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Multi-Robot Localization via GPS and Relative Measurements in the Presence of Asynchronous and Lossy Communication

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Abstract— **This work addresses the problem of distributed multi-agent localization in presence of heterogeneous measurements and wireless communication. The proposed algorithm integrates low precision global sensors, like GPS and compasses, with more precise relative position (i.e., range plus bearing) sensors. Global sensors are used to reconstruct the absolute position and orientation, while relative sensors are used to retrieve the shape of the formation. A fast distributed and asynchronous linear least-squares algorithm is proposed to solve an approximated version of the non-linear Maximum Likelihood problem. The algorithm is provably shown to be robust to communication losses and random delays. The use of ACK-less broadcast-based communication protocols ensures an efficient and easy implementation in real world scenarios. If the relative measurement errors are sufficiently small, we show that the algorithm attains a solution which is very close to the maximum likelihood solution. The theoretical findings and the algorithm performances are extensively tested by means of Monte-Carlo simulations.**

I. INTRODUCTION

In the last decades, the hardware cost reduction and the appearance of dedicated software for rapid prototyping have made mobile autonomous robotics becoming a growing business involving many start-ups. In particular, the advances in cooperative robotics using multiple vehicles have achieved results performance in controlled environments, e.g., [1], [2].

Global and relative localization of the vehicles is one fundamental task that needs to be accomplished in order to accomplish many more complex tasks. Most of the remarkable results have been obtained in indoor controlled environments where multiple cameras are able to track and estimate the vehicles location and orientation [3]. These estimates are computed at a central location and then forwarded to the different vehicles. However, such architecture is not replicable in outdoor unstructured environments. Although global position system (GPS) sensors and compass sensors are available, their accuracy might be insufficient for many tasks, e.g., tight formation control, map-building. As so, additional sensors, able to measure relative position and orientation among vehicles, e.g., stereo cameras, ultrasonic rangers, if paired with GPS and compass could dramatically improve the accuracy of outdoor robot absolute location.

Localization in unstructured environments has a long history and a whole area of research, named simultaneous localization and mapping (SLAM), has been devoted to the topic [4]. The main goal is to reconstruct the location and past trajectory of a vehicle based on sensory data collected along its travelling. The problem is challenging since estimating the location from multiple poses is a highly non-linear problem which might have multiple solutions [5], specifically if bearing-only sensors [6] or range-only sensors are used [7]. Many advances have been made when both types of sensors are available. Although these solutions are often batch-based and cooperation among agents is absent [8].

This work addresses the problem of *robust distributed real-time multi-vehicle localization via wireless communication*. We propose to integrate less precise global sensors (GPS and compass) with more precise relative positioning sensors (range and bearing sensors) to achieve global high accuracy. Intuitively, precise relative sensors allow for the reconstruction of a relative formation. Differently, compass and GPS installed in multiple vehicles can provide estimation of the global centroid and orientation of the whole formation. Another challenge that we want to address is to provide a solution which is totally distributed, asynchronous and robust to communication losses. In fact, a centralized solution is not scalable and subject to a *single-point-of-failure*. Moreover, synchronous communication is difficult to enforce and is prone to failuers. Recent research [9]–[12] propose distributed yet synchronous solutions for multi-robot localization. Asynchronous approaches are proposed in [13]–[15] where robustness of the solutions to communication delays is either theoretically or numerically assessed. However, robustness to lossy communication is never addressed.

We propose an asynchronous distributed algorithm for multi-robot localization that integrates GPS, compass, range and bearing measurements and is robust to packet losses and random delays. We show that if the range and bearing errors are sufficiently small, it is possible to linearize the localization problem achieving a performance which is very close to the exact maximum likelihood solution. The algorithm is based on a broadcast communication protocols not requiring ACK packets. Therefore is fast and easy to implement.

II. MATHEMATICAL PRELIMINARIES

Resorting to standard graph theory, the estimation problem can be associated with an *undirected measurement graph* $\mathbf{G} = (\mathbf{V};\mathbf{E})$ where $\mathbf{V} \in \{1,\dots,N\}$ represents the nodes and $\mathbf{E} \subset \mathbf{V}\times\mathbf{V}$ ($|\mathbf{E}| = M$) contains the unordered pairs of nodes $\{i, j\}$ which are connected to, measure and communicate with each other. $\mathcal{N}_i \subseteq \mathbf{V}$ denotes the set $\{j \mid \{i, j\} \in \mathbf{E}\}$, i.e. the neighboring set of node i. An undirected graph $\mathbf{G}$ is said to be connected if for any pair of vertices $\{i, j\}$ a path exists, connecting i to j. In the problem at hand, we consider a

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communication graph among the nodes which coincides with the measurements graph $\mathbf{G}$. Moreover, broadcast and asynchronous communications are assumed among the nodes. We denote with $|\cdot|$ the modulus of a scalar. The incidence matrix $A \in R^{M\times N}$ of $\mathbf{G}$ is defined as $A = [a_{ei}]$, where $a_{ei} = \{1, -1, 0\}$, if edge e is incident on node i and directed away from it, is incident on node i and directed toward it, or is not incident on node i, respectively. We denote with the symbol $\|\cdot\|$ the vector 2-norm and with $[\cdot]^T$ the transpose operator. The symbol $\odot$ represents the *Hadamard* product. Given a vector $\mathbf{v} \in \mathbb{R}^2$, the function $\operatorname{atan2}(\cdot) : \mathbb{R}^2 \to [0, 2\pi]$ returns its angle, i.e., $\mathbf{v} = \|\mathbf{v}\| e^{j\operatorname{atan2}(\mathbf{v})}$. Given a matrix $\mathbf{v} \in \mathbb{R}^{m\times n}$, with $v_{\text{ctr.}}$ we denote the vector centroid, i.e., $v_{\text{crt.}} = \frac{1}{n}\sum_{i=1}^{n} v_i$, where v_i is the i- th row of the matrix. The symbol σ_x denotes the standard deviation of the generic measurement x. The operator $\mathbb{E}[\cdot]$ denotes the expected value. With $\mathcal{N}(\cdot,\cdot)$ we denote the normal distribution. The symbol $\operatorname{proj}(\cdot) : \mathbb{R} \mapsto \mathbb{R}^2$ denotes the function $\operatorname{proj}(\theta) = \begin{bmatrix}\cos\theta & \sin\theta\end{bmatrix}^T$. Finally, $\mathbb{I}$ denotes the identity matrix of suitable dimensions.

III. PROBLEM FORMULATION

Consider the problem of estimating the 2D positions, expressed in a common reference frame, of N nodes of a sensor network. Each node is endowed with a set of sensors that provide both relative and absolute measurements.

In the following, first we introduce the statistical models used for each type of measurements. Second, we formulate the non linear *Maximum-Likelihood* estimation problem. Third, we introduce a suitable linear and convex reformulation.

A. Measurement Models

We assume the N nodes are provided with a GPS module, a compass and a relative range and bearing sensors. We denote with $p_i = (x_i, y_i),\ i \in \mathbf{V}$, the 2D position of node i in a common inertial frame, and with θ_i its orientation with respect to the inertial North axis, assumed to coincide with the x-axis. Each sensor is described by the following statistical model:

- $p_i^{\text{GPS}} = (x_i^{\text{GPS}}, y_i^{\text{GPS}})$ represents a noisy GPS measurement of $p_i = (x_i, y_i)$. We assume $p_i^{\text{GPS}} \sim \mathcal{N}(p_i, \sigma_p^2\mathbb{I})$.
- θ_i^C is the compass noisy measurements of θ_i. We assume an angular Gaussian distribution [16] $\operatorname{proj}(\theta_i^C) \sim \mathcal{N}\left(\operatorname{proj}(\theta_i), \sigma_\theta^2\mathbb{I}\right)$, which approximates the *Langevin* distribution [17].
- r_{ij} is the range sensor measurement of the distance between i and j. We assume $r_{ij} \sim \mathcal{N}(\|p_i - p_j\|, \sigma_r^2)$.
- δ_{ij} is the noisy measurements of the bearing angle of the node j in the local frame of node i. For δ_{ij} we adopt an angular Gaussian distribution model that is $\operatorname{proj}(\delta_{ij}) \sim \mathcal{N}\left(\operatorname{proj}(\operatorname{atan2}(p_j - p_i) - \theta_i), \sigma_\delta^2\mathbb{I}\right)$.

We are aware the assumption of Gaussian noises might be restrictive and represent a limit in practical scenarios where noises are characterized by non Gaussian distributions.

Remark III.1. Observe that, to reduce the set-up cost, each node has access to highly noisy absolute measurements together with precise relative measurements. In particular, the GPS sensors are usually characterized by $\sigma_p = 2$ [m] [18], while the compass by $\sigma_\theta = 0.05$ [rad] [19]. To retrieve information about range and bearing different sensors can be used, e.g., depth-camera, laser, ultrasound. Acceptable values might be $\sigma_r = 0.1$ [m] and $\sigma_\delta = 0.03$ [rad]. Due to the variability in the accuracy of the available sensors, we will test our algorithm in a wide range of standard deviation values. Finally, observe that all the results hold even if a subset of nodes are provided with a GPS unit. However, for simplicity, we consider all the nodes endowed with a GPS.

B. Maximum-Likelihood Estimator

We assume that all the measurements are independent and their probability distributions are given in the previous section. It is possible to formulate the localization problem as a *Maximum-Likelihood* (ML) estimation problem. Let us define the state and measurements sets, respectively, as

$$\begin{aligned}
\mathbf{x} &= \{\mathbf{p}, \theta\} = \{p_i,\, \theta_i \text{ with } i \in \mathbf{V}\}, \\
\mathbf{y} &= \left\{p_i^{\text{GPS}},\, \theta_i^C,\, r_{hk},\, \delta_{hk} \text{ with } i \in \mathbf{V},\ (h,k) \in \mathbf{E}\right\},
\end{aligned}$$

where $\mathbf{p} := [p_1, \ldots, p_N]^T$ and $\theta := [\theta_1, \ldots, \theta_N]^T$. Then, the negative log-likelihood cost function can be written as

$$J(\mathbf{x}) := -\log f\left(\mathbf{y}\,|\,\mathbf{x}\right) = J_p + J_\theta + J_r + J_\delta + c, \tag{1}$$

where

$$\begin{aligned}
J_p &= \sum_{i=1}^{N} \frac{\|p_i - p_i^{\text{GPS}}\|^2}{2\sigma_p^2}, \quad J_r = \sum_{(i,j)=1}^{M} \frac{(r_{ij} - \|p_i - p_j\|)^2}{2\sigma_r^2}, \\
J_\theta &= \sum_{i=1}^{N} \frac{\left\|\operatorname{proj}(\theta_i^C) - \operatorname{proj}(\theta_i)\right\|^2}{2\sigma_\theta^2}, \\
J_\delta &= \sum_{(i,j)=1}^{M} \frac{\left\|\operatorname{proj}(\delta_{ij}) - \operatorname{proj}\left(\operatorname{atan2}(p_j - p_i) - \theta_i\right)\right\|^2}{2\sigma_\delta^2},
\end{aligned}$$

and c is a constant term that does not depend on $\mathbf{x}$ and $\mathbf{y}$. The minimization of the cost (1) would provide the ML estimator for the nodes' absolute positions and orientations, i.e.:

$$\widehat{\mathbf{x}}^{\text{ML}} = \operatorname{argmin}_{\mathbf{x}}\ J(\mathbf{x}). \tag{2}$$

Now let us consider the following equivalent parametrization of agents' positions using their centroid $p_{\text{ctr.}}$ and corresponding deviation Δp_i. This reads as $p_i = p_{\text{ctr.}} + \Delta p_i$, $\sum_i \Delta p_i = 0$. Let us also define $\mathbf{\Delta p} = (\Delta p_1, \ldots, \Delta p_N)$. Thanks to the new parametrization, equation (2) is equivalent to:

$$\begin{aligned}
\left\{\widehat{p}_{\text{ctr.}}^{\text{ML}}, \Delta\widehat{\mathbf{p}}^{\text{ML}}, \widehat{\theta}^{\text{ML}}\right\} = \ &\underset{\{p_{\text{ctr.}}, \Delta\mathbf{p}, \theta\}}{\operatorname{argmin}} \quad J(p_{\text{ctr.}}, \Delta\mathbf{p}, \theta), \\
&\text{s.t.} \qquad \sum_i \Delta p_i = 0.
\end{aligned} \tag{3}$$

The reformulation (3) lets us show how the ML estimator exploits the GPS information to solve for the absolute positioning of the formation's centroid. Moreover, we characterize the limit behavior of the estimator when range, bearing and compass noises are very large or very small. The proofs of the following Lemmas can be found in the technote [20].

Lemma III.1. *Consider the negative* log*-likelihood cost* (1)*. Then, the ML solution* $\widehat{\mathbf{x}}^{\text{ML}}$ *which solves* (3) *is such that*

$$\widehat{p}_{\text{ctr.}}^{\text{ML}} = p_{\text{ctr.}}^{\text{GPS}}, \tag{4}$$

where $\widehat{p}^{\text{ML}}_{\text{ctr.}} := \frac{1}{N}\sum_{i=1}^{N}\widehat{p}_i$ *and* $p^{\text{GPS}}_{\text{ctr.}} := \frac{1}{N}\sum_{i=1}^{N} p^{\text{GPS}}_i$.

Lemma III.2. *For fixed GPS variance* σ_p *we have*

1) $\lim_{\max\{\sigma_\theta,\sigma_r,\sigma_\delta\}\to 0} \widehat{p}^{\text{ML}}_i = p^{\text{GPS}}_{\text{ctr.}} + \Delta p_i$,
2) $\lim_{\min\{\sigma_r,\sigma_\delta\}\to +\infty} \widehat{p}^{\text{ML}}_i = p^{\text{GPS}}_i$.

Statement 1) of Lemma III.2 states that if $\max\{\sigma_\theta,\sigma_r,\sigma_\delta\} \to 0$, the shape of the formation is perfectly retrieved. Then, the only source of error between the estimated formation and the ground-truth is given by the error between GPS centroid and the true centroid. Statement 2) states that if the relative measurements accuracies deteriorate, the ML estimator will "trust" the GPS measurements only.

Unfortunately problem (2) is highly non-linear and hard to solve. In particular, if the angles are noise-free, the problem is linear [9]. Conversely, if the angles are not known, the problem presents many local minima [8]. One possible way to tackle it, is using a standard gradient descent approach since the gradient vector of the log-likelihood function can be computed in closed form using (1). However, such approach heavily suffers of bad initialization. In fact, the presence of multiple local minima in (1) causes the algorithm to stop on the wrong minimizer.

In the following, we resort to a suitable approximation to reformulate problem (2) in a linear-least square framework.

C. An Approximated Linear Least-Squares Formulation

Here, the idea is to move from the polar coordinate system to the equivalent Cartesian representation. Indeed, assuming a perfect knowledge of range, bearing and compass, it is possible to express the *displacement* d_{ij} as

$$d_{ij} := p_i - p_j = r_{ij}\begin{bmatrix}\cos(\delta_{ij}+\theta_i)\\ \sin(\delta_{ij}+\theta_i)\end{bmatrix}. \tag{5}$$

Since the measurements are affected by noise, it is necessary to map the noise of range, bearing and compass into the equivalent noise in Cartesian coordinates. Namely, given the noisy version of (5), that is

$$d_{ij} = p_i - p_j + n_{ij}, \tag{6}$$

where n_{ij} is the noise in Cartesian coordinate, we want to find the expression for its covariance, $\mathbb{E}[n_{ij}n_{ij}^T] = \Sigma_{ij}$, in terms of the statistical description of range, bearing and compass measurements noises. After a first order expansion we obtain

$$\Sigma_{ij} = \begin{bmatrix}\sigma_x^2(i,j) & \sigma_{xy}(i,j)\\ \sigma_{yx}(i,j) & \sigma_y^2(i,j)\end{bmatrix}, \tag{7}$$

where

$$\begin{aligned}\sigma_x^2(i,j) &= \sigma_r^2\cos^2(\delta_{ij}+\theta_i) + r_{ij}^2(\sigma_\delta^2+\sigma_\theta^2)\sin^2(\delta_{ij}+\theta_i),\\ \sigma_y^2(i,j) &= \sigma_r^2\sin^2(\delta_{ij}+\theta_i) + r_{ij}^2(\sigma_\delta^2+\sigma_\theta^2)\cos^2(\delta_{ij}+\theta_i),\\ \sigma_{xy}(i,j) &= \left(\sigma_r^2 - r_{ij}^2(\sigma_\delta^2+\sigma_\theta^2)\right)\sin(\delta_{ij}+\theta_i)\cos(\delta_{ij}+\theta_i).\end{aligned}$$

Remark III.2. Since the linear approximation introduced is based on a first order expansion, its validity holds under the assumption of sufficiently small measurement errors.

Remark III.3. Note that Σ_{ij} in (7) is a function of the true values of range, bearing and compass. Since it is not possible to have access to these data, in a real setup these quantities must be replaced by their corresponding measured values.

Once computed the displacements (6), it is possible to define the weighted residuals as

$$J_d = \frac{1}{2}\sum_{\{i,j\}\in\mathbf{E}} \|p_i - p_j - d_{ij}\|^2_{\Sigma_{ij}^{-1}}.$$

Then, it is possible to define an approximation of (1) accounting for the GPS measurements and the displacements, as

$$J_{\text{LS}}(\mathbf{p}) = J_p + J_d. \tag{8}$$

The minimization problem, which is a linear least-squares (LLS), hence solvable in closed form, becomes

$$\widehat{\mathbf{p}}^{\text{LS}} = \operatorname{argmin}_{\mathbf{p}}\ J_{\text{LS}}(\mathbf{p})\ . \tag{9}$$

By assuming $\mathbf{G}$ connected, the optimal estimate is given by

$$\widehat{\mathbf{p}}^{\text{LS}} = (\Sigma_{\text{GPS}}^{-1} + A^T\Sigma^{-1}A)^{-1}(\Sigma_{\text{GPS}}^{-1}\mathbf{p}^{\mathbf{GPS}} + A^T\Sigma^{-1}\mathbf{d}), \tag{10}$$

where $\Sigma_{\text{GPS}} = \sigma_p^2\mathbb{I}$, Σ is the matrix which accounts for all the Σ_{ij}, and $\mathbf{d}$ and $\mathbf{p}^{\text{GPS}}$ are the vectors obtained stacking together all the relative distances defined in (6) and the GPS absolute positions, respectively.

Remark III.4. Note that $\widehat{\mathbf{p}}^{\text{LS}}$ gives only an estimate of the absolute positions $\mathbf{p}$ without providing any estimate of the absolute orientations. These are retrieved using the compass and exploited to project the noise in rectangular coordinates.

Remark III.5. Observe that, even if the LLS problem returns an approximate solution for the problem of equation (2), since the problem (9) is convex, its solution is unique.

For the LS estimator it is possible to show an optimality result similar to the one stated in Lemmas III.1 and III.2 for the ML estimator. We state the following Lemma whose proof follows the arguments used for Lemmas III.1 and III.2.

Lemma III.3. *Consider the cost function* (8). *Then, the optimal solution* $\widehat{\mathbf{p}}^{\text{LS}}$ *which solves* (9) *is such that*

$$\widehat{p}^{\text{LS}}_{\text{ctr.}} = p^{\text{GPS}}_{\text{ctr.}}. \tag{11}$$

Moreover, for fixed GPS variance σ_p *we have*

$$\lim_{\max\{\sigma_\theta,\sigma_r,\sigma_\delta\}\to 0} \widehat{p}^{\text{LS}}_i = p^{\text{GPS}}_{\text{ctr.}} + \Delta p^{\text{LS}}_i\ ,$$

$$\lim_{\min\{\sigma_r,\sigma_\delta\}\to +\infty} \widehat{p}^{\text{LS}}_i = p^{\text{GPS}}_i\ .$$

Observe that, to compute $\widehat{\mathbf{p}}^{\text{LS}}$ as in equation (9), one needs all the measurements, their covariances and the topology of $\mathbf{G}$ to be available to a central computation unit.

IV. DISTRIBUTED AND ASYNCHRONOUS ALGORITHM

In this section we present a distributed and asynchronous solution for the minimization problem (9), which is robust to communication delays and packet losses. In the following, by *distributed*, we mean that there is no central unit gathering all the measurements $\mathbf{p}^{\text{GPS}}$ and $\mathbf{d}$, having global knowledge of the graph $\mathbf{G}$ and computing $\widehat{\mathbf{p}}^{\text{LS}}$ directly; instead, each node has limited computational and memory resources, and can communicate only with $j \in \mathcal{N}_i$. By *asynchronous*, we mean that there is no common reference time (generated, e.g., by a centralized clock source) which keeps all the updating/transmitting actions synchronized among the nodes.

The implementation presented is inspired by [21], [22], where it is shown that this strategy is efficient both in terms of number of iterations and number of sent packets per communication round, compared to existing alternative strategies. The algorithm we propose, which we refer to as the *asynchronous gradient-based localization* (a-GL), is based on a standard gradient descent strategy employing an *asynchronous broadcast* communication protocol. Namely, during each iteration there is only one node transmitting information to its neighbors. Moreover, the time between two consecutive iterations does not have to be constant. For ease of notation, hereafter we drop the superscript LS for single node estimates.

We assume that every node has access to its own measurements and to those of its neighbors nodes, as well as the associated covariances. Additionally we assume that node i, $i \in \mathbf{V}$, stores in memory an estimate $\widehat{p}_\mathrm{i}$ of p_i and, for $j \in \mathcal{N}_i$, an estimate $\widehat{p}_j^{(i)}$ of $\widehat{p}_j$.

The a-GL is shown in Algorithm 1. Let $t_0, t_1, t_2, \ldots$ be the time instants in which the iterations of the algorithm occur.

Algorithm 1: a-GL Algorihtm

Require: Node $i \in \mathbf{V}$ store in memory the measurements p_i^{GPS}, d_{ij}, $j \in \mathcal{N}_i$, the variances σ_p, N_{ij} and the neighbors estimates $\widehat{p}_j^{(i)}$, $j \in \mathcal{N}_i$.

```
1: for t = t_0, t_1, t_2, ... do
   # Random node selection
2:   Node i ∈ V wakes-up
   # Node i self update
3:   p̂_i ← p̂_i − α(i) ⊙ ∂J_LS/∂p_i
   # Self-update broadcasting
4:   p̂_i broadcast to j, j ∈ N_i
   # neighbors memory update
5:   p̂_i^(j) ← p̂_i, ∀ j ∈ N_i
6: end for
```

In Algorithm 1, $\alpha(i) = [\alpha_x(i) \;\; \alpha_y(i)]^T$ is a suitable scaling factor for the gradient step. Through standard algebraic computations, one can see that:

$$\frac{\partial J_{\mathrm{LS}}}{\partial p_i} = \frac{p_i - p_i^{\mathrm{GPS}}}{\sigma_p^2} + \sum_{j \in \mathcal{N}_i} \Sigma_{ij}^{-1} (p_i - p_j - d_{ij}) \ .$$

Observe that in order to compute $\frac{\partial J_{\mathrm{LS}}}{\partial p_i}$, node i requires information coming only from its neighbors. This makes the algorithm amenable for a distributed implementation. Since every node has available in memory a copy of the neighbors estimate, a natural way to evaluate the gradient is

$$\frac{\partial J_{\mathrm{LS}}}{\partial p_i} = \frac{\widehat{p}_i(t) - p_i^{\mathrm{GPS}}}{\sigma_p^2} + \sum_{j \in \mathcal{N}_i} \Sigma_{ij}^{-1} (\widehat{p}_i(t) - \widehat{p}_j^{(i)}(t) - d_{ij}) \ ,$$

It is possible to show that J_{LS} does not increase if

$$0 < \alpha_x(i) \leq \left(\frac{1}{\sigma_p^2} + \sum_{j \in \mathcal{N}_i} (\gamma_x(i,j) + \gamma_x(j,i)) \right)^{-1} , \quad \text{(12a)}$$

$$0 < \alpha_y(i) \leq \left(\frac{1}{\sigma_p^2} + \sum_{j \in \mathcal{N}_i} (\gamma_y(i,j) + \gamma_y(j,i)) \right)^{-1} , \quad \text{(12b)}$$

where $\gamma_x(h,k)$ and $\gamma_y(h,k)$ represent the diagonal elements of Σ_{ij}^{-1}. In particular, if $\alpha(i)$ coincides with the RHSs of (12) then the minimum of J_{LS} is attained.

In the following we analyze the robustness of the a-GL.

A. Convergence Analysis in Presence of Packet Losses and Communication Delays

Given Algorithm 1 to compute the LLS solution (9) in a distributed and asynchronous fashion, here we consider a more realistic scenario: *presence of delays and packet losses in the communication channel*. Convergence of the a-GL algorithm to the optimal LS solution is proven, provided the network is uniformly persistent communicating as defined in Definition 1, and the transmission delays and the frequencies of communication failures satisfy the following assumptions.

Definition 1 (Uniformly persistent comm. network)**.** *A network of N nodes is said to be* uniformly persistent communicating *if there exists a positive integer number τ such that, for all $t \in \mathbb{N}$, each node performs lines 3 and 4 of the a-GL algorithm at least once within the iteration-interval $[t, t+\tau)$.*

Assumption IV.1 (Bounded packet losses)**.** There exists an integer $L > 0$ such that the number of consecutive communication failures between every pair of neighboring nodes in the graph $\mathbf{G}$ is less than L.

Assumption IV.2 (Bounded delays)**.** Assume node i broadcasts its estimate to its neighbors during iteration t, and that the communication link (i,j) does not fail. Then, there exists an integer $D > 0$ such that $\widehat{p}_i(t+1)$ is used by node j to perform its local update not later than iteration $t + D$.

Next we characterize the robustness of the a-GL algorithm to delays and packet losses. The proof can be found in [21].

Proposition 1 (Proposition V.3 in [21])**.** *Consider a* uniformly persistent communicating network *of N nodes running the a-GL algorithm over a connected measurement graph* $\mathbf{G}$*. Let Assumptions IV.1 and IV.2 be satisfied. Assume the weights $\alpha(i)$ satisfy Equations* (12a)–(12b)*. Moreover, assume that $\widehat{p}_i$, $i \in \{1, \ldots, N\}$, $\widehat{p}_j^{(i)}$, $j \in \mathcal{N}_i$, be initialized to* $\mathbf{p}^{\mathrm{GPS}}$*. Then the following facts hold true*

1) *the evolution $t \to \widehat{\mathbf{p}}(t)$ asymptotically converges to the optimal estimate $\widehat{\mathbf{p}}^{\mathrm{LS}}$, i.e.,*

$$\lim_{t \to \infty} \widehat{\mathbf{p}}(t) = \widehat{\mathbf{p}}^{\mathrm{LS}};$$

2) *the convergence is exponential, namely, there exists $C > 0$ and $0 \leq \rho < 1$ such that*

$$\|\widehat{\mathbf{p}}(t) - \widehat{\mathbf{p}}^{\mathrm{LS}}\| \leq C \rho^t \|\widehat{\mathbf{p}}(0) - \widehat{\mathbf{p}}^{\mathrm{LS}}\|. \quad (13)$$

V. SIMULATIONS

Here, we test the effectiveness of the proposed algorithm. We consider a regularly spread group of robots placed on a $2D$ lattice with an inter-node distance of 4 [m]. Each agent is endowed with sensors characterized by $\sigma_p = 2$ [m] [18]; $\sigma_\theta = 0.05$ [rad] [19]; if not differently specified, $\sigma_r = 0.1$

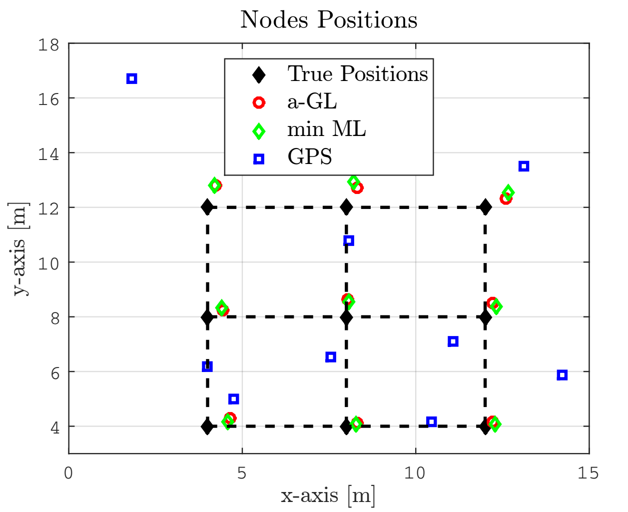

Fig. 1: Absolute positions for a formation of $N=9$ robots, $\sigma_r=0.1$ [m] and $\sigma_\delta=0.03$ [rad]. The black dashed line highlights the shape of the real formation as well as the communication graph.

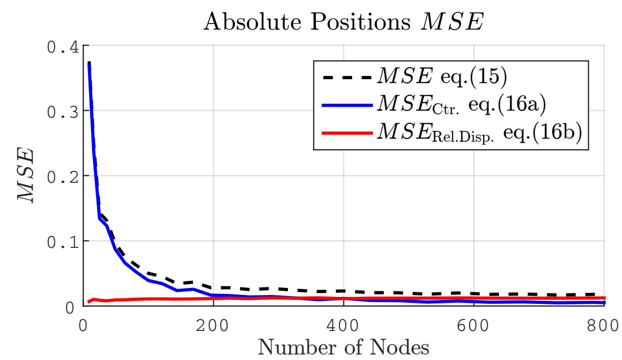

Fig. 2: Absolute positions MSE, computed via Monte Carlo simulations, as function of N for $\sigma_r=0.1$ [m] and $\sigma_\delta=0.03$ [rad].

[m] and $\sigma_\delta=0.03$ [rad]. However, due to their variability, the algorithm is tested for different values of σ_r and σ_δ.

A. Performance Measures

Here we introduce some performance measures used in the following sections. In Section V-B, the estimated positions are compared with the ground truth, i.e., the true positions $\mathbf{p}$, in terms of Mean Squared Error (MSE) defined as

$$MSE(\widehat{\mathbf{p}},\mathbf{p})=\mathbb{E}\left[\|\widehat{\mathbf{p}}-\mathbf{p}\|^2\right], \tag{14}$$

where $\widehat{\mathbf{p}}$ denotes the generic vector of positions estimates. We denote with $x_{\text{ctr.}}, y_{\text{ctr.}}, \widehat{x}_{\text{ctr.}}, \widehat{y}_{\text{ctr.}}$ the centroids of the true and of the estimated x and y coordinates, respectively. Moreover, it is convenient to define, for the x coordinate,

$$\Delta x_i := x_i - x_{\text{ctr.}}, \qquad \Delta\widehat{x}_i := \widehat{x}_i-\widehat{x}_{\text{ctr.}}, \qquad \Delta x_{\text{ctr.}} := \widehat{x}_{\text{ctr.}}-x_{\text{ctr.}},$$

and similarly Δy_i, $\Delta\widehat{y}_i$ and $\Delta y_{\text{ctr.}}$ for the y coordinate. Then, by recalling the fact that

$$\sum_{i=1}^N \Delta x_i=\sum_{i=1}^N \Delta y_i=\sum_{i=1}^N \Delta\widehat{x}_i=\sum_{i=1}^N \Delta\widehat{y}_i=0 .$$

after some algebraic manipulations, it is possible to see that $MSE(\widehat{\mathbf{p}},\mathbf{p})=MSE_{\text{Ctr.}}+MSE_{\text{Rel.Disp.}}$, where

$$MSE_{\text{Ctr.}} := \mathbb{E}\left[\Delta x_{\text{ctr.}}^2+\Delta y_{\text{ctr.}}^2\right]=\frac{\sigma_p^2}{N}\overset{N\to\infty}{\longrightarrow} 0 , \tag{15a}$$

$$MSE_{\text{Rel.Disp.}} := \mathbb{E}\left[\sum_{i=1}^N(\Delta\widehat{x}_i-\Delta x_i)^2+(\Delta\widehat{y}_i-\Delta y_i)^2\right], \tag{15b}$$

represent the MSE of the centroids, which scales with N, and of the relative displacement from the centroid, respectively. In Section V-C, we compare, in log-scale, the estimates of the a-GL algorithm with the steady state estimate obtained with the LS centralized algorithm, i.e.,

$$\log\|\widehat{\mathbf{p}}(t)-\widehat{\mathbf{p}}^{\text{LS}}\|. \tag{16}$$

From (13), we expect $\widehat{\mathbf{p}}(t)$ to exponentially converge to $\widehat{\mathbf{p}}^{\text{LS}}$.

Remark V.1 (Numeric MSE)**.** Observe that the theoretic MSE cannot be exactly computed. In the following, we plot the numeric MSE computed via Monte Carlo simulations.

Remark V.2 (Dependence between σ_r and σ_δ)**.** In the following we test the proposed algorithm as a function of σ_r and σ_δ. We vary only σ_r since it is assumed $\sigma_\delta = atan2(\sigma_r,\frac{4}{3})$. This lets us approximately draw samples in a ball centered in the true positions.

B. Steady State Analysis

Here, we analyze the steady state behavior of the a-GL algorithm as function of N, σ_r and σ_δ. Figure 1 shows the GPS measurements $\mathbf{p}^{\text{GPS}}$, the a-GL estimates $\widehat{\mathbf{p}}\equiv\widehat{\mathbf{p}}^{\text{LS}}$ and the minimizer of the log-likelihood $\widehat{\mathbf{p}}^{\text{ML}}$, respectively. Due to the many local minima characterizing the ML problem, $\widehat{\mathbf{p}}^{\text{ML}}$ is computed by exhaustive search around $\mathbf{p}$. From the plot, it can be seen how, thanks to the additional relative information, the estimates outperforms the GPS measurements.
Figure 2, for increasing N, shows the behavior of the MSE of Eq. (14) and of its components $MSE_{\text{Ctr.}}$ and $MSE_{\text{Rel.Disp.}}$ of Eqs. (15a)–(15b), respectively. As expected, $MSE_{\text{Ctr.}}\to 0$ for $N\to\infty$, while $MSE_{\text{Rel.Disp.}}$ remains almost constant and comparable to σ_r^2. It is clear that two are the main sources of error: one depending on the relative information, which lets to reconstruct the shape of the formation with an error comparable to that of the relative measurements. The other related to the absolute position reconstruction, obtained from the GPS information, which for small number of agents is the greater source of error, but improves with the number of robots as $1/N$. Observe that, when only a subset of $\bar{N}<N$ nodes is equipped with a GPS the error scales as $1/\bar{N}$.
Figure 3 shows the MSE for increasing σ_r. The plot shows in red the behavior of the a-GL algorithm and the ML estimator in green. Moreover, some limit behaviors are plotted: the MSE of the GPS measurements (blue dashed line); the MSE of the mean of the GPS measurements (black dashed line). These, according to Lemma III.3, are due to the following facts: (i) for increasing σ_r the relative sensors become useless and the estimator "trusts" mainly the GPS; (ii) for small σ_r the shape of the formation is "perfectly" known so, the error is due to the displacement of the GPS mean from the ground truth mean. Figure 3 shows how the a-GL algorithm behaves similarly to the ML estimator for the whole range of

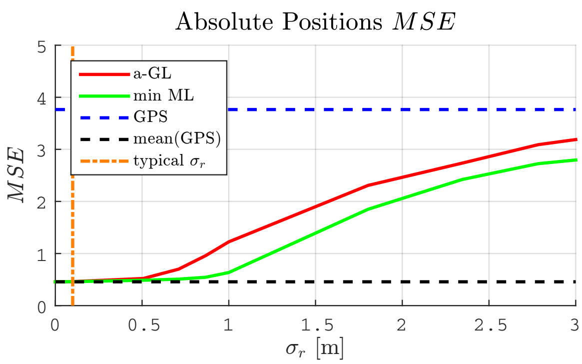

Fig. 3: Absolute positions *MSE*, computed via Monte Carlo simulations, as function of σ_r $\left(\sigma_\delta = atan2(\sigma_r, \frac{4}{3})\right)$ for $N = 9$. The dark orange vertical dashed-dotted line highlights the behavior corresponding to $\sigma_r = 0.1$ [m].

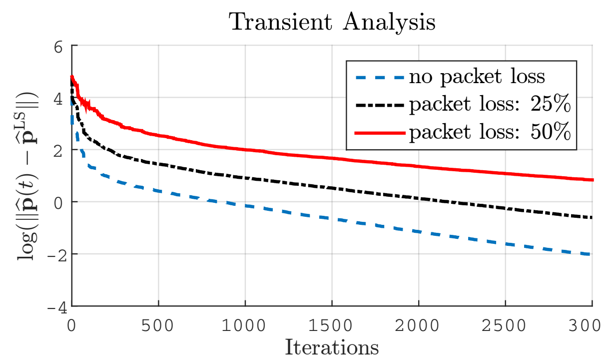

Fig. 4: Comparison between the a-GL solution and the optimal centralize LS solution, for different percentages of packet losses.

σ_r. Moreover, the difference is almost null for $\sigma_r \in [0.1, 0.5]$ [m], which characterize practical operating sensors range.

C. Transient Analysis

Here we analyze the robustness of our algorithm. At each iteration, a node, randomly chosen, wakes up, updates its state and communicates its estimate to $j \in \mathcal{N}_i$. We assume independent communication links between neighboring nodes, each of them characterized by a certain failure probability. Figure 4 plots the error in (16) for different percentages of packet losses. As expected, the higher the losses the slower the convergence. Note that, in a real set-up, different nodes could wake up and update their estimates at the same time. This could increase the probability of communication collision but, at the same time, speed up the convergence rate.

VI. CONCLUSIONS AND FUTURE WORK

In this work we considered the problem of absolute position reconstruction of a multi-robot formation assuming each agent is endowed with noisy GPS and compass modules and finer relative range and bearing sensors. We presented a fast, robust, distributed and asynchronous LLS algorithm, which, combining absolute and relative information to reconstruct the global formation, solves for an approximation of the corresponding ML estimation problem. Exhaustive simulations show how, for sufficiently small relative errors, the approximated solution behaves like the ML estimator.
Future avenues, regards the investigation of the impact of the formation shape and of the communication graph on the relative formation reconstruction. Moreover, solutions to better estimate the robots absolute rotations will be investigated.

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