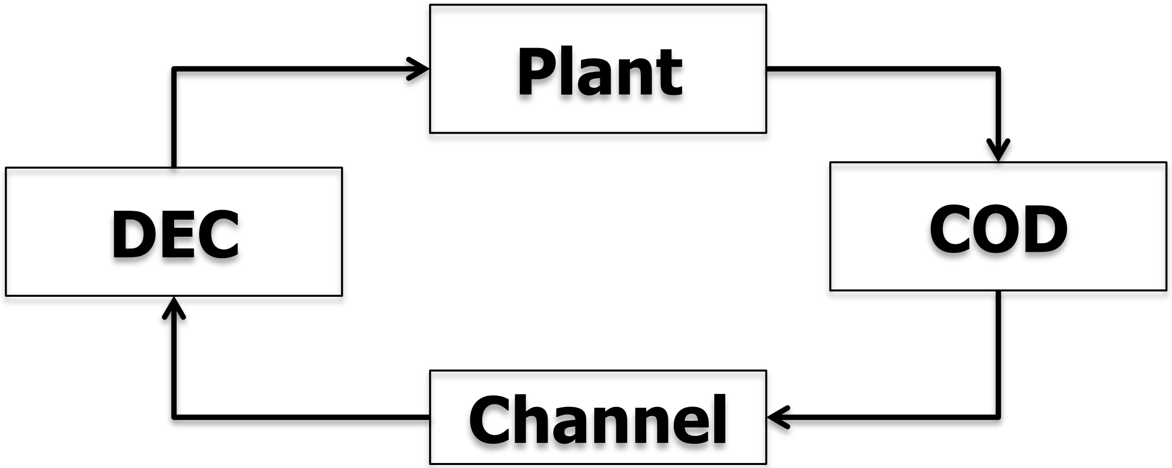
Control over finite capacity channels: the role of data loss, delay and signal-to-noise limitations



Luca Schenato
University of Padova

Control Seminars, Berkeley, 2014

M.Ag.I.C.
Multi Agent Intelligent Control

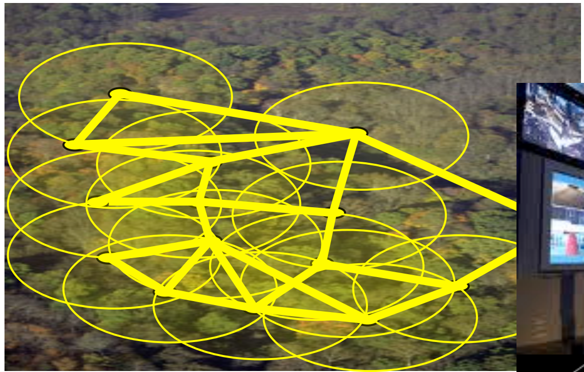
DEPARTMENT OF INFORMATION ENGINEERING
UNIVERSITY OF PADOVA

University of Padova

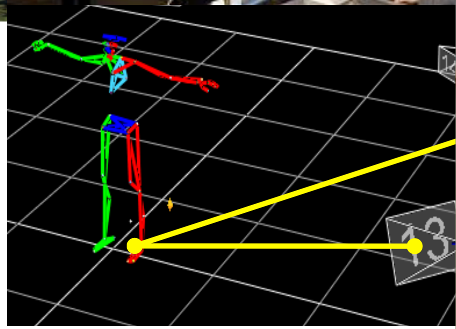
- Founded 1222: 2nd oldest university
- 60K students out of 200K citizens
- First Ph.d. woman in 1678: Elena Piscopia
- Alumni: Galileo, Copernicus, Riccati, Bernoulli
- Department of Information Engineering (EE&CS&BIOENG) 3K students

Applications: MAgIC Lab

Wireless Sensor Actuator Networks

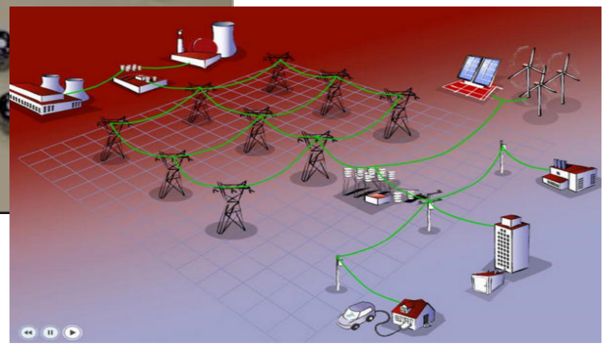


Smart Camera Networks



Robotic Networks

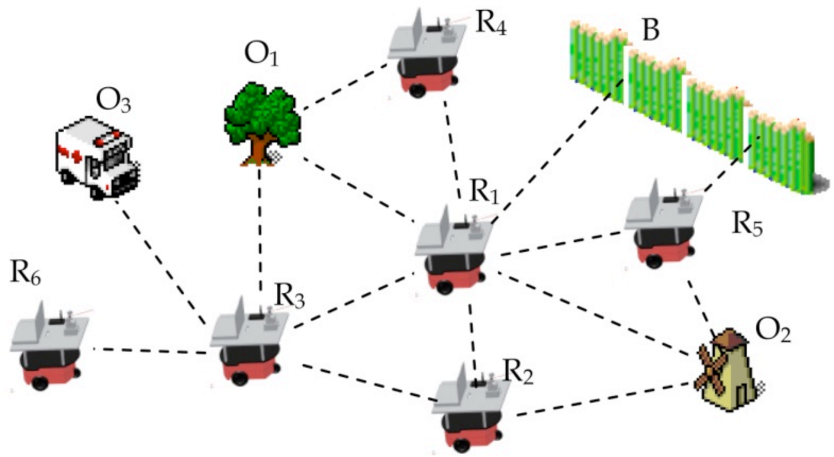
Smart Energy Grids



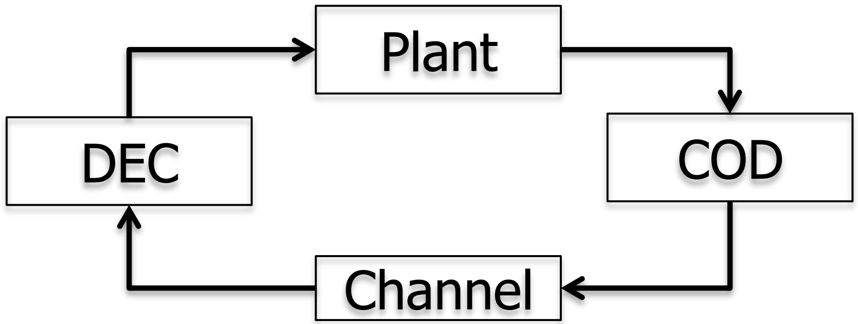
Networked Control Systems: physically distributed dynamical systems interconnected by a communication network

Research lines

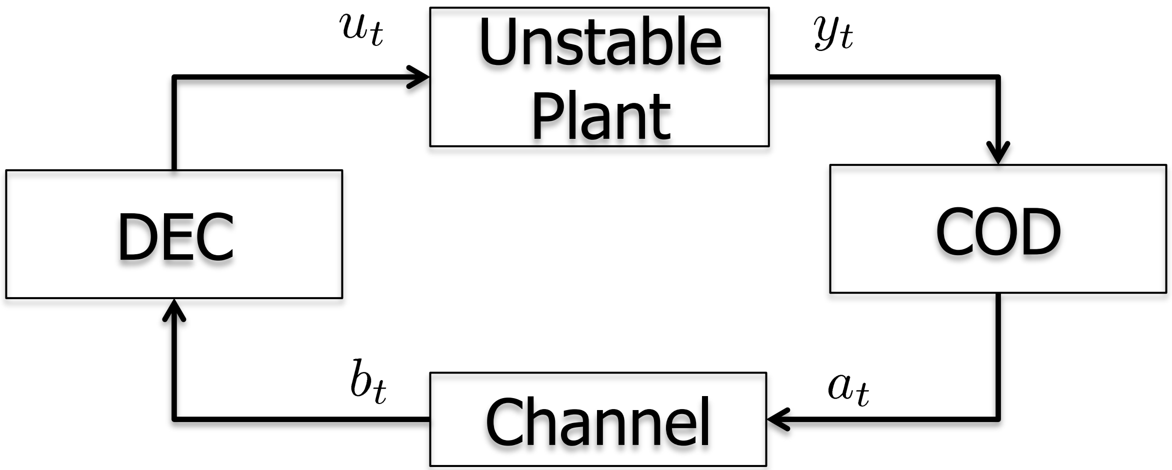
- Research line 1: multi-agent systems:
  - Consensus algorithms
  - Distributed estimation
  - Distributed optimization



- Research line 2: control subject to communication constraints:
  - Packet loss
  - Random delay
  - Sensor fusion



Motivation



Control Theory: unstable sources, perfect channels ,'60s
Communication/Information Theory: stable sources, realistic channels, '60s
Convergence of Control and Communication: unstable sources with realistic channels , '00s

Joint work with:

Alessandro Chiuso

Andrea Zanella

Nicola Laurenti

Subhrakanti Dey
Uppsala Univ., Sweden

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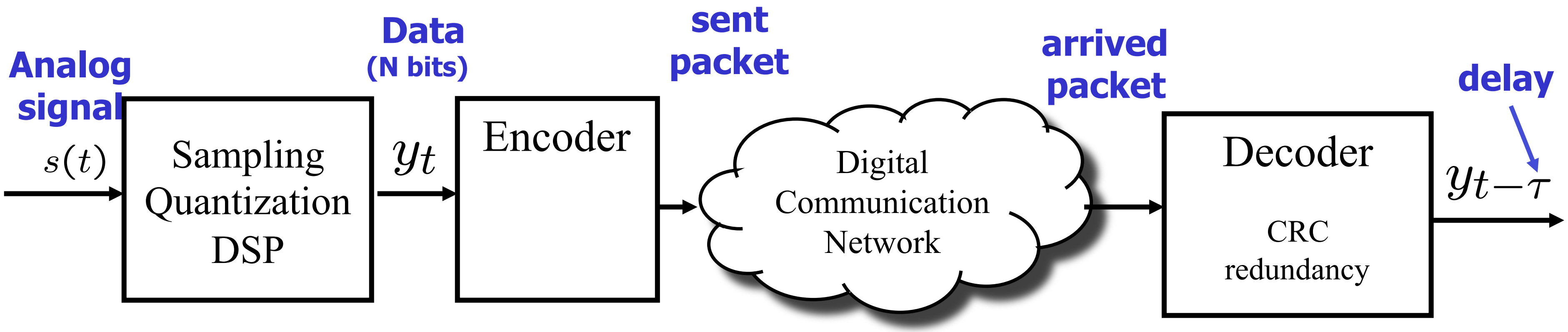
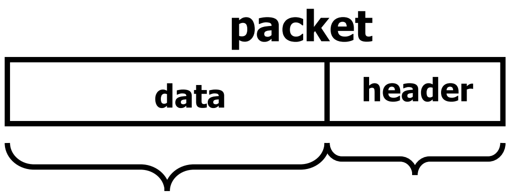
10 years ago in Berkeley….





	data	header
ATM	384 bits	40 bits
Ethernet	>368 bits	112 bits
Bluetooth	>499 bits	~100 bits
Zigbee	<1000 bits	128 bits

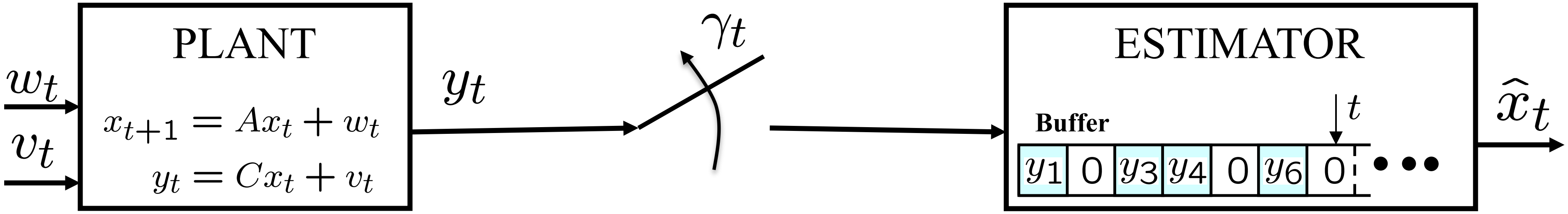
Assumptions:

(1) Quantization noise<<sensor noise
(2) Packet-rate limited (≠ bit-rate)
(3) No transmission noise (data corrupted=dropped packet)

Packet loss at receiver & Unit delay (τ=1)

10 years ago in Berkeley….

$$\hat{x}_t = \mathbb{E}[x_t | \{y_k\} \text{ available at estimator at time } t]$$



$$\gamma_t = \begin{cases} 1 & \textit{if } y_t \text{ received at time } t \\ 0 & \textit{otherwise} \end{cases}$$

$$\tilde{y}_t = \gamma_t(Cx_t + v_t) = C_t x_t + u_t$$

Time-varying Kalman filter

$$\hat{x}_t = \mathbb{E}[x_t \,|\, \tilde{y}_t, \ldots, \tilde{y}_t, \gamma_t, \ldots, \gamma_1]$$

10 years ago in Berkeley….

B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M.I. Jordan, and S. Sastry. **Kalman filtering with intermittent observations**. *IEEE Transactions on Automatic Control*, 49(9):1453–1464, September 2004

$$\begin{aligned} \widehat{x}_{t+1|t} &= A\widehat{x}_{t|t-1} + \gamma_t A K_t(y_t - C\widehat{x}_{t|t-1}) \\ K_t &= f(P_{t|t-1}) \\ P_{t+1|t} &= \Phi_{\gamma_t}(P_{t|t-1}) \end{aligned}$$

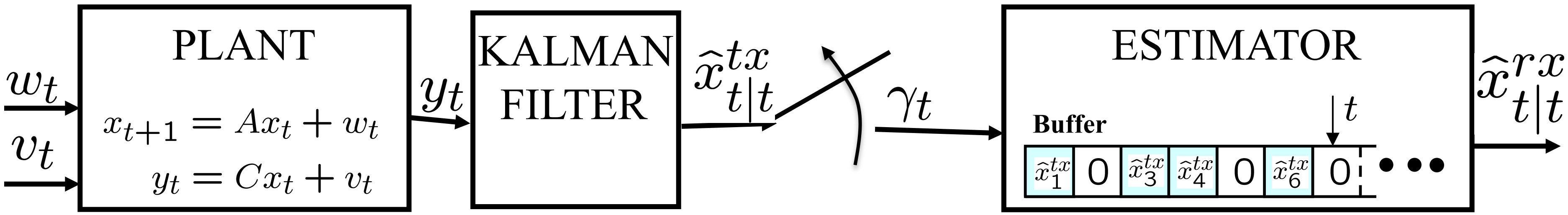
$$\Phi_\lambda(P) = APA^T + Q - \lambda\, APC^T(CPC^T + R)^{-1}CPA^T$$

Modified Algebraic Riccati Equation (MARE) (Φ_1(P)=ARE)

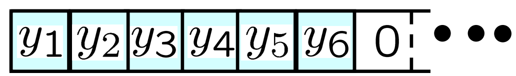
- Simple to understand but not trivial
- Critical packet loss probability function of eigenvalues of A
- Some new mathematical techniques
- Estimator designed for performance not only stability
- Many open questions remained unanswered

One open question

V. Gupta, D. Spanos, B. Hassibi, and R. M. Murray. **Optimal LQG control across a packet-dropping link.** *Systems and Control Letters*, 56(6):439–446, 2007



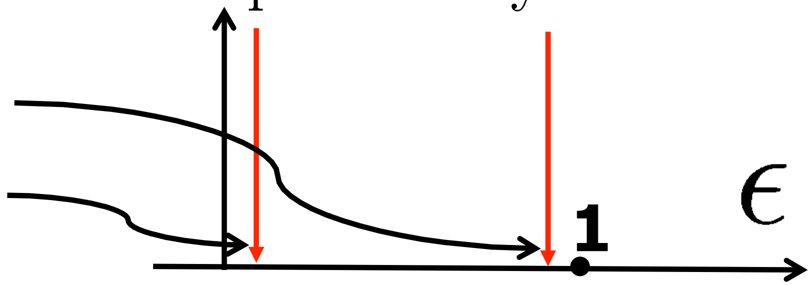
$$\hat{x}^{rx}_{t|t} = \begin{cases} \hat{x}^{tx}_{t|t} & \gamma_t = 1 \\ A\hat{x}^{rx}_{t-1|t-1} & \gamma_t = 0 \end{cases}$$



If $y \in \mathbb{R}, x \in \mathbb{R}^n$, then critical packet loss probability ϵ.

$\epsilon < \epsilon^c_x = \frac{1}{|\lambda_{max}(A)|^2}$: transmit $\hat{x}_t$

$\epsilon < \epsilon^c_y = \frac{1}{|\prod \lambda^u_i(A)|^2}$: transmit y_t



If n=10000 is it better to send the quantized state rather than the quantized measurement? ==> need to include quantization

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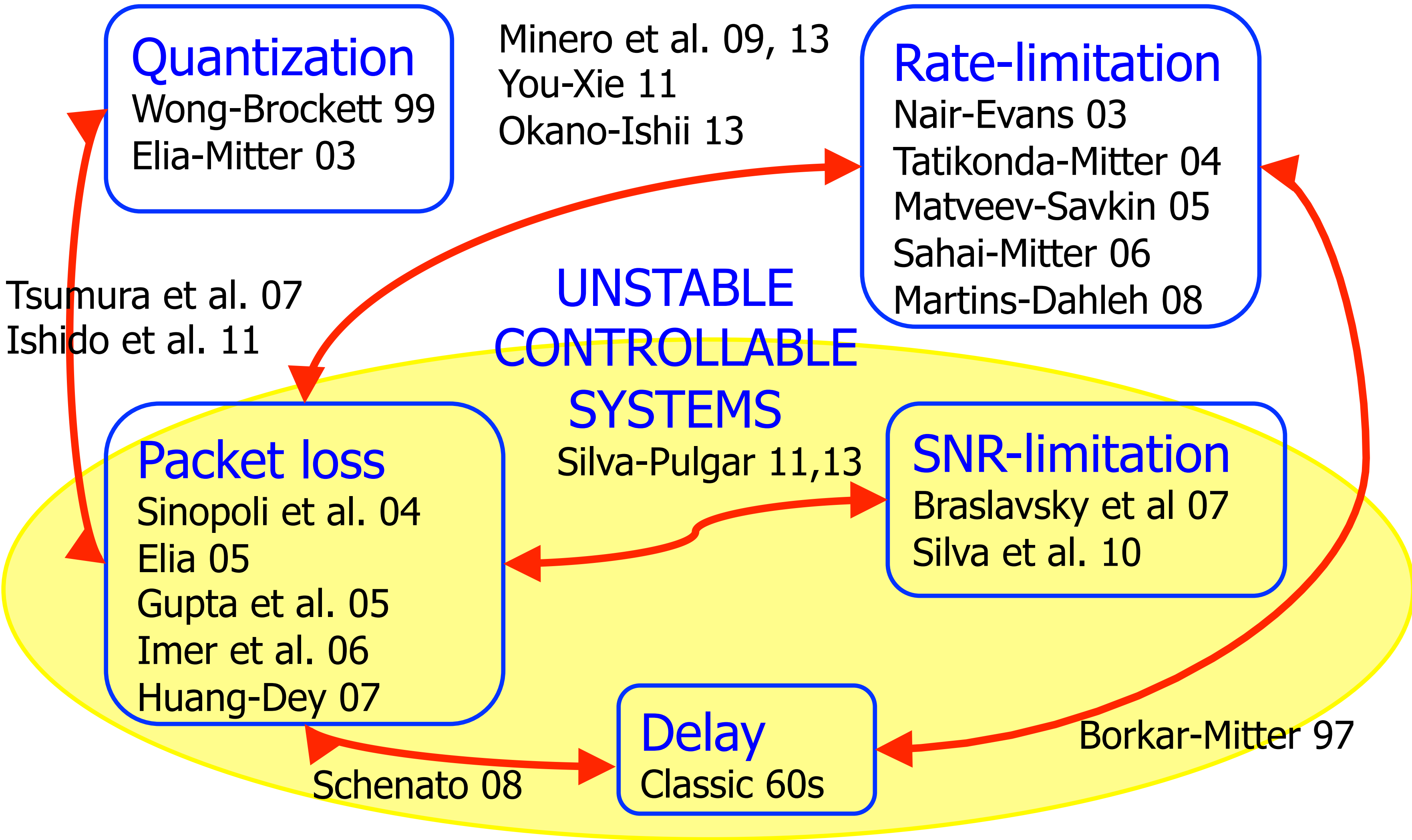
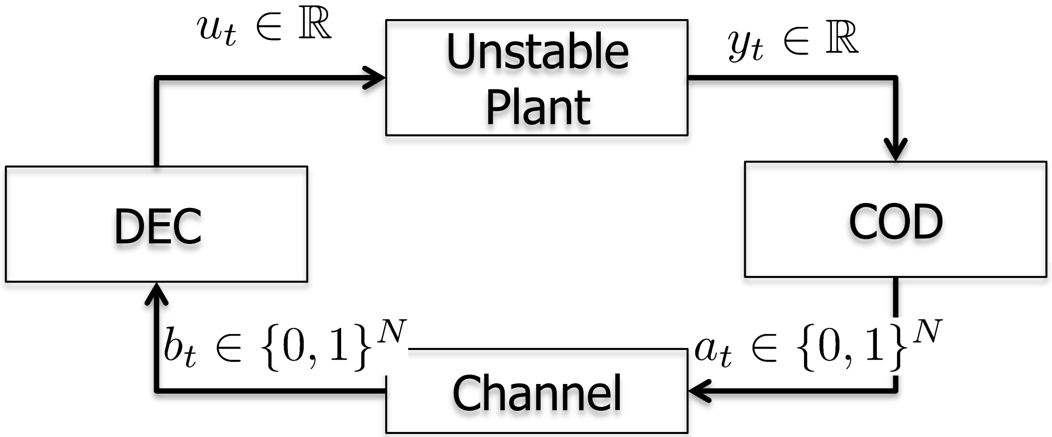
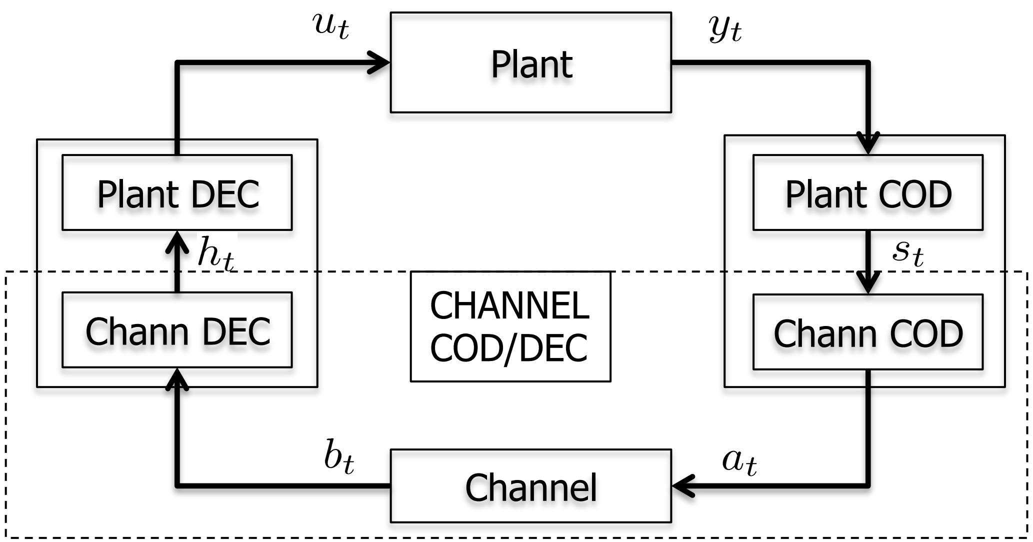
Previous work



Modeling



$$
\begin{aligned}
x_{t+1} &= a x_t + u_t + w_t \\
y_t &= x_t + v_t \\
|a| &> 1, \\
w_t &\sim N(0, \sigma_w^2), v_t \sim N(0, \sigma_v^2)
\end{aligned}
$$

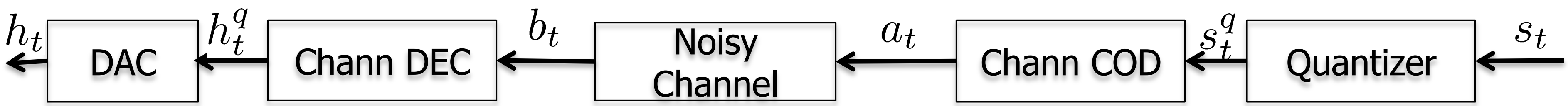


Proposed approach:

1) Separate control/estimation design from communication design.
2) Use of traditional coding with finite block-length (different from any-time coding of Sahai-Mitter 07 !!)

Ideally: $h_t \approx s_t \in \mathbb{R}$

About coding modeling



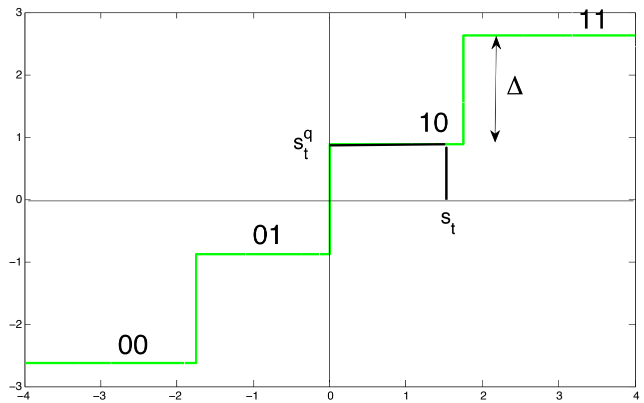
A naïve coding/decoding scheme:

[10]: symbol to be sent

[10|1]: add parity check bit

a_t=[111|000|111]: add redundancy

Noisy Channel: recovery via majority bits



RECEIVED (b_t)	RECOVERY	DECODED
[101|100|011]	[10|1]	correct decoding: [10] ($h_t^q = s_t^q$)
[111|110|111]	[11|1]	erasure
[111|000|001]	[10|0]	erasure
[100|110|111]	[01|1]	wrong decoding: [01] ($h_t^q \neq s_t^q$)

Receiver knows Δ and therefore maps [10] into the real number h_t

About coding modeling

Role of code lenght:

s_t^q=[10]: 2-bits of information per period

a_t=[111|000|111]: 9-bit word per period over the channel

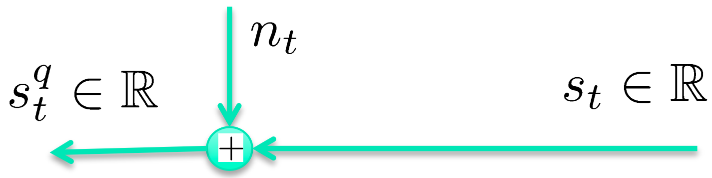
(s_t^q, s_{t-1}^q)=[11,10]->a_t=[xxx|xxx|xxx|xxx|xxx|xxx] smarter coding

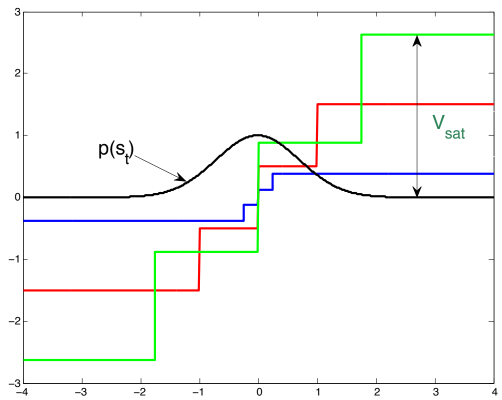
18-bit blocklength over 2 period => 9-bits/period

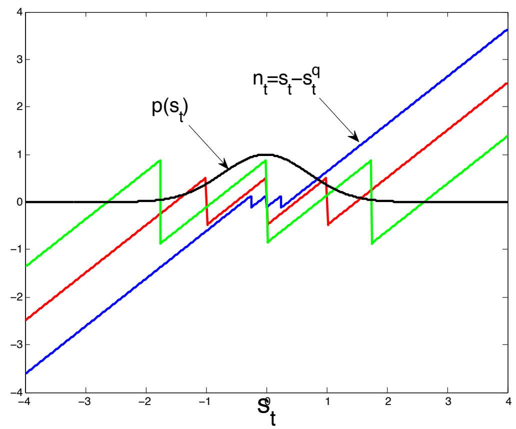
Longer block-length:

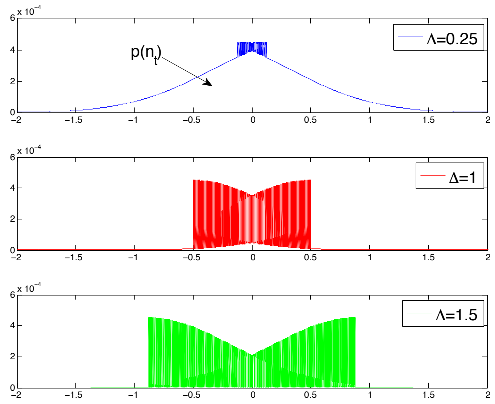
- Same channel rate (bits/period)
- Smaller erasure probability
- Larger delay

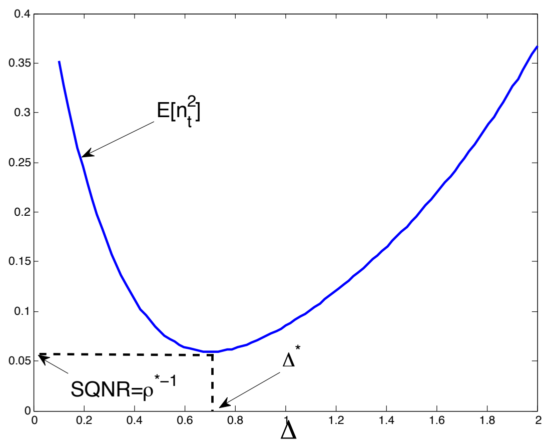
About quantization modeling











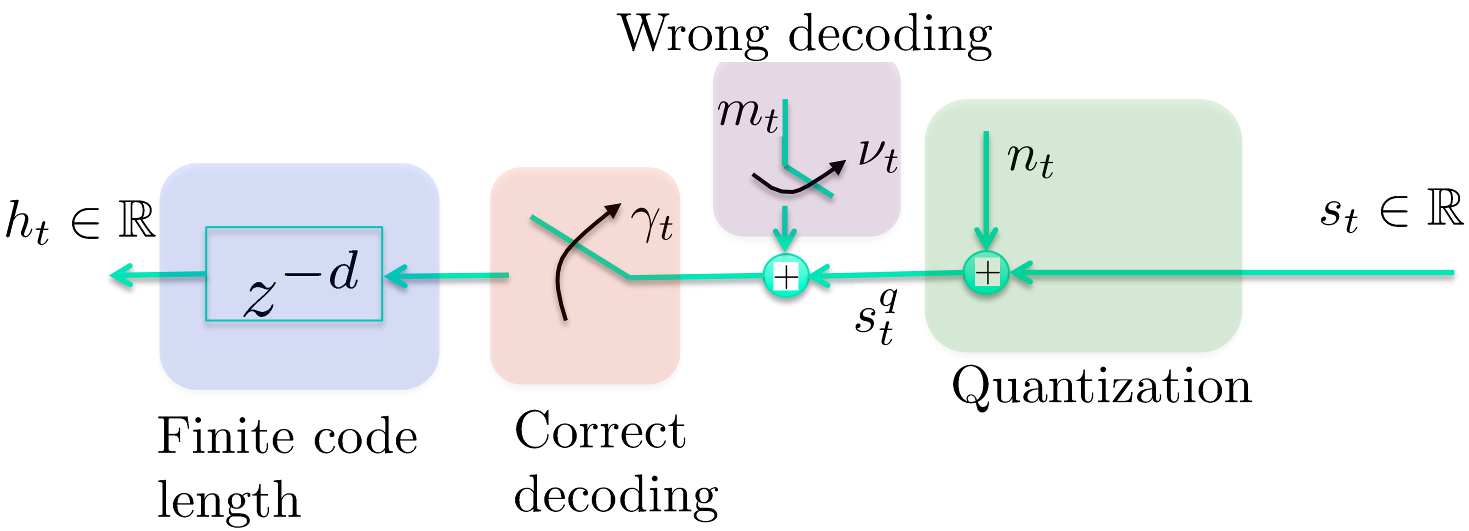
$$\mathbb{E}[n_t^2] = \frac{1}{\rho}\mathbb{E}[s_t^2],\ \rho\text{: SNR}$$

$$n_t \perp s_t \ ?$$

D. Marco and D. Neuhoff, "**The validity of the additive noise model for uniform scalar quantizers**," *IEEE Trans. Info. Theory*, vol. 51, no. 5, pp. 1739–1755, 2005

A. Leong, S. Dey, and G. Nair, "**Quantized filtering schemes for multi- sensor linear state estimation: Stability and performance under high rate quantization**," *IEEE Trans. Sig. Proc.*, vol. 61, no. 15, pp. 3852–3865, 2013.

"Analog" channel COD/DEC model



n_t: quantization noise

$\gamma_t = 0, \nu_t = \{0,1\}$: undecoded word (erasure)

$\gamma_t = 1, \nu_t = 0$: correctly decoded word

$\gamma_t = 1, \nu_t = 1$: wrongly decoded word

d: decoding delay (integer)

$P[\gamma_t = 0] = \varepsilon$: erasure probability

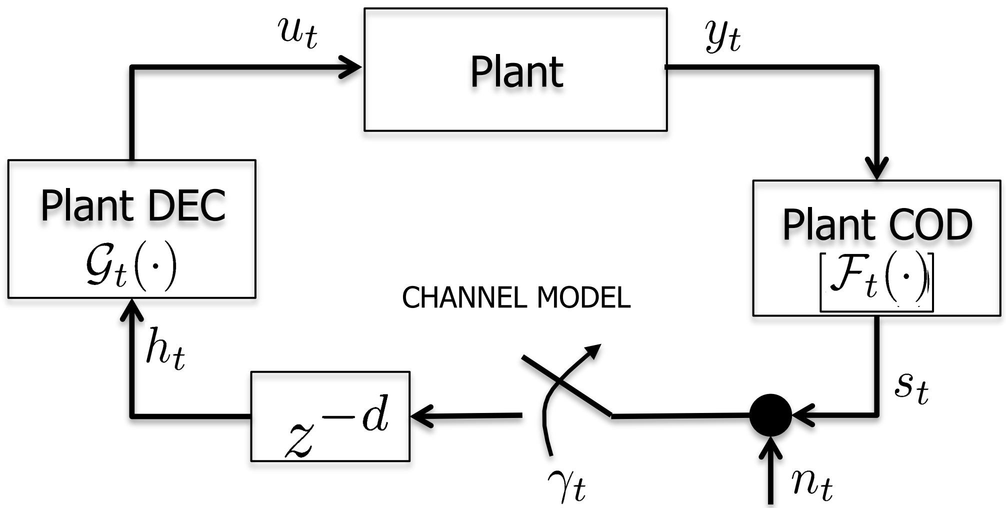
$P[\nu_t = 1] = \varepsilon_w$: undetected error probability

$\varepsilon_w \ll \varepsilon$

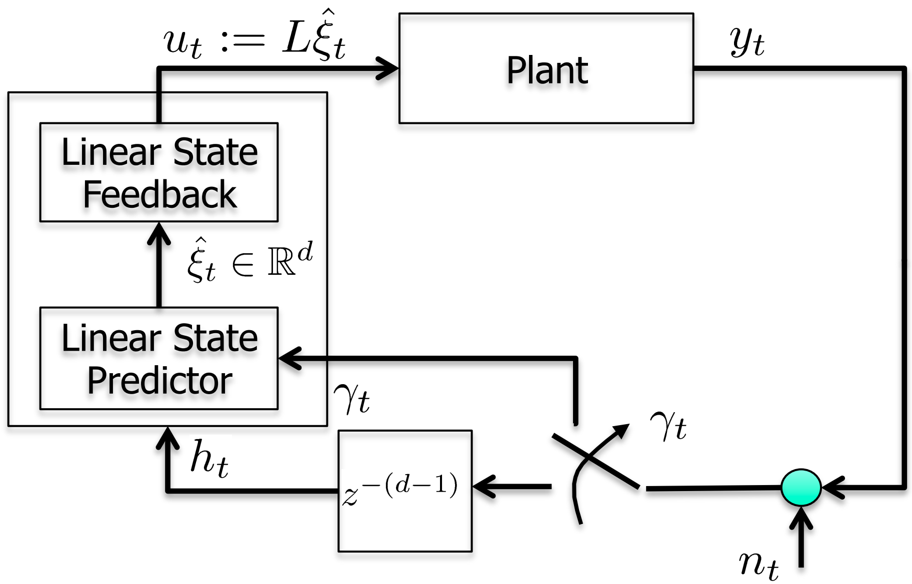
$E[n_t^2] = \frac{1}{\rho} E[s_t^2]$, ρ: SNR

$E[m_t^2] \approx E[s_t^2]$

Problem formulation

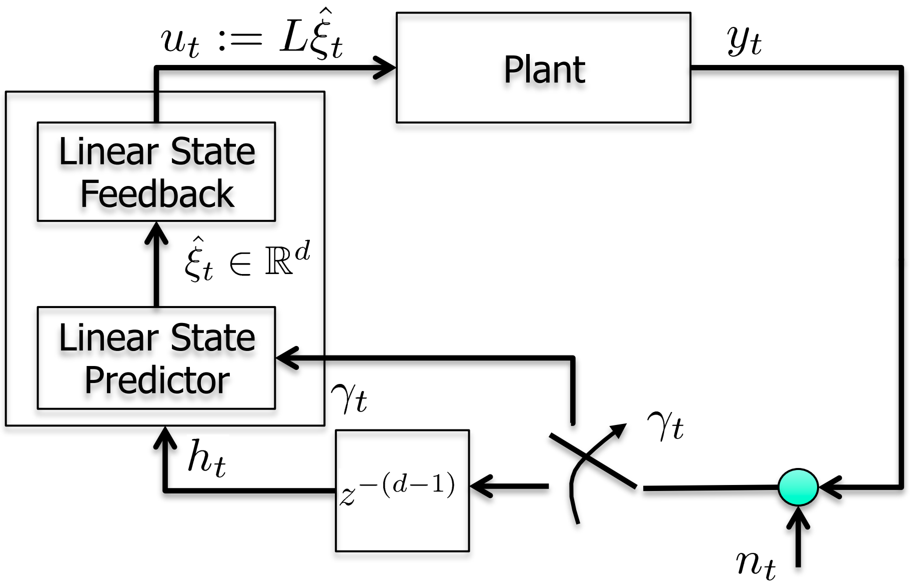


$$
\begin{aligned}
x_{t+1} &= a x_t + u_t + w_t \\
y_t &= x_t + v_t
\end{aligned}
$$



1. Scalar dynamics
2. No transmission pre-processing
3. Estimator+ state feedback architecture

Problem formulation (cont'd)



$$
\begin{aligned}
x_{t+1} &= a x_t + u_t + w_t \\
y_t &= x_t + v_t
\end{aligned}
$$

Augmented System dynamics

$$
\underbrace{\begin{bmatrix} x_{t-d+2} \\ \vdots \\ \vdots \\ x_{t+1} \end{bmatrix}}_{\xi_{t+1}} = \underbrace{\begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & \ddots & 1 \\ 0 & \cdots & 0 & a \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_{t-d+1} \\ \vdots \\ \vdots \\ x_t \end{bmatrix}}_{\xi_t} + \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{B} (u_t + w_t)
$$

$$
\begin{aligned}
y_t &= \underbrace{\begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}}_{C} \xi_t + v_t \\
h_t &= \gamma_{t-d+1}(\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}}_{H} \xi_t + v_{t-d+1} + n_{t-d+1})
\end{aligned}
$$

Linear estimator + linear controller

$$
\begin{aligned}
\hat{\xi}_{t+1} &= A\hat{\xi}_t + Bu_t + \gamma_{t-d+1} G (h_t - H\hat{\xi}_t) \\
u_t &= L\hat{\xi}_t
\end{aligned}
$$

LQG performance optimization

$$
\begin{aligned}
(G^*, L^*) &:= \text{argmin}_{G,L} \mathbb{E}[y_t^2] + r\mathbb{E}[u_t^2] \\
\text{s.t.} &\quad \mathbb{E}[n_t^2] = \frac{1}{\rho}\mathbb{E}[y_t^2], \quad n_t \perp y_t
\end{aligned}
$$

Problem solution

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Augmented System dynamics

$$\begin{aligned} \xi_{t+1} &= A\xi_t + B(u_t + w_t) \\ y_t &= C\xi_t + v_t \\ h_t &= \gamma_{t-d+1} H(\xi_t + v_{t-d+1} + n_{t-d+1}) \end{aligned}$$

Linear estimator + linear controller

$$\begin{aligned} \hat{\xi}_{t+1} &= A\hat{\xi}_t + Bu_t + \gamma_{t-d+1} G(h_t - H\hat{\xi}_t) \\ u_t &= L\hat{\xi}_t \end{aligned}$$

LQG performance optimization

$$\begin{aligned} (G^*, L^*) &:= \arg min_{G,L} J(G,L) = \mathbb{E}[y_t^2] + r\mathbb{E}[u_t^2] \\ \text{s.t.} \quad & \mathbb{E}[n_t^2] = \alpha \mathbb{E}[y_t^2] \end{aligned}$$

$$P := \text{Var}\left\{ \begin{bmatrix} \hat{\xi}_t \\ \xi_t - \hat{\xi}_t \end{bmatrix} \right\}$$

$$\begin{aligned} min_{G,L} \quad & J(P,G,L) \\ \text{s.t.} \quad & P = \mathcal{M}(P,G,L) \end{aligned}$$

J and $\mathcal{M}$: linear in P

"quadratic" in G, L

$$P = \underbrace{(1-\epsilon)\bar{A}_1 P \bar{A}_1^\top + \epsilon \bar{A}_0 P \bar{A}_0^\top + \sigma_w^2 \bar{B}\bar{B}^\top + \alpha(1-\epsilon)\bar{G}\bar{C}P\bar{C}^\top\bar{G}^\top + (1-\epsilon)(1+\alpha)\bar{G}\sigma_v^2\bar{G}^\top}_{\mathcal{M}(P,G,L)}$$

Problem solution

Solve via Lagrangian

$$\begin{array}{ll} min_{P,\Lambda,G,L} & J(P,G,L) + \mathrm{trace}\big(\Lambda(P - \mathcal{M}(P,G,L))\big) := \mathcal{L}(P,\Lambda,G,L) \\ \text{s.t.} & P \geq 0, \Lambda \geq 0 \end{array}$$

Necessary optimal conditions

$$\frac{\partial \mathcal{L}}{\partial P} = 0\,, \quad \frac{\partial \mathcal{L}}{\partial \Lambda} = 0\,, \quad \frac{\partial \mathcal{L}}{\partial L} = 0\,, \quad \frac{\partial \mathcal{L}}{\partial G} = 0$$

Coupled Riccati-like Equations

$$\begin{aligned} P &= \Phi_1(P,\Lambda) \\ \Lambda &= \Phi_2(P,\Lambda) \\ G &= \Psi_1(P) \\ L &= \Psi_2(\Lambda) \end{aligned}$$

W.L. De Koning. **Compensatability and optimal compensation of systems with white parameters**. *IEEE Transactions on Automatic Control*, 37(5):579–588, 1992

Further simplification

$$\underbrace{\begin{bmatrix} x_{t-d+2} \\ \vdots \\ \vdots \\ x_{t+1} \end{bmatrix}}_{\xi_{t+1}} = \underbrace{\begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & \ddots & 1 \\ 0 & \cdots & 0 & a \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_{t-d+1} \\ \vdots \\ \vdots \\ x_t \end{bmatrix}}_{\xi_t} + \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{B} (u_t + w_t)$$

$$y_t = \underbrace{\begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}}_{C} \xi_t + v_t$$

$$h_t = \gamma_{t-d+1}(\underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}}_{H} \xi_t + v_{t-d+1} + n_{t-d+1})$$

Coupled Riccati-like Equations

$$\begin{aligned} P &= \Phi_1(P, \Lambda) \\ \Lambda &= \Phi_2(P, \Lambda) \\ G &= \Psi_1(P) \\ L &= \Psi_2(\Lambda) \end{aligned}$$

$$\begin{aligned} L &= \begin{bmatrix} 0 & 0 & \cdots & 0 & \ell \end{bmatrix} \\ G &= \begin{bmatrix} g & ag & \cdots & a^{d-1}g \end{bmatrix}^T \end{aligned}$$

For $r = 0$ problem equivalent to the solution of a scalar Riccati-like equation:

$$p = a^2 p + \sigma_w^2 - \delta \frac{a^2 p^2}{p^* + \bar{r}(d)}$$

$$\delta := \frac{1-\epsilon}{1+\alpha a^{2d}}$$

Further simplification

$$p = a^2 p + \sigma_w^2 - \delta \frac{a^2 p^2}{p + \bar{r}(d)}$$

$$\delta := \frac{1-\epsilon}{1+\alpha a^{2d}}$$

B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M.I. Jordan, and S. Sastry. **Kalman filtering with intermittent observations**. *IEEE Transactions on Automatic Control*, 49(9):1453–1464, September 2004

Necessary and sufficient stability for r≥0:

$$\frac{1-\epsilon}{1+\alpha a^{2d}} > 1 - \frac{1}{a^2}$$

d: decoding delay

ϵ: erasure probability

$\alpha = \frac{1}{SNR}$: noise-to-signal ratio

Discussion w/ related works

$$\frac{1-\epsilon}{1+\alpha a^{2d}} > 1 - \frac{1}{a^2}$$

1) Infinite resolution (α=0) and no delay (d=0):

$$1 - \epsilon > 1 - \frac{1}{a^2}$$

B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M.I. Jordan, and S. Sastry. **Kalman filtering with intermittent observations**. *IEEE Transactions on Automatic Control*, 49(9):1453–1464, September 2004

2) Infinite resolution (α=0) and with delay (d>0):

$$1 - \epsilon > 1 - \frac{1}{a^2}$$

L. Schenato. **Kalman filtering for networked control systems with random delay and packet loss**. *IEEE Transactions on Automatic Control*, 53:1311–1317, 2008

3) No packet loss (ε =0) and no delay (d>0):

$$SNR = \frac{1}{\alpha} > a^2 - 1$$

J.H. Braslavsky, R.H. Middleton, and J.S. Freudenberg. **Feedback stabilization over signal-to-noise ratio constrained channels**. *IEEE Transactions on Automatic Control*, 52(8), 2007

Recalling the rate $R = \frac{1}{2}\log(1 + SNR)$ and $R < \mathcal{C}$:

$$\mathcal{C} > \log|a|$$

S. Tatikonda and S. Mitter. **Control under communication constraints**. *IEEE Transaction on Automatic Control*, 49(7):1056–1068, July 2004.

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Discussion w/ related works

$$\frac{1-\epsilon}{1+\alpha a^{2d}} > 1 - \frac{1}{a^2}$$

4) No packet loss (ε=0) and delay (d=1):

$$SNR = \frac{1}{\alpha} > a^4 - a^2$$

J.H. Braslavsky, R.H. Middleton, and J.S. Freudenberg. **Feedback stabilization over signal-to-noise ratio constrained channels**. *IEEE Transactions on Automatic Control*, 52(8), 2007

5) Infinite resolution (α=0), packet loss as SNR-limitation + delay

$$\frac{1-\epsilon}{1+\epsilon(a^{2d}-1)} > 1 - \frac{1}{a^2}$$

E.I. Silva and S.A. Pulgar. **Performance limitations for single-input LTI plants controlled over SNR constrained channels with feedback**. *Automatica*, 49(2), 2013

$$1 - \epsilon > 1 - \frac{1}{a^2}$$

Our condition less stringent and independent of delay

6) Rate-limited with delay (d=1):

$$R = \frac{1}{2}\log(1 + SNR)$$

$$\mathbb{E}\left[\left(\frac{a^2}{2^{2R_t}}\right)^n\right] < 1$$

$$R_t = R\delta_t, \delta_t \sim \mathcal{B}(1-\epsilon)$$

$$\frac{a^2}{1+\rho}(1-\epsilon) + a^2\epsilon < 1$$

P. Minero, L. Coviello, and M. Franceschetti**. Stabilization over Markov feedback channels: The general case**. *Transactions on Automatic Control*, 58(2):349–362, 2013

Discussion w/ related works

$$\frac{1-\epsilon}{1+\alpha a^{2d}} > 1 - \frac{1}{a^2}$$

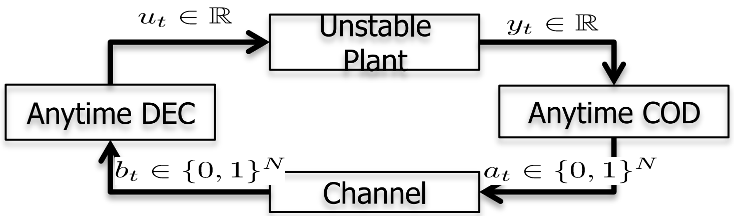
6) Down-sampling: equivalent to a $\longleftarrow$ a^d, d $\longleftarrow$ 1

$$\frac{1-\epsilon}{1+\alpha a^{2d}} > 1 - \frac{1}{a^{2d}}$$

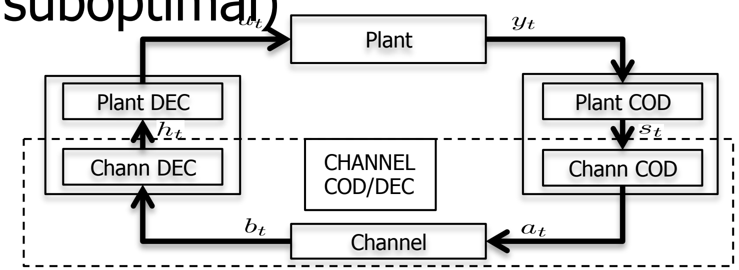
More stringent constraint

7) Relation with sequential coding (any-time capacity)

Anytime coding/decoding



Fixed-length codes (our approach is suboptimal)



Necessary for optimality:

A. Sahai and S. Mitter. **The necessity and sufficiency of anytime capacity for control over a noisy communication link: Part I**. *IEEE Transaction on Information Theory*, 2006

What is the role of capacity?

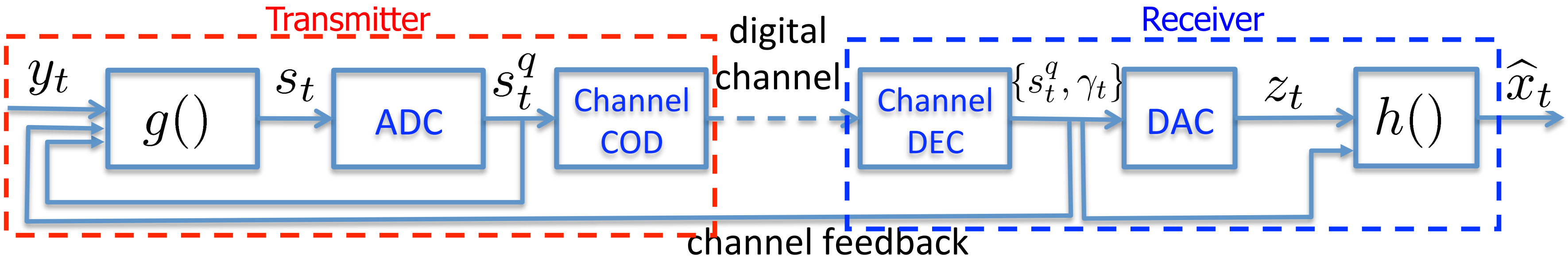
$SNR(=\frac{1}{\alpha}), d, \epsilon$ are not abritrary but are function of the channel

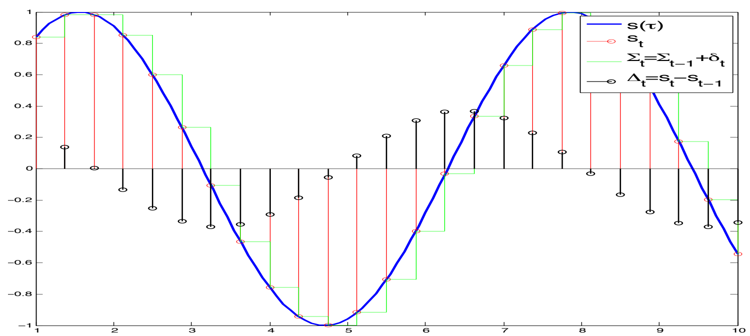
$$
\begin{array}{rcl}
|a^*(\mathcal{C})| & := & \max_{a,\alpha,d,\epsilon} |a| \\
 & s.t. & \frac{1-\epsilon}{1+\alpha a^{2d}} > 1 - \frac{1}{a^2} \\
 & & (\alpha, d, \epsilon) \in \Omega(\mathcal{C})
\end{array}
$$

Feasible set which depends on channel parameters

Y. Polyanskiy, H.V. Poor, and S. Verdu. **Channel coding rate in the finite blocklength regime.** *IEEE Transactions on Information Theory*, 56(5):23072359, 2010

Remote estimation subject to quantization and packet loss





"Delta-Sigma" modulation:

$\Delta_t = y_t - y_{t-1}$ at the transmitter

$\Sigma_t = \Sigma_{t-1} + \Delta_t$ at the receiver

If $\Sigma_0 = y_0$ then $\Sigma_t = y_t$ for all t

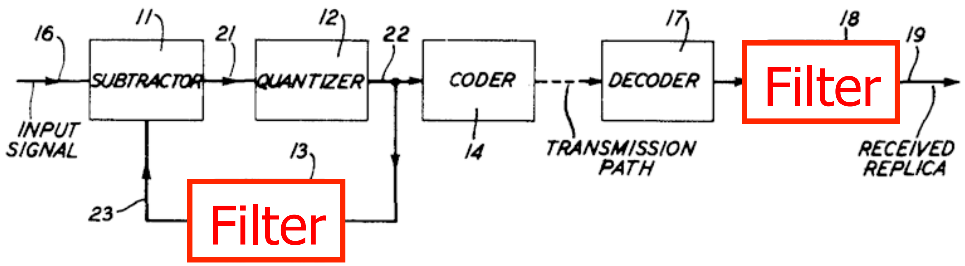
Differential pulse-code modulation (DPCM)

July 29, 1952 C. C. CUTLER 2,605,361

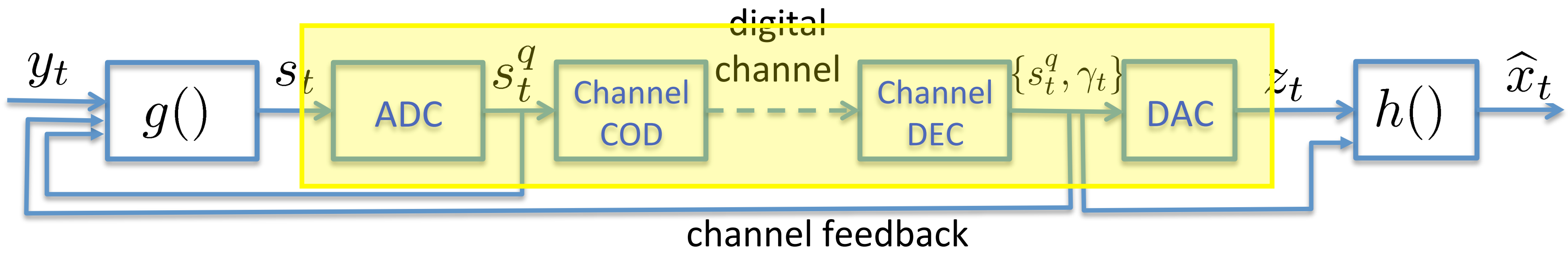
DIFFERENTIAL QUANTIZATION OF COMMUNICATION SIGNALS

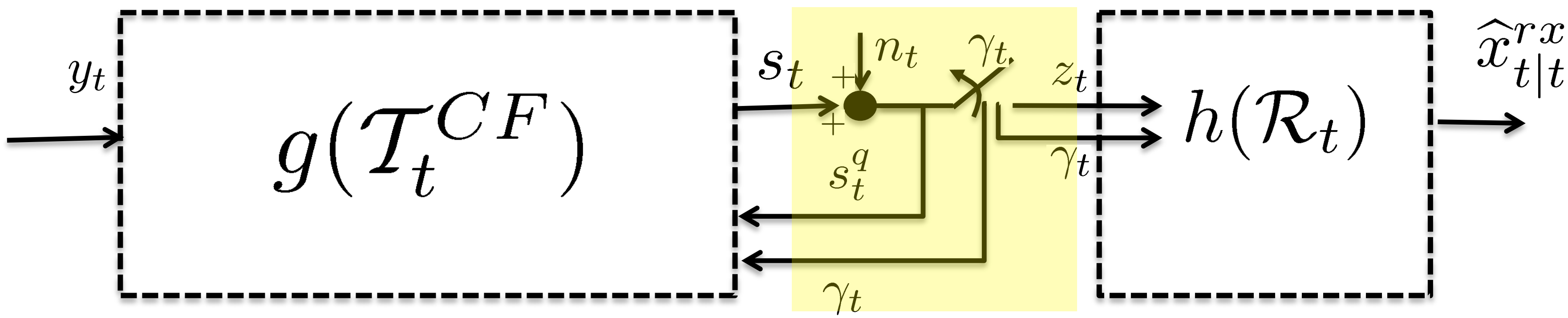
Filed June 29, 1950 3 Sheets-Sheet 1

FIG. 1



Remote estimation subject to quantization and packet loss





Information set with channel feedback (ACK/NACK)

$$\mathcal{T}_t^{CF} = \{y_t, .., y_0, s_{t-1}, .., s_0, n_{t-1}, .., n_0, \gamma_{t-1}, .., \gamma_0\}$$

Information set without channel feedback (ACK/NACK)

$$\mathcal{T}_t^{NCF} = \{y_t, .., y_0, s_{t-1}, .., s_0, n_{t-1}, .., n_0\}$$

Information set at receiver

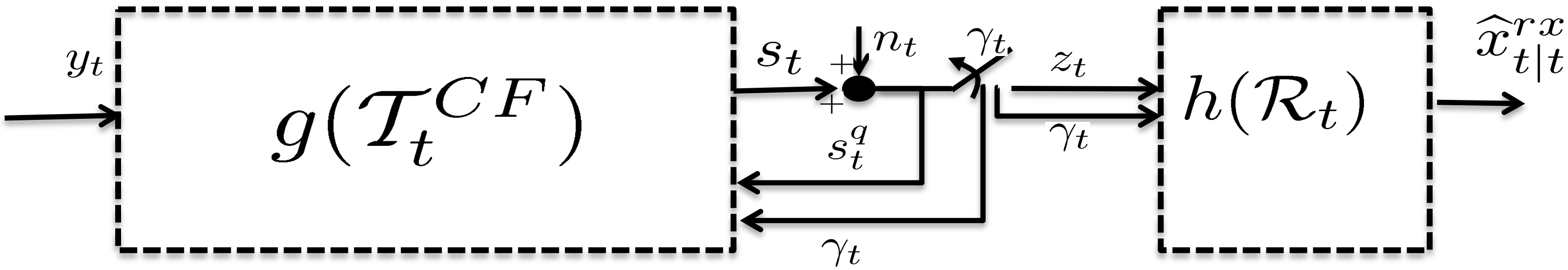
$$\mathcal{R}_t := \{z_t, \ldots, z_0, \gamma_t, \ldots \gamma_0\}$$

Goal: minimize error variance $\mathbb{E}[(x_{t+1} - \hat{x}_{t+1|t}^{rx})^2]$

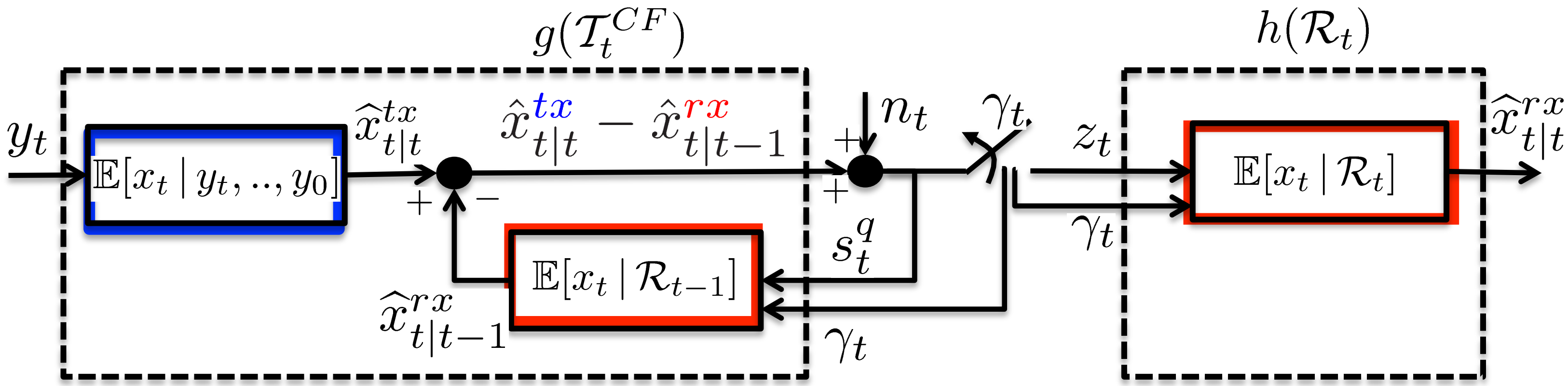
What is the optimal strategy with channel feedback ?

$$
\begin{aligned}
x_{t+1} &= a x_t + w_t, \quad \text{scalar system} \\
y_t &= x_t + v_t \\
|a| &< 1, \quad \text{stable source}
\end{aligned}
$$

$$\mathcal{T}_t^{CF} \supset \mathcal{R}_{t-1}$$



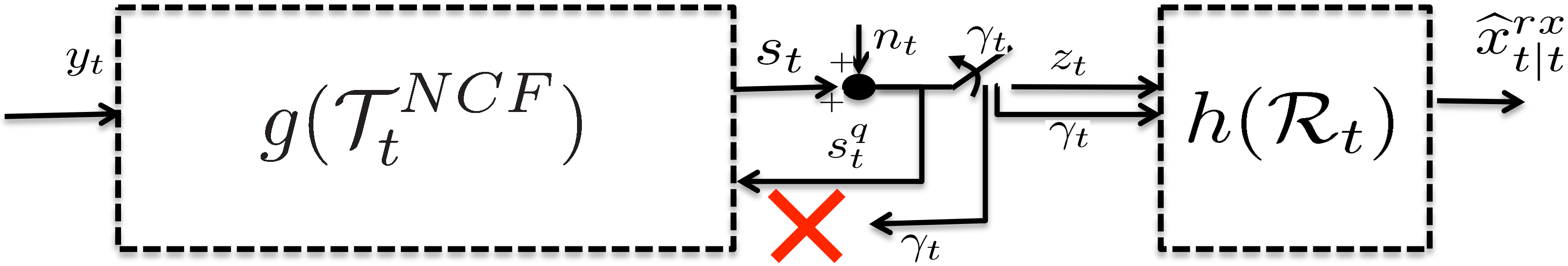
Optimal strategy (among linear strategies): send innovation



What is the optimal strategy with no channel feedback ?

$$
\begin{aligned}
x_{t+1} &= a x_t + w_t, \quad \text{scalar system} \\
y_t &= x_t + v_t \\
|a| &< 1, \quad \text{stable source}
\end{aligned}
$$

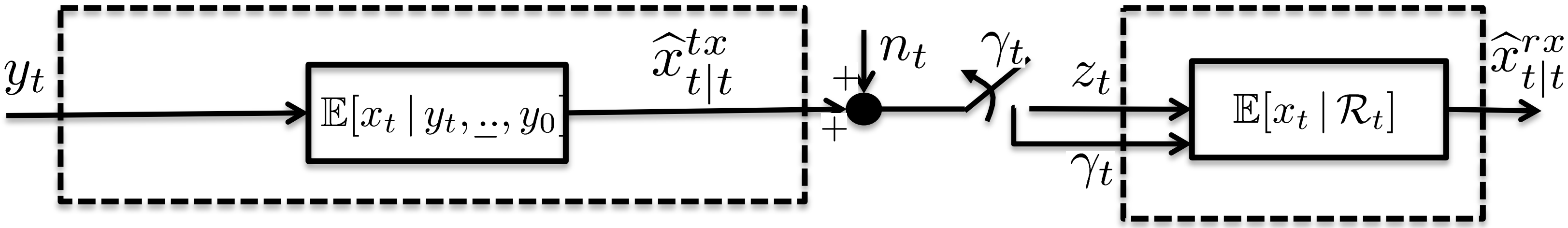
$$\mathcal{T}_t^{NCF} \not\supset \mathcal{R}_{t-1}$$



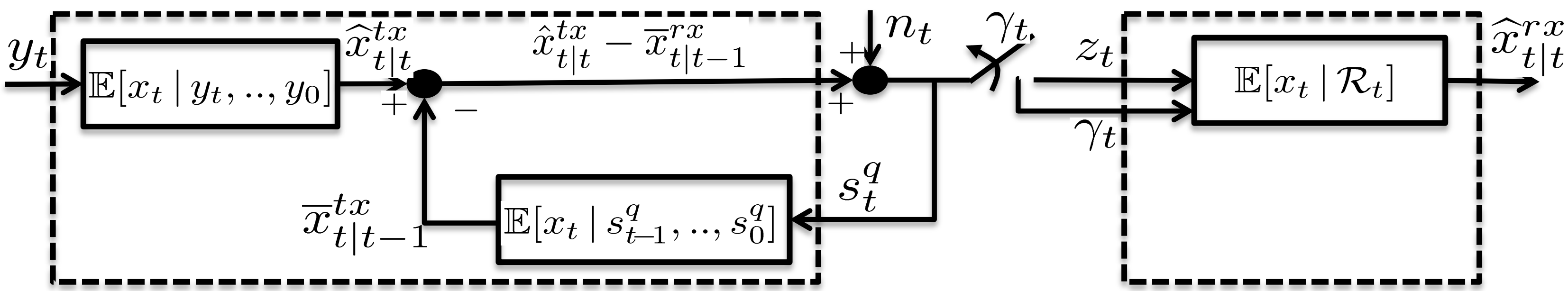
Optimal strategy ? not clear, likely non-linear
Approach: reasonable suboptimal strategies

Suboptimal strategies

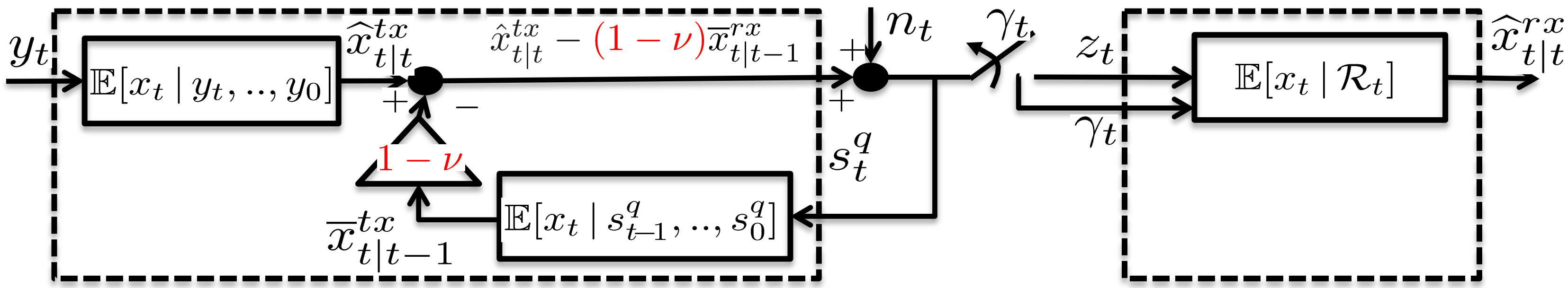
1) Estimated state forwarding (Kalman estimate)



2) Innovation forwarding assuming no packet loss

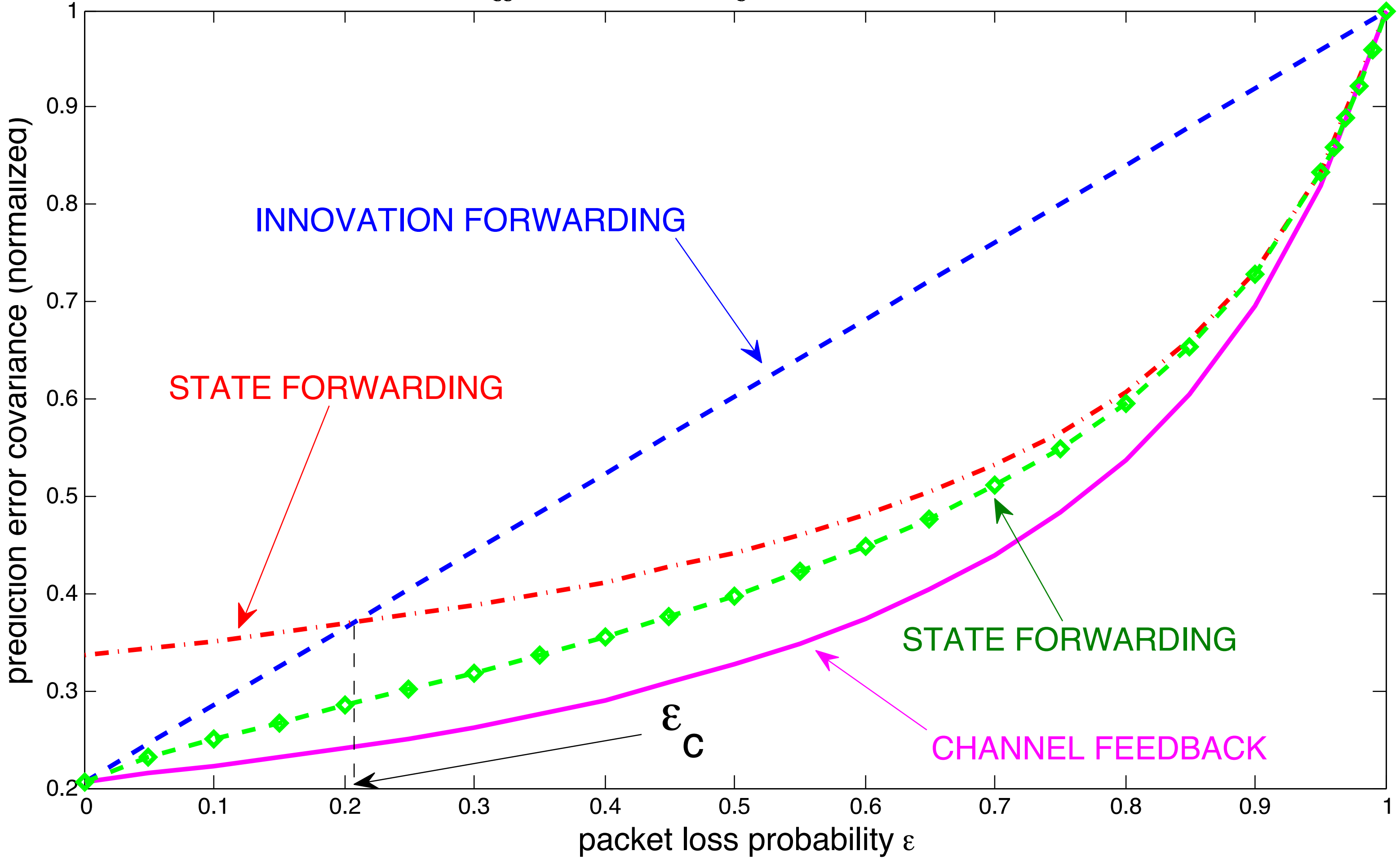


3) Hybrid strategy: soft innovation forwarding



Analytical results

$$SNR = 3, \sigma_w^2 = 0.1, \sigma_v^2 = 0.05, a = 0.95$$

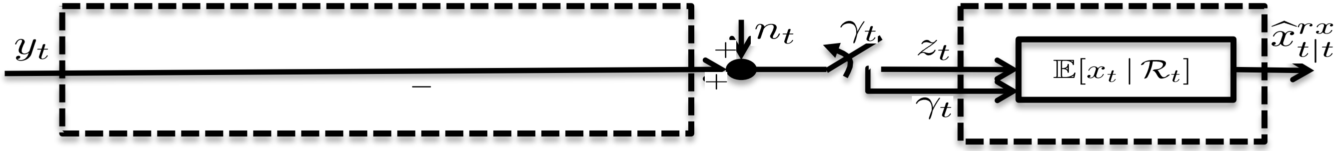


For any choice of parameters $\epsilon_c < 0.5$

A unexpected result

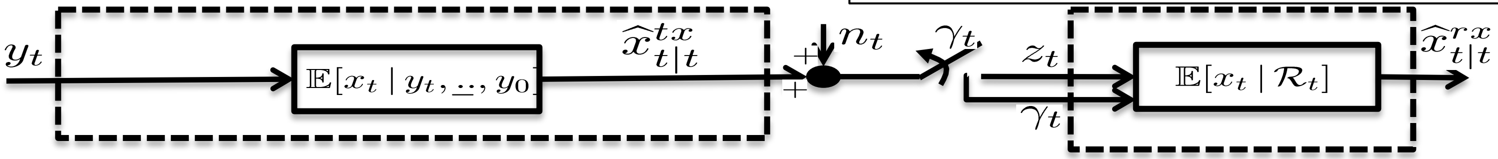
1) Measurement forwarding

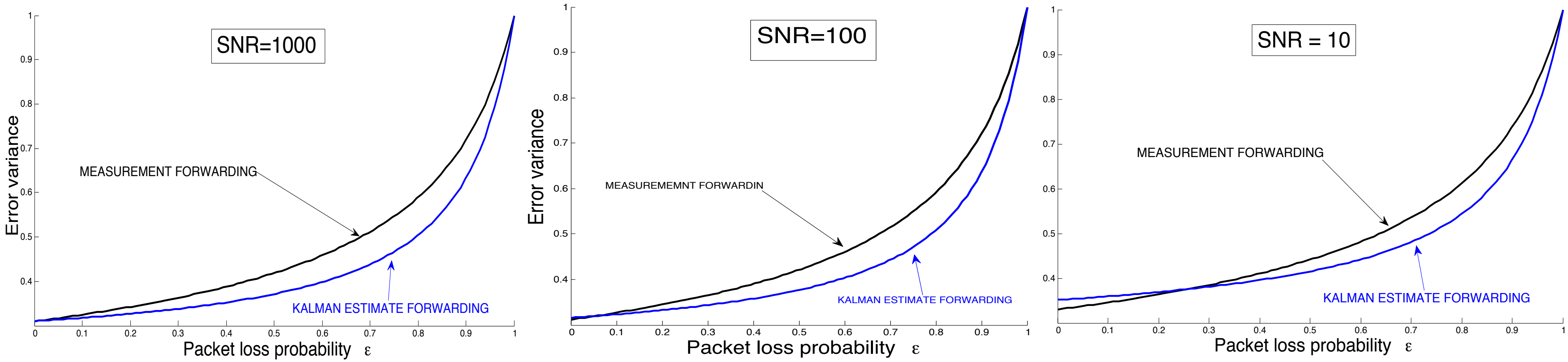
B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M.I. Jordan, and S. Sastry. **Kalman filtering with intermittent observations**. *IEEE Transactions on Automatic Control*, 49(9):1453–1464, September 2004



2) Kalman estimate forwarding

V. Gupta, D. Spanos, B. Hassibi, and R. M. Murray. **Optimal LQG control across a packet-dropping link.** *Systems and Control Letters*, 56(6):439–446, 2007





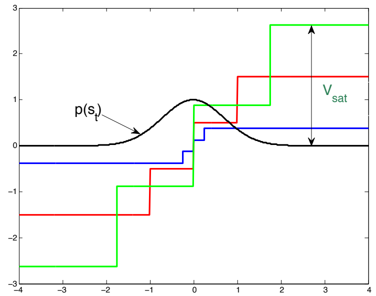
Takehome messages

- "Analog" model that takes into account rate-limitation, delay and packet loss
- Stability is often useless without performance
- Available information at receiver/transmitter has heavy impact on estimator/controller design
- Some unexpected results when SNR and packet loss are jointly considered

(Many) Open Problems

- Characterization of $\Omega(\mathcal{C})$
- Adaptive scaling for quantization to enforce

$$\mathbb{E}[n_t^2] = \alpha \mathbb{E}[s_t^2], \quad \forall t$$



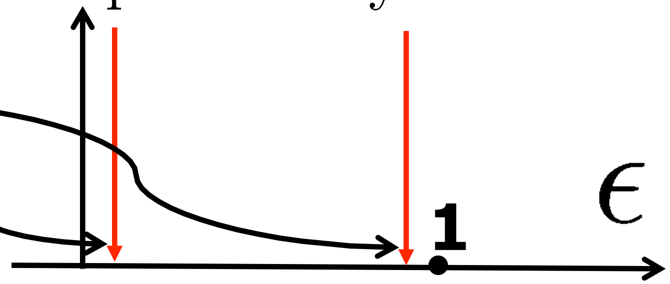
- SISO dynamical systems
- MIMO dynamical systems
- Quantization model for vector signals $s_t \in \mathbb{R}^n$
- Explicit computation of $a^*(\mathcal{C})$ for realistic codes
- Evaluation of control performance (stability is not enough)

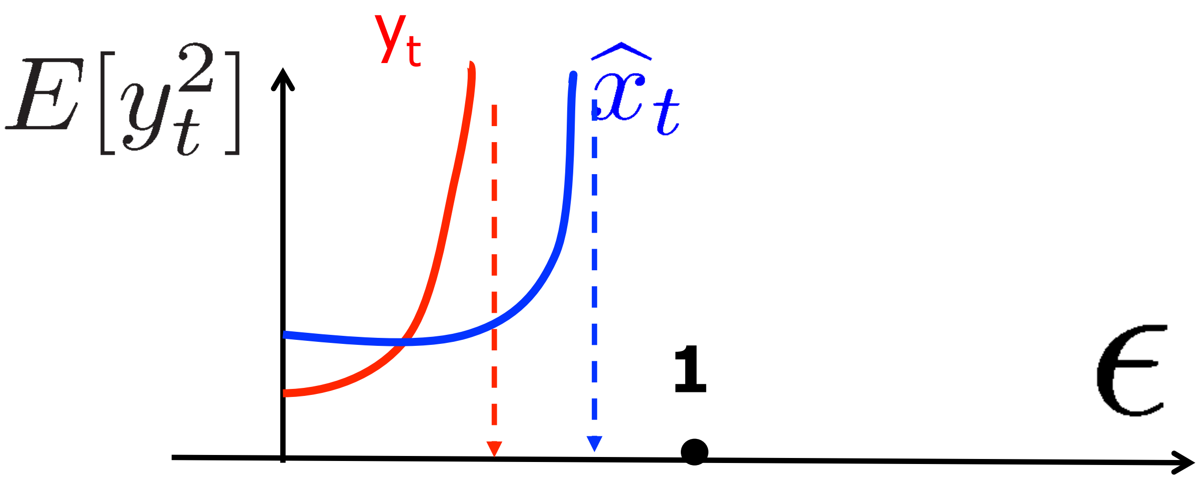
(Many) Open Problems

If $y \in \mathbb{R}, x \in \mathbb{R}^n$, then critical packet loss probability ϵ.

$\epsilon < \epsilon_x^c = \frac{1}{|\lambda_{max}(A)|^2}$: transmit $\hat{x}_t$

$\epsilon < \epsilon_y^c = \frac{1}{|\prod \lambda_i^u(A)|^2}$: transmit y_t





Questions ?

URL: http://automatica.dei.unipd.it/people/schenato.html

Chiuso, N. Laurenti, L. Schenato, A. Zanella. **LQG control over finite capacity channels: the role of data losses, delays and SNR limitations**. *Automatica (under review)*

A. Chiuso, N. Laurenti, L. Schenato, A. Zanella. **Analysis of delay-throughput-reliability tradeoff in a multihop wireless channel for the control of unstable systems**. *Technical Report, 2013*

S. Dey, A. Chiuso, L. Schenato. **Remote estimation with noisy measurements subject to packet loss and quantization noise**. *IEEE Transactions on Control of Network Systems (under review)*, 20XX

Theoretical vs Sampling Error variance

$$\hat{P}_y := \frac{1}{10000}\sum_{t=1}^{10000} y_t^2 \quad P_y = \begin{bmatrix} C & C \end{bmatrix} P \begin{bmatrix} C^\top \\ C^\top \end{bmatrix} + \sigma_v^2$$

$N_b = 3$ bits/sample ($\rho = 12,\ d_{max}(a,\varepsilon,\rho) = 2$)

d	1	2	3
P_y	21.81	702.5	∞
$\hat{P}_y$	21.25	429.1	$\to \infty$

$N_b = 4$ bits/sample ($\rho = 48,\ d_{max}(a,\varepsilon,\rho) = 4$)

d	1	2	3	4
P_y	13.67	36.20	136.84	∞
$\hat{P}_y$	13.42	38.10	149.58	$\to \infty$

$N_b = 5$ bits/sample ($\rho = 192,\ d_{max}(a,\varepsilon,\rho) = 5$)

d	1	2	3	4	5	6
P_y	12.47	28.87	70.71	199.77	1012.86	∞
$\hat{P}_y$	12.35	29.15	72.82	194.3	1414.15	$\to \infty$

$N_b = 6$ bits/sample ($\rho = 768,\ d_{max}(a,\varepsilon,\rho) = 7$)

d	1	2	3	4	5	6	7	8
P_y	12.20	27.45	62.80	145.25	366.95	1122.34	11412.34	∞
$\hat{P}_y$	12.31	28.09	62.12	147.97	373.46	1149.14	12100.01	$\to \infty$

Table : Sample vs. Population variances as a function of the delay d. Each table refer to a different number of bits/sample (equivalently $SQNR$ ρ.)