N} \nu_i(k) \tag{13}$$

where

$$U_{[0,K-1]} = \{\nu_i(k)\ i = 1 \cdots, N\ k = 0, \cdots, K-1\} \tag{14}$$

and μ is a regularization parameter that can be tuned to set the desired trade-off between the estimate performances (i.e the error covariance) and the communication cost (i.e. the number of transmissions). Minimizing the metric for different values of μ corresponds to minimizing the error covariance under different constraints on the mean number of transmissions. The optimal schedule is then

$$U^*(P_0) = \arg\min_{U} J\left(U, P_0\right). \tag{15}$$

The problem is essentially a stochastic control problem, with the scheduling variables as the control sequence, and can be efficiently solved e.g. through Dynamic Programming. However, the structure of the optimal solution is difficult to characterize, as we will show later.

In the following we show some structural results for the two simplest cases: a scalar system and a 2-dimensional system with decoupled process noise, 2 sensors and 1-step horizon ($K = 1$). For these particular cases, we can denote $U = (\nu_1, \nu_2)$. We consider $C_1 = C_2 = 1$ without loss of generality. We say that channels are identical if $P_1^{\mathrm{tx}} g_1 = P_2^{\mathrm{tx}} g_2$, which implies that $q_{10} = q_{01}$ and $p_{10} = p_{01}$, while we say that the sensors are identical if $R_1 = R_2$. Please note that the following propositions hold true both with and without SIC once the probabilities $p_{11},\ p_{10}$, and p_{01} are substituted by their counterparts with SIC.

4.1 Scalar system

Lemma 2. *If channels and sensors are identical, then* $J((1,0),P) = J((0,1),P)\ \forall P \geq 0$*. If channels or sensors are not identical, it holds that*

$$J((1,0),P) < J((0,1),P)\ \forall P > 0 \Longleftrightarrow \frac{R_1}{q_{10}} < \frac{R_2}{q_{01}}$$

and

$$J((1,0),\ P) < J((0,1),\ P)\ \text{for}\ P > \hat{P} \Longleftrightarrow q_{10} > q_{01}$$

where

$$\hat{P} = \frac{q_{01}R_1 - q_{10}R_2}{q_{10} - q_{01}}.$$

Roughly speaking, the previous Lemma states that the sensor with the best channel is better than the other for at least some error covariances. The sensor whose measurements have the best quality (lowest R) is preferred for low error covariance, while the sensor with the best channel (highest q) is preferred for high error covariance. Interestingly, there are configurations of the parameters for which a given sensor is never preferred to the other. In that case, without multi-packet reception, this sensor may be useless. The following Proposition shows that even in this case such a sensor can be exploited when the receiver has multi-packet reception capabilities.

Proposition 3 (Scalar system, 1-step horizon)**.** *Suppose* $\mu > 0$*. Then, for a scalar system:*

1. $\exists \underline{P} > 0$ *s.t. if* $0 \leq P < \underline{P}$ *then* $U^*(P) = (0,0)$*, while if* $P > \underline{P}$ *then* $U^*(P) \neq (0,0)$
2. $\exists \overline{P} > 0$ *s.t. if* $P > \overline{P}$ *then* $U^*(P) = (\nu_1^*, \nu_2^*) = \arg\min_{\nu_1,\nu_2} \boldsymbol{P}(\gamma_1 = 0, \gamma_2 = 0\,|\,\nu_1, \nu_2)$*, namely the scheduling that gives the lowest probability of no new delivered packets. In particular, when condition* (12) *holds, multiple simultaneous transmissions are optimal, i.e.* $U^*(P) = (1,1)$ *for* $P > \overline{P}$
3. *There are no additional thresholds in the interval* $(\underline{P},\ \overline{P})$ *in the following cases:*
   - *the sensors and channels are identical*

- *the sensors are identical and* $p_{00} < \min\{1 - q_{10},\, 1 - q_{01}\}$
- *if* $R_1 > R_2$ *but scheduling* $(1, 0)$ *is always better than scheduling* $(0, 1)$ *and* $p_{00} < 1 - q_{10}$*, i.e.*

$$R_1 > R_2 \quad q_{10} > q_{01} \quad \frac{R_1}{q_{10}} < \frac{R_2}{q_{01}} \quad p_{00} < 1 - q_{10}$$

The previous Proposition shows that the optimal scheduling has a threshold-type behaviour. In particular, $\underline{P}$ defines the threshold before which no transmission is optimal, while $\overline{P}$ is the largest threshold, since for $P > \overline{P}$ the optimal scheduling is fixed. In general, it is possible that $\underline{P} = \overline{P}$.

Note that there exist cases in which $\frac{R_1}{q_{10}} < \frac{R_2}{q_{01}}$ but condition (12) holds, for example with $R_1 < R_2$ and identical channels. In that case for $P > \overline{P}$ both sensors transmit, despite the fact the first sensor would be always preferred to second sensor when multiple transmission are not allowed. It is worth mentioning that defining structural properties of the optimal schedule in the range $(\underline{P}, \overline{P})$ is complicated even in the scalar case with only two sensors. This is due to the fact that the equation $J\left((1,1), P\right) = J\left((1,0), P\right)$ is a quartic equation and finding the positive roots (that are the thresholds of the policy) in closed-form is prohibitive. Two interesting behaviours of the optimal cost and of the optimal scheduling are reported in Fig. 2. In particular, the bottom panel represents the case when sensors and channels are very different and external noise is high. We can see that, without SIC, in a case multiple simultaneous transmissions are optimal for $P > \overline{P}$, while in the other it is optimal for $P \in (\overline{\overline{P}}, \overline{P})$. A receiver equipped with SIC achieves a lower cost for $P > \overline{P}_{\text{SIC}}$ or for $P \in (\overline{\overline{P}}_{\text{SIC}}, \overline{P}_{\text{SIC}})$ ($\overline{P}_{\text{SIC}} > 5$ is not depicted for sake of clarity).

4.2 2-dimensional decoupled system

Consider a 2-dimensional system where the state transition matrix is decoupled. For easy of notation, denote the system by

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \qquad C_1 = [1 \;\; 0] \qquad C_2 = [0 \;\; 1] \qquad Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \qquad P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

Lemma 4. *At the point* $P = (P_1, P_2)$*,*

$$J((1,0),\, P) < J((0,1),\, P) \iff \frac{q_{10} A_1^2 P_1^2}{P_1 + R_1} < \frac{q_{01} A_2^2 P_2^2}{P_2 + R_2}.$$

The curve Γ *:* $J((1,0),\, P) = J((0,1),\, P)$ *divides the positive quadrant of the plane* $(P_1,\, P_2)$ *in two regions. It passes through* $(0, 0)$*, asymptotically tends to the line r:* $q_{10} A_1^2 P_2 = q_{01} A_2^2 P_2$ *while always lying underneath it if* $q_{10} A_1^2 R_1 > q_{01} A_2^2 R_2$*, or always above it otherwise.*

The previous Lemma shows that, as illustrated in Fig. 3 for a decoupled system, for each sensor there always exists a set of the error covariances for which it is preferred to the other, independently of the quality of the channels, the quality of the sensors, and the magnitude of the eigenvalues.

Proposition 5 (2D decoupled system, 1-step horizon)**.** *For the two-dimensional system, if* $\mu > 0$*, the following holds:*

1. $\exists \underline{P}_1 > 0,\; \underline{P}_2 > 0$ *that define the region*
$$R_{00} = \{(P_1, P_2) \,:\, 0 \le P_1 < \underline{P}_1 \text{ and } 0 \le P_2 < \underline{P}_2\}$$
s.t. if $P \in R_{00}$*, then* $U^*(P) = (0, 0)$*, while if* $P \notin R_{00}$ *then* $U^*(P) \neq (0, 0)$*. The point* $(\underline{P}_1, \underline{P}_2)$ *belongs to the curve* Γ*. Moreover, there exists a neighbourhood of the origin in which region* R_{00} *is strictly included where* $U^*(P) \neq (1, 1)$*.*

2. $\forall P_1\, \exists \overline{P}_2(P_1) > 0$ *s.t. if* $P_2 > \overline{P}_2(P_1)$ *then* $U^*((P_1, P_2)) = (1, 0)$
$\forall P_2\, \exists \overline{P}_1(P_2) > 0$ *s.t. if* $P_1 > \overline{P}_1(P_2)$ *then* $U^*((P_1, P_2)) = (1, 0)$

3. $U^*(P) = (1, 1)$ *for* $P = (P_1, P_2)$ *belonging to the non-empty region* R_{11}*:*
$$\begin{cases} (q_{10} - p_{11} - p_{10}) \dfrac{A_1^2 P_1^2}{P_1 + R_1} + \nu < (p_{11} + p_{01}) \dfrac{A_2^2 P_2^2}{P_2 + R_2} \\ (q_{01} - p_{11} - p_{01}) \dfrac{A_2^2 P_2^2}{P_2 + R_2} + \nu < (p_{11} + p_{10}) \dfrac{A_1^2 P_1^2}{P_1 + R_1} \end{cases}$$
The curve Γ *intersects the bound of the region* R_{11} *at a unique point* $(\hat{P}_1, \hat{P}_2)$*, so that any point* $(P_1,\, P_2)$ *s.t.* $P_1 > \hat{P}_1,\, P_2 > \hat{P}_2$ *belonging to the curve* Γ *belongs to* R_{11}*. For* $P_1 \to +\infty$*, the boundaries of* R_{11} *tend to two straight lines.*

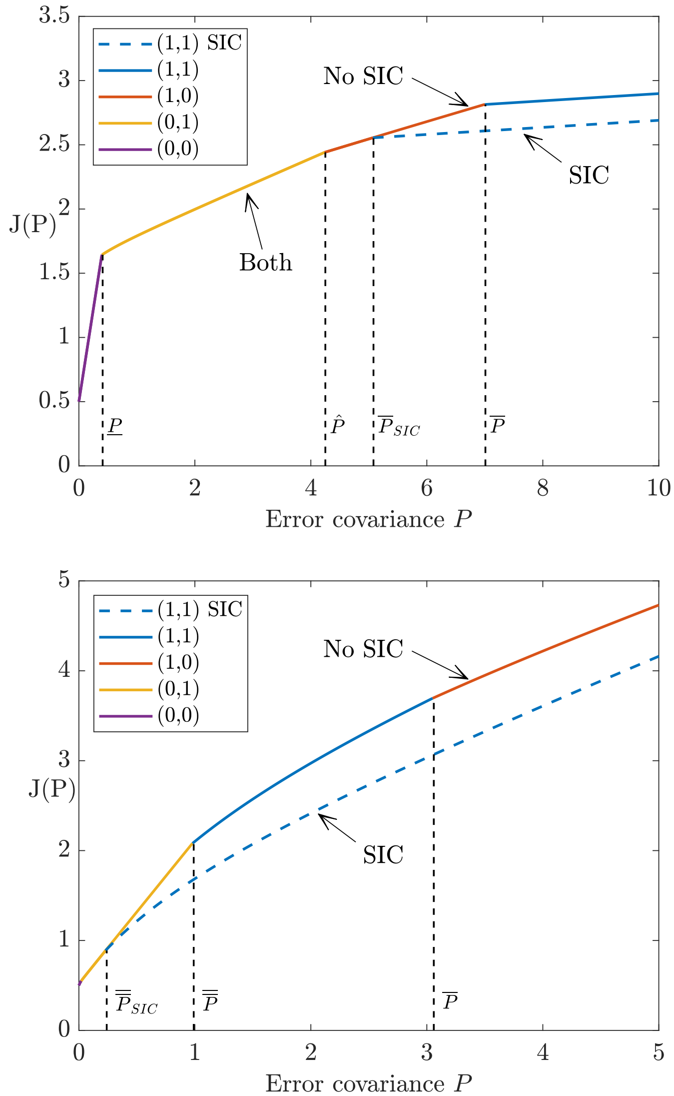

Figure 2: Examples of optimal scheduling. $A = 1.7,\ Q = 0.5$. Top: $g_1 = 1.5,\ g_2 = 1,\ \sigma = 0.1,\ R_1 = 0.2,\ R_2 = 0.1,\ \mu = 1$ Bottom: $g_1 = 5,\ g_2 = \sigma = 1,\ R_1 = 1,\ R_2 = 0.01,\ \mu = 0.01$

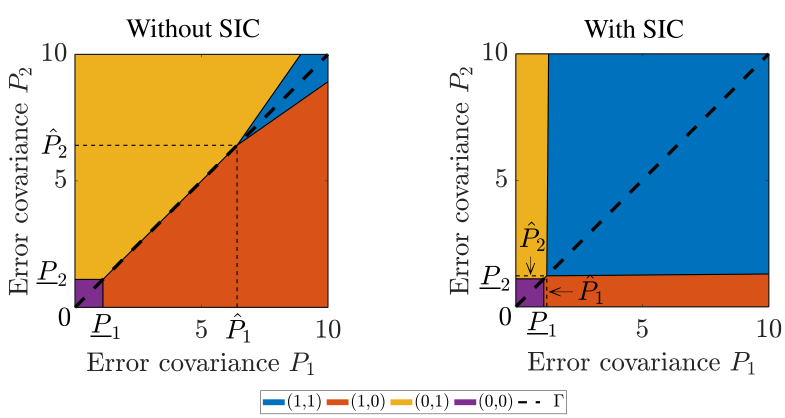

Figure 3: Example of optimal schedule for a 2D system. Left: without SIC. Right: with SIC. $A_1 = A_2 = 1.7,\ Q_1 = Q_2 = 0.5,\ g_1 = g_2 = 1,\ \sigma = 0.1,\ R_1 = R_2 = 0.1,\ \mu = 3$.

According to point (1) of the previous Proposition, as illustrated in Fig. 3 there exists a rectangular region R_{00} with the origin as the bottom-left vertex in which no transmission is the optimal scheduling. According to (2), we have that, keeping a component of the error covariance fixed and making the other bigger, the optimal scheduling eventually schedules only transmissions to observe the most uncertain state. Scheduling $(1, 1)$ is optimal in a region R_{11} (partially) containing the curve that divides the region where scheduling $(1, 0)$ is better than scheduling $(0, 1)$ to the region where the opposite holds true. This region does not touch the axis. As can be seen with a typical policy reported in Fig. 3, while R_{00}, namely, the rectangular region on the bottom left of the plane, is the same with or without SIC, R_{11} is definitely larger with SIC. This shows that, with a decoupled system, SIC can provide a great improvement for estimation purposes when multiple transmissions are allowed.

5 NUMERICAL SIMULATIONS

In this section, we fix the system parameters as follows:

$$A = 1.7 \qquad Q = 0.5 \qquad C_1 = C_2 = 1 \qquad R_1 = R_2 = 0.1$$
$$P_1^{\text{tx}} = P_2^{\text{tx}} = 1 \qquad g_1 = g_2 = 1 \qquad \sigma^2 = 0.1$$

Where not explicitly indicated, $\alpha = 0.7$ and $\mu = 0.1$. The resulting arrival probabilities are:

$$q_{10} = q_{01} = q = 0.932 \qquad p_{11} = 0.110 \qquad p_{10} = p_{01} = 0.438$$
$$p_{11}^{\text{SIC}} = 0.862 \qquad p_{10}^{\text{SIC}} = p_{01}^{\text{SIC}} = 0.062 \qquad p_{00}^{\text{SIC}} = p_{00} = 0.014$$

Since channels are identical, according to Proposition 3, the optimal policy is:

$$U^*(P) = \begin{cases} (0, 0) \text{ if } P < \underline{P} \\ (1, 0) \text{ if } \underline{P} \le P < \overline{P} \\ (1, 1) \text{ if } P \ge \overline{P} \end{cases}$$

This section aims to explore the improvement that can be achieved employing a receiver with multi-packet reception capabilities. We compare the optimal (1-step horizon) policies devised in Sec. 4 where multiple simultaneous transmissions are allowed with the optimal policy for the case where at most one single transmission is possible, that is the variance-based optimal (1-step horizon) policy given by [7].

First we consider the time evolution of the error covariance for the different policies over the same realisations of the processes P_1^{rc} and P_2^{rc}. Results are shown in Fig. 4. We can see that multiple transmissions achieve a lower error covariance in the cases where both transmissions are successfully completed. At the same time, peaks (corresponding to no new packets) are less frequent since $p_{00} < 1 - q$ with identical channel. It is clear that the mean error covariance is further improved using SIC. Indeed, error covariance without SIC is an upper-bound of the error covariance with SIC.

It is interesting to see how the optimal policies behave for different α. A low α indicates that a packet can be correctly received also with a low SINR, i.e. communication is robust against interference and noise. Different α values can be achieved by choosing appropriate modulation and coding schemes, e.g. a low-order modulation and a high coding rate return a low α at the price of a low data-rate. Fig. 5 reports the mean error covariance for the different policies with varying α. Simultaneous multiple transmissions always achieve lower mean error covariances. It is worth mentioning that for small α, the improvement given by SIC is minor, due to the fact that mutual interference affects the packet reception in a negligible way. On the other hand, when α is close to one, without SIC the improvement given by the multiple transmissions is minor. Interestingly, for the given parameters, the plot corresponding to SIC is almost flat for $\alpha \in (0, 0.7)$. In Fig. 6 the behaviour of $\underline{P}$ and $\overline{P}$ with varying μ is reported. We can see that $\underline{P}$ is not very affected by α, while the dependence of $\overline{P}$ on α is more evident. We can see that for large α, $\overline{P}$ without SIC becomes very large: multiple transmissions are rarely scheduled implying that the mean error covariance is very close to the case where multiple transmissions are not allowed (as shown in Fig. 5). On the other hand, when α is small, $\overline{P}$ with and without SIC are coincident, explaining the same performance achieved for α close to 0 in Fig. 5.

In Fig. 7 we show how $\underline{P}$ and $\overline{P}$ behave with varying μ. We report only the plots for the case without SIC, since the case with SIC is analogous. While α depends on the robustness of the communication links and is a parameter of the network, μ is a design parameter that penalizes the number of transmissions. It is related to the actual energy constraint given by the battery life of the sensors or it can be used to set a trade-off between communication cost and estimation performance. We can see that both $\underline{P}$ and $\overline{P}$ are increasing function of μ: this means that, as expected, when the cost of the transmission increases, transmissions are penalized more and only when the error covariance is high, a transmission is scheduled. Also, for large values of μ, multiple simultaneous transmissions are optimal only for very high error covariances, thus rarely used.

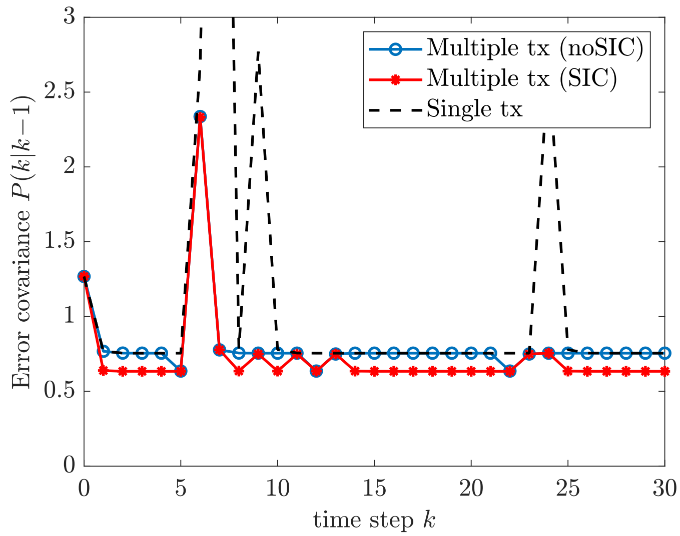

Figure 4: Evolution of the error covariance

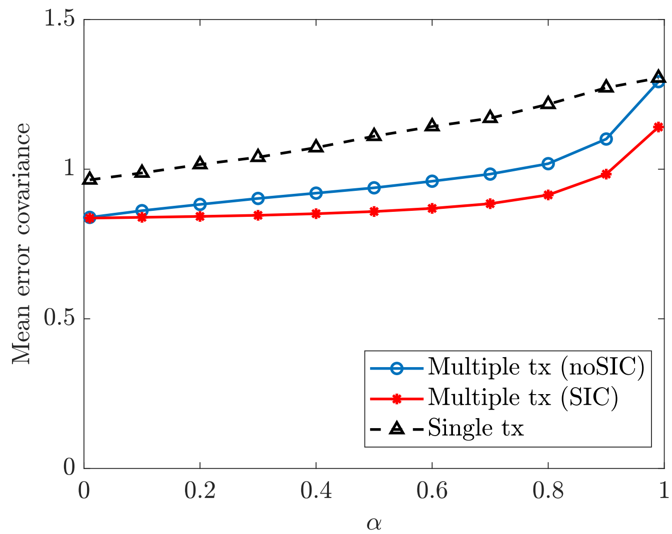

Figure 5: Error covariance with varying α

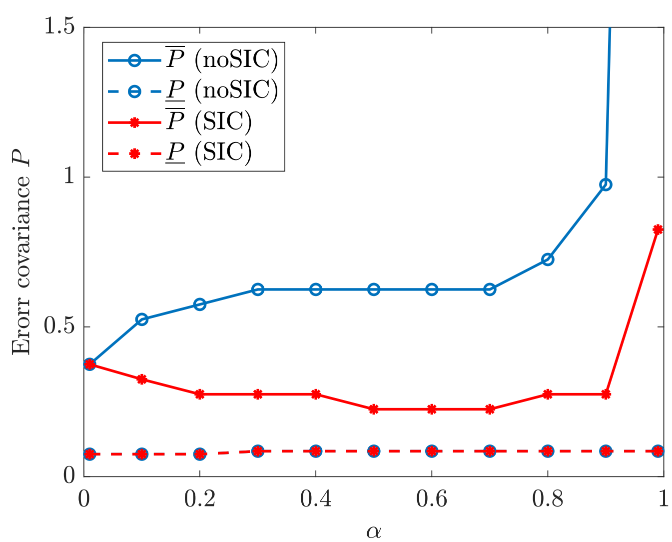

Figure 6: Thresholds with varying α

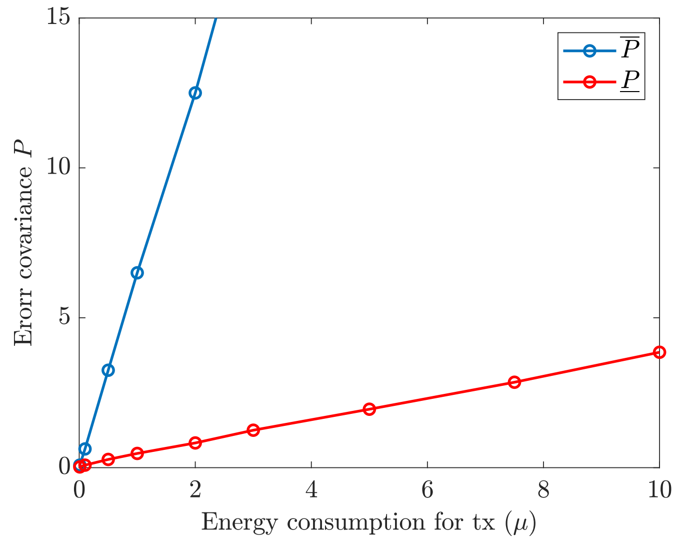

Figure 7: Thresholds with varying μ

6 CONCLUSIONS

In this paper we considered a sensor scheduling problem for remote estimation when the receiver is able to decode multiple simultaneous incoming packets, as with modern wireless technologies. We consider a suitable model for multi-packet reception that takes into account interference and two different decoding algorithms, i.e. with and without SIC. We provide the problem formulation for the general case, then we study the structural properties of the optimal schedule in the special case of a scalar system and a 2-dimensional decoupled system, illustrating that characterizing the optimal scheduling policies even in these simple cases is difficult. Through numerical results, we illustrate that multiple simultaneous transmissions can be beneficial, especially with SIC. Future research directions comprise the infinite-horizon case and the characterization of the optimal scheduling for general multi-dimensional systems.

7 APPENDIX

Proof (Lemma 2)**.** *Consider*

$$\begin{aligned}\Delta J(P) &:= J((1,0),P) - J((0,1),P)\\ &= A^2\left(q_{10}(P^{-1}+R_1^{-1})^{-1} + (1-q_{10})P\right) + Q + \mu - A^2\left(q_{01}(P^{-1}+R_2^{-1})^{-1} + (1-q_{01})P\right) - Q - \mu\\ &= A^2P^2\frac{(q_{10}-q_{01})P - q_{10}R_2 + q_{01}R_1}{(P+R_2)(P+R_1)}.\end{aligned}$$

Then if we solve the inequality $\Delta J(P) < 0$ subject to the condition $P \geq 0$, we can prove the proposition.

Proof (Proposition 3)**.** *Define $R_{eq}^{-1} = R_1^{-1} + R_2^{-1}$. The set of possible schedules is $\{(0,0),\ (1,0),\ (0,1)\ (1,1)\}$, thus we have:*

$$\begin{aligned}J((0,0),P) &= A^2P + Q\\ J((0,1),P) &= A^2\Big(q_{01}(P^{-1}+R_2^{-1})^{-1} + (1-q_{01})P\Big) + Q + \mu\\ J((1,0),P) &= A^2\Big(q_{10}(P^{-1}+R_1^{-1})^{-1} + (1-q_{10})P\Big) + Q + \mu\\ J((1,1),P) &= A^2\Big(p_{10}(P^{-1}+R_1^{-1})^{-1} + p_{01}(P^{-1}+R_2^{-1})^{-1} + p_{11}(P^{-1}+R_{eq}^{-1})^{-1} + p_{00}P\Big) + Q + 2\mu\end{aligned}$$

For $P = 0$, with $\mu \neq 0$, $J((0,0),\ P)$ is lower than the cost evaluated for all the other schedules. Since $J(U,P)$ is continuous w.r.t. P for any fixed U, there exists a $\underline{P}$ s.t. $J((0,0),\ P)$ is strictly the smallest $\forall P \in [0,\overline{P})$. Denoting by

$\mathrm{d}J(\cdot)$ *the derivative w.r.t.* P *of* $J(\cdot)$*, we have*

$$\begin{aligned}
\mathrm{d}J((0,0),P) &= A^2 \\
\mathrm{d}J((0,1),P) &= A^2\left(q_{01}R_2^2(P+R_2)^{-2}+1-q_{01}\right) \\
\mathrm{d}J((1,0),P) &= A^2\left(q_{10}R_1^2(P+R_1)^{-2}+1-q_{10}\right) \\
\mathrm{d}J((1,1),P) &= A^2\left(p_{00}+p_{11}R_{eq}^2(P+R_{eq})^{-2}+p_{10}R_1^2(P+R_1)^{-2}+p_{01}R_2^2(P+R_2)^{-2}\right)
\end{aligned}$$

Since the terms between parenthesis are positive and smaller than 1 for $P>0$*,* $\mathrm{d}J((0,0),P)$ *is the largest for any* $P>0$*, so once* $J((0,0),P)$ *is no longer the lowest (i.e.* $P>\underline{P}$*), it will remain so always. Taking the limit of J, with a slight abuse of notation, we have*

$$\begin{aligned}
J((0,0),\infty) &= A^2 = A^2\boldsymbol{P}(\gamma_1=0,\gamma_2=0\,|\,U=(0,0)) \\
J((0,1),\infty) &= A^2(1-q_{01}) = A^2\boldsymbol{P}(\gamma_1=0,\gamma_2=0\,|\,U=(0,1)) \\
J((1,0),\infty) &= A^2(1-q_{10}) = A^2\boldsymbol{P}(\gamma_1=0,\gamma_2=0\,|\,U=(1,0)) \\
J((1,1),\infty) &= A^2p_{00} = A^2\boldsymbol{P}(\gamma_1=0,\gamma_2=0\,|\,U=(1,1))
\end{aligned}$$

By continuity, there exists a $\overline{P}>0$ *s.t. for* $\forall P>\overline{P}$

$$\arg\min_{\nu_1,\nu_2} J((\nu_1,\nu_2),P) = \arg\min_{\nu_1,\nu_2} \boldsymbol{P}(\gamma_1=0,\gamma_2=0\,|\,\nu_1,\nu_2).$$

The complete characterization of the optimal scheduling may be carried out by finding all the intersection points between $J((0,1),P)$ *and* $J((1,1),P)$*. In general this is prohibitive because it requires the solution of a* 4*-th order equation, for which the expressions for the roots are quite involved. Furthermore, the roots needed to be compared to sort the optimality range. It is still possible to further characterize the optimal scheduling with identical sensors, i.e.* $R_{eq}=R_1/2$*. Without loss of generality assume that* $q_{10}>q_{01}$*, so that* $J((1,0),P)<J((0,1),P)\ \forall P>0$ *according to Lemma 2. Consider*

$$\begin{aligned}
\Delta\mathrm{d}J(P) &:= \mathrm{d}J((1,1),P)-\mathrm{d}J((1,0),P) \\
&= p_{11}R_1^2/4\,(P+R_1/2)^{-2}+p_{00}-1+q_{10}+(p_{10}+p_{01}-q_{10})R_1^2(P+R_1)^{-2} \\
&< p_{00}-1+q_{10}+(p_{11}+p_{10}+p_{01}-q_{10})R_1^2(P+R_1)^{-2} \\
&= p_{00}-1+q_{10}-(p_{00}-1+q_{10})R_1^2(P+R_1)^{-2}
\end{aligned}$$

that is lower than zero $\forall P$ *if* $p_{00}<1-q_{10}$*. It follows that in this case at most the thresholds* $\underline{P}$ *and* $\overline{P}$ *are present. This procedure can be further generalized to the case where* $R_1>R_2$ *but* $R_1q_{01}<R_2q_{10}$*. In that case, the first equality becomes an inequality, while the subsequent ones still hold. By Lemma 2,* $J((1,0),P)<J((0,1),P)\ \forall P>0$*, thus intersections between* $J((1,1),P)$ *and* $J((0,1),P)$ *do not affect the optimal scheduling, concluding the proof.*

Proof (Proposition 5)**.** *Denote* $P=(P_1,P_2)$*. The set of possible schedules is* $\{(0,0),\ (1,0),\ (0,1)\,(1,1)\}$*, thus we have:*

$$\begin{aligned}
J((0,0),P) &= A_1^2P_1+A_2^2P_2+Q_1+Q_2 \\
J((0,1),P) &= q_{01}A_2^2(P_2^{-1}+R_2^{-1})^{-1}+(1-q_{01})A_2^2P_2+A_1^2P_1+Q_1+Q_2+\mu \\
J((1,0),P) &= q_{10}A_1^2(P_1^{-1}+R_1^{-1})^{-1}+(1-q_{10})A_1^2P_1+A_2^2P_2+Q_1+Q_2+\mu \\
J((1,1),P) &= p_{11}\left(A_1^2(P_1^{-1}+R_1^{-1})^{-1}+A_2^2(P_2^{-1}+R_2^{-1})^{-1}\right)+p_{10}\left(A_1^2(P_1^{-1}+R_1^{-1})^{-1}+A_2^2P_2\right) \\
&\quad +p_{01}\left(A_2^2(P_2^{-1}+R_2^{-1})^{-1}+A_1^2P_1\right)+p_{00}(A_1^2P_1+A_2^2P_2)+Q_1+Q_2+2\mu
\end{aligned}$$

Consider

$$\begin{aligned}
\Delta J'(P) &:= J((1,0),\,P)-J((0,0),\,P) \\
&= q_{10}A_1^2(P_1^{-1}+R_1^{-1})^{-1}-q_{10}A_1^2P_1+\mu \\
&= -\frac{q_{10}A_1^2P_1^2-P_1\mu-R_1\mu}{P_1+R_1}.
\end{aligned}$$

Then if we solve the inequality $\Delta J'(P) < 0$ subject to the condition $P_1 > 0$ we see that $J((1,0),P) < J((0,0),P)$ for $P_1 \in [0, \underline{P_1})$ where $\underline{P}_1 > 0 \,:\, \Delta J'(P) = 0$. We can repeat the same analysis for $\Delta J''(P) := J((0,1),P) - J((0,0),P)$, proving the same results for $\underline{P}_2$. Consider

$$
\begin{aligned}
\Delta J(P) &:= J((1,1),\,P) - J((0,0),\,P) \\
&= (p_{11}+p_{10})A_1^2(P_1^{-1}+R_1^{-1})^{-1} - (p_{11}-p_{01})A_1^2P_1 + \mu(p_{11}+p_{01})A_2^2(P_2^{-1}+R_2^{-1})^{-1} - (p_{11}-p_{10})A_2^2P_2 + \mu \\
&> q_{10}A_1^2(P_1^{-1}+R_1^{-1})^{-1} - q_{10}A_1^2P_1 + \mu + q_{01}A_2^2(P_2^{-1}+R_2^{-1})^{-1} - q_{01}A_2^2P_2 + \mu \\
&= \Delta J'(P) + \Delta J''(P)
\end{aligned}
$$

where the inequality is due to Lemma 1. It follows that, in the region R_{00}, $J((1,1),P) > J((0,0),P)$. We can conclude that $U^*(P) = (0,0)$ if and only if $P \in R_{00}$. Taking the limit of P_1, with a slight abuse of notation

$$
\begin{aligned}
J((0,1),\,(\infty,P_2)) &= (1-q_{01})A_2^2P_2 \\
J((1,0),\,(\infty,P_2)) &= A^2P_2 \\
J((1,1),\,(\infty,P_2)) &= A^2(p_{00}+p_{10})P_2.
\end{aligned}
$$

Since $1-q_{01} < p_{00}+p_{10}$ by Lemma 1, by continuity, for $\forall P_1\ \exists \overline{P}_2(P_1) > 0$ s.t. if $P_2 > \overline{P}_2(P_1)$ then $U^*((P_1,P_2)) = (1,0)$. The same can be proved for $\overline{P}_1(P_2)$. The set where $U = (1,1)$ is optimal can be found imposing $J((1,1),\,P) - J((1,0),\,P) < 0$ and $J((1,1),\,P) - J((0,1),\,P) < 0$. Simple algebraic manipulations result in the set R_{11}. Now we compute the intersections of $J((1,1),P) - J((1,0),P) = 0$ and $J((1,0),P) - J((0,1),P) = 0$. Isolating P_2 from the second and plugging it in the first, we get

$$
(q_{10} - p_{11} - p_{10}(p_{11}+p_{01})\frac{q_{10}}{q_{01}})A_1^2P_1^2(P_1+R_1)^{-1} = -\mu
$$

The left hand side is monotonically decreasing w.r.t. P_1 and zero at $P_1 = 0$. It follows that there is a unique intersection. With the same reasoning, we show that the intersection of $J((1,1),P) - J((0,1),P) = 0$ and $J((1,0),P) - J((0,1),P) = 0$ is unique. Denote this point $\hat{P} = (\hat{P}_1,\hat{P}_2)$. Since $J((1,1),(\hat{P}_1,\hat{P}_2)) = J((0,1),(\hat{P}_1,\hat{P}_2))$ and $J((1,0),(\hat{P}_1,\hat{P}_2)) = J((0,1),(\hat{P}_1,\hat{P}_2))$, by construction, we have that $J((1,1),(\hat{P}_1,\hat{P}_2)) = J((1,0),(\hat{P}_1,\hat{P}_2))$, i.e. the two intersection points coincide, proving the claim.

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