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Distributed Localization from Relative Noisy Measurements: a Robust Gradient Based Approach

Marco Todescato   Andrea Carron   Ruggero Carli   Luca Schenato

Abstract— **In this work we address the problem of optimal estimating the position of each agent in a network from relative noisy vectorial distances with its neighbors. Although the problem can be cast as a standard least-squares problem, the main challenge is to devise scalable algorithms that allow each agent to estimate its own position by means of only local communication and bounded complexity, independently of the network size and topology. We propose a gradient based algorithm that is guaranteed to have exponentially convergence rate to the optimal centralized least-square solution. Moreover we show the convergence also in presence of bounded delays and packet losses. We finally provide numerical results to support our work.**

I. Introduction

The recent progress in MEMS technology, digital electronics and wireless communication has made possible the development of small and cheap devices capable of communicating, computing, sensing, interacting with the environment and storing information. These devices open the way for a countless number of novel applications as swarm robotics, wireless sensor networks, smart energy grid, smart traffic networks and smart camera networks. Among all the technical challenges to face, scalability is the major one. For a network, the scalability is the ability to handle a growing number of nodes in a capable manner, i.e. without requiring to increase the hardware resources and to adapt the software algorithms.

In this paper we face the problem of designing a distributed and scalable algorithm that, given relative distance noisy measurements, is capable to reconstruct the optimal estimate of the location of a network of devices. In particular by *distributed* we mean that the devices have to exploit the advantages of sharing information with other devices, but with the constraints of a limited communication capability, i.e. a node is allowed to exchange data only with its neighbours. By *scalable* we mean that the computational complexity, bandwidth, memory requirements should be independent of the network size. The problem at hand in this work can be casted as the following unconstrained optimization problem

$$\min_{x_1,\dots,x_N} \sum_{(i,j)\in\mathcal{E}}^{|\mathcal{E}|} \| x_i - x_j - z_{ij} \|^2 \qquad (1)$$

where $x_i, z_{ij} \in \mathbb{R}^\ell$ are the unknown position and the relative noisy measurement, respectively, and $\mathcal{E}$ represents all the pair of nodes for which are available relative measurements.

The solution of this optimization problem becomes a least-square problem. In the literature are available many distributed solutions. The authors of [1], [2] propose a distributed Jacobi solution which requires a synchronous implementation. Similarly, in [3] is proposed a coordinate descent strategy which is suitable for asynchronous implementation however it requires the updating node to receive all the estimated positions of its neighbours. Differently, in [4] the authors propose a broadcast consensus-based algorithm but the local estimates exhibits an oscillatory behavior around the true value. A similar approach has been proposed in [5] where the local ergodic average of the gossip asynchronous algorithm is proved to converge to the optimal value as $1/k$, where k is the number of iterations. An alternative approach based on the Kaczmarz method for the solution of linear systems has been suggested in [6], however the proposed algorithms either oscillate or converge to the optimal value as $1/k$. Another asynchronous algorithm is proposed in [7], moreover the authors proposes some heuristics [8] to get the transient performances of [4].

The contribution of this work is to provide an asynchronous algorithm which is scalable, robust to delays and have proven exponential convergence rate under mild assumption. The algorithm is based on a standard gradient descent strategy. To compute the gradient each node is required to store in memory only a copy of the estimate of all its neighbors. The proposed algorithm is similar to the algorithm presented in [9], which requires bidirectional communication among nodes; on the contrary our strategy is based on broadcast protocols which require no acknowledge from the neighbors.
Thorough numerical simulations our solution is shown to outperform the performance of the other algorithms presented in the literature in terms of speed of convergence to the optimal solutions and in terms of number of packets required to be transmitted.

II. Problem Formulation

The problem we consider in this paper is that of estimating N variables $x_1, \dots, x_N$ from noisy measurements of the form

$$z_{ij} := x_i - x_j + n_{ij}, \qquad i, j \in \{1, \dots, N\}, \qquad (2)$$

where n_{ij} is zero-mean measurement noise. Though the variables are often vector-valued, for simplicity, in this paper we assume that $x_i \in \mathbb{R}$, $i \in \{1, \dots, N\}$.

R. Carli, A. Carron, L. Schenato and M. Todescato are with the Department of Information Engineering, University of Padova, Via Gradenigo 6/a, 35131 Padova, Italy {carlirug|carronan|schenato|todescat}@dei.unipd.it.

This estimation problem can be naturally associated with a undirected *measurement graph* $\mathcal{G} = (V; \mathcal{E})$ where

(i) V denotes the set of nodes which are labeled 1 through N, being N the number of nodes, i.e., $V = \{1, \ldots, N\}$;
(ii) $\mathcal{E}$ is the edge set and consists of all the pairs of nodes (i, j) such that a noisy measurement of the form (2) between i and j is available to both node i and node j.

In the sequel it is convenient to assume that, if z_{ij} is the measurement available at node i then $z_{ji} = -z_{ij}$ is the measurement available at node j. Basically we are assuming that the measurements are symmetrical, meaning that both agents of a pair know the measurement, with a reverse sign.

Assume that there are M available measurements, i.e., $|\mathcal{E}| = M$ and assume that the measurements errors on distinct edges are uncorrelated.

Next we formally state the problem we aim at solving. To do so we first need some preliminary definitions. Let $\mathbf{x} \in \mathbb{R}^N$ be the vector obtained stacking together all the variables $x_1, \ldots, x_N$, i.e., $\mathbf{x} = [x_1, \ldots, x_N]^T$, where given a vector v with v^T we denote its transpose, and let $\mathbf{z} \in \mathbb{R}^M$ and $\mathbf{n} \in \mathbb{R}^M$ be the vectors obtained stacking together all the measurements z_{ij} and the noises n_{ij}, respectively. Additionally, let $R_{ij} > 0$ denote the covariance of the zero mean error n_{ij}, i..e, $R_{ij} = \mathbb{E}[n_{ij}^2]$, where $\mathbb{E}$ denotes the expectation operator, and let $R \in \mathbb{R}^{M \times M}$ be the diagonal matrix collecting in its diagonal the covariances of the noises n_{ij}, $(i, j) \in \mathcal{E}$, i.e., $R = \mathbb{E}[\mathbf{n}\mathbf{n}^\mathbf{T}]$. Finally let $\mathbf{1}$ be the column vector with all components equal to one.

Now, on each edge, let us choose an orientation, that is, let us define a starting node and an ending node, in order to encode the measurements by using the *incidence matrix* $A \in \mathbb{R}^{M \times N}$ of $\mathcal{G}$ defined as $A = [a_{ei}]$, where $a_{ei} = 1, -1, 0$, if edge e is incident on node i and directed away from i, is incident on node i and directed toward it, or is not incident on node i, respectively. Observe that equation (2) can be rewritten in a vector form as

$$\mathbf{z} = A\mathbf{x} + \mathbf{n}.$$

Consider the function $J : \mathbb{R}^{N+M} \to \mathbb{R}$, defined as

$$J(\mathbf{x}, \mathbf{z}) = \frac{1}{2} \sum_{(i,j) \in \mathcal{E}} \frac{(x_i - x_j - z_{ij})^2}{R_{ij}}.$$

Observe that

$$J(\mathbf{x}, \mathbf{z}) = \frac{1}{2} (\mathbf{z} - A\mathbf{x})^T R^{-1} (\mathbf{z} - A\mathbf{x}).$$

Define the set

$$\chi := \underset{\mathbf{x} \in \mathbb{R}^N}{\operatorname{argmin}} \; J(\mathbf{x}, \mathbf{z}).$$

The goal is to construct an optimal estimate $\mathbf{x}^*$ of $\mathbf{x}$ in a least square sense, namely, to compute

$$\mathbf{x}^* \in \chi \tag{3}$$

Assume the measurement graph $\mathcal{G}$ to be *connected*, then it is well known that

$$\chi = \left\{ \left(A^T R^{-1} A\right)^\dagger A^T R^{-1} \mathbf{z} + \alpha \mathbf{1}, \; \alpha \in \mathbb{R} \right\}.$$

Moreover let

$$\mathbf{x}^*_{\text{opt}} = \left(A^T R^{-1} A\right)^\dagger A^T R^{-1} \mathbf{z},$$

then $\mathbf{x}^*_{\text{opt}}$ is the minimum norm solution of (3), i.e.,

$$\mathbf{x}^*_{\text{opt}} = \min_{\mathbf{x}^* \in \chi} \; \| \mathbf{x}^* \| \, .$$

Remark II.1 Observe that, just with relative measurements, determining the x_i's is only possible up to an additive constant. This ambiguity might be avoided by assuming that a node (say node 1) is used as reference node, i.e., $x_1 = 0$.

III. An asynchronous gradient-based localization algorithm

To compute an optimal estimate $\mathbf{x}^*$ directly, one needs all the measurements and their covariances and the topology of the measurement graph $\mathcal{G}$. In this section the goal is to compute the optimal solution in a distributed fashion, employing only local communication. In particular we assume that a node i and another node j can communicate with each other only if $(i, j) \in \mathcal{E}$. Accordingly a node i is said to be a neighbor of another node j (and viceversa) if $(i, j) \in \mathcal{E}$. For $i \in \{1, \ldots, N\}$, by $\mathcal{N}_i$ we denote the set of neighbors of node i, namely,

$$\mathcal{N}_i = \{j \in V \text{ such that } (i, j) \in \mathcal{E}\}.$$

In this paper we are interested into solutions with the following two features:

(i) They are *distributed* as opposed to centralized solutions, namely, there is no a central unit gathering all the measurements z_{ij}, having global knowledge of the graph $\mathcal{G}$ and computing $\mathbf{x}^*$ directly; instead each node has at its disposal computational and memory resources and is allowed to communicate only with its neighbors in the graph $\mathcal{G}$.
(ii) They are *asynchronous*, as opposed to synchronous solutions, namely, there is no a common reference time which keeps all the updating and transmitting actions synchronized among all the nodes.

In what follows we introduce a distributed algorithm which is based on a standard gradient descent strategy and which employs an *asynchronous broadcast* communication protocol; specifically during each iteration of the algorithm there is only one node which transmits information to all its neighbors in the graph $\mathcal{G}$. We refer to this algorithm as the *asynchronous gradient-based localization* algorithm (denoted hereafter as a-GL algorithm).

We assume that every node has access to the measurements on the edges that are incident to it, as well as the associated covariances. Additionally we assume that node i, $i \in V$, stores in memory an estimate $\hat{x}_i$ of x_i and, for $j \in \mathcal{N}_i$, an estimate $\hat{x}_j^{(i)}$ of x_j.

Next we formally describes the a-GL algorithm. Let $t_0, t_1, t_2, \dots$ be the time instants in which the iterations of the a-GL algorithm occur. Assume at time t_h node i is activated. The following actions are performed in order.

(i) Node i updates its estimate $\hat{x}_i$ in the following way

$$\hat{x}_i \leftarrow \hat{x}_i - \alpha_i \sum_{j \in \mathcal{N}_i} \frac{\hat{x}_i - \hat{x}_j^{(i)} - z_{ij}}{R_{ij}},$$

where α_i is a suitable positive real number;

(ii) Node i broadcasts the updated value of the estimate $\hat{x}_i$ to all its neighbors j, $j \in \mathcal{N}_i$;

(iii) Node j, $j \in \mathcal{N}_i$, updates the estimate $\hat{x}_i^{(j)}$ setting it equal to the value $\hat{x}_i$ it has received from node i, i.e.,

$$\hat{x}_i^{(j)} \leftarrow \hat{x}_i.$$

Some explanations are now in order. Observe that the quantity $\sum_{j \in \mathcal{N}_i} \left(\hat{x}_i - \hat{x}_j^{(i)} - z_{ij}\right)/R_{ij}$ represents the gradient computed with the respect to $\hat{x}_i$ of the function

$$J_i = \frac{1}{2} \sum_{j \in N_i} \frac{\left(\hat{x}_i - \hat{x}_j^{(i)} - z_{ij}\right)^2}{R_{ij}}.$$

Basically, node i updates the value of $\hat{x}_i$ moving along a descent direction of the function J_i. Notice that J_i does not increase if

$$0 < \alpha_i \leq \left(\sum_{j \in \mathcal{N}_i} \frac{1}{R_{ij}} \right)^{-1}$$

and, in particular, if $\alpha_i = \left(\sum_{j \in \mathcal{N}_i} 1/R_{ij}\right)^{-1}$ then the minimum of J_i is attained. Indeed in this case we have that

$$\hat{x}_i \leftarrow \left(\sum_{j \in \mathcal{N}_i} \frac{1}{R_{ij}} \right)^{-1} \left(\sum_{j \in \mathcal{N}_i} \frac{\hat{x}_j^{(i)} + z_{ij}}{R_{ij}} \right)$$

which corresponds to the unique solution of the problem

$$\underset{\hat{x}_i}{\operatorname{argmin}} \frac{1}{2} \sum_{j \in N_i} \frac{\left(\hat{x}_i - \hat{x}_j^{(i)} - z_{ij}\right)^2}{R_{ij}}.$$

Next we provide a convenient vector form description of the a-GL algorithm. To do so, we introduce the following definitions. Let $\hat{x}_i(h)$ and $\hat{x}_j^{(i)}(h)$, $j \in \mathcal{N}_i$, denote the estimates that node i has of x_i and of x_j, $j \in \mathcal{N}_i$, respectively, just before time instant t_h. Since we are assuming that there are no communication delays and packet losses, it follows that $\hat{x}_j^{(i)}(h) = \hat{x}_j(h)$, $j \in \mathcal{N}_i$. Then

$$\hat{x}_i(h+1) = \left(1 - \alpha_i \sum_{j \in \mathcal{N}_i} \frac{1}{R_{ij}} \right) \hat{x}_i(h) + \alpha_i \sum_{j \in \mathcal{N}_i} \frac{\hat{x}_j(h) + z_{ij}}{R_{ij}},$$

while $\hat{x}_k(h+1) = \hat{x}_k(h)$, $k \neq i$. Let us rewrite the above equation as

$$\hat{x}_i(h+1) = p_{ii}\hat{x}_i(h) + \sum_{j \in \mathcal{N}_i} p_{ij}\hat{x}_j(h) + u_i, \qquad (4)$$

where

$$p_{ij} = \begin{cases} 1 - \alpha_i \sum_{j \in \mathcal{N}_i} 1/R_{ij} & \text{if } j = i, \\ \alpha_i/R_{ij} & \text{if } j \neq i, j \in \mathcal{N}_i, \\ 0 & \text{otherwise,} \end{cases}$$

and where

$$u_i = \alpha_i \sum_{j \in \mathcal{N}_i} \frac{z_{ij}}{R_{ij}}.$$

Let $P \in \mathbb{R}^{N \times N}$ be the matrix defined by the weights p_{ij} above introduced. Then the updating step at time t_h can be written in vector form as

$$\hat{\mathbf{x}}(h+1) = \left(I + e_i e_i^T (P - I)\right) \hat{\mathbf{x}}(h) + U_i \qquad (5)$$

where $\hat{\mathbf{x}}(h+1) = [\hat{x}_1(h+1), \dots, \hat{x}_N(h+1)]^T$ and where the vector $U_i \in \mathbb{R}^N$ is defined as $U_i = u_i e_i$.

Next we show that, by a proper change of variable, equation (5) can be rewritten as the iteration of a time-varying linear consensus algorithm. To do so, let

$$Q_i = I + e_i e_i^T (P - I),$$

and observe that, if

$$0 < \alpha_i \leq \left(\sum_{j \in \mathcal{N}_i} \frac{1}{R_{ij}} \right)^{-1}, \qquad \forall\, i \in V$$

then the matrix Q_i is a stochastic matrix for all $i \in V$. Indeed all the elements of Q_i are nonnegative and, moreover, one can see that $Q_i \mathbf{1} = \mathbf{1}$. Now let us introduce the auxiliary variable

$$\xi(h) = \hat{\mathbf{x}}(h) - \hat{\mathbf{x}}^*_{\text{opt}}.$$

By exploiting the fact that, for $i \in \{1, \dots, N\}$,

$$\mathbf{x}^*_{\text{opt}} = Q_i \mathbf{x}^*_{\text{opt}} + U_i, \qquad (6)$$

we have that the variable ξ satisfies the following recursive equation

$$\xi(h+1) = Q_i \xi(h). \qquad (7)$$

Observe that $\hat{\mathbf{x}}(h) \to \mathbf{x}^*_{\text{opt}} + \gamma\mathbf{1}$ if and only if $\xi(h) \to \gamma\mathbf{1}$. Moreover, since Q_i is a stochastic matrix for any $i \in \{1, \dots, N\}$, we have that (7) represents a N-dimensional time-varying consensus algorithm.

Remark III.1 It is worth to stress that the a-GL algorithm is a modified version of the algorithm proposed in [9]. The main differences are related to the communication protocol. Specifically, in [9] when a node is activated, say i, firstly it interrogates its neighbors to obtain their estimates $\{\hat{x}_j\}_{j \in \mathcal{N}_i}$; secondly, based on the information received, it updates its own estimate $\hat{x}_i$. This implies that during this iteration of the algorithm there are $|\mathcal{N}_i|+1$ transmitted packets (one packet is related to the broadcast request by node i while the other $|\mathcal{N}_i|$

packets are related to $\{\hat{x}_j\}_{j\in\mathcal{N}_i}$ responses). Instead in the a-GL algorithm, there is just one packet broadcasted during each iteration of the algorithm. This leads to a lighter, faster and energy-saving solution. Additionally in [9] there is no robustness analysis against packet losses.

In next sections, we analyze the convergence properties and the robustness to delays and packet losses of the a-GL algorithm by studying system (7) resorting to the mathematical tools developed in the literature of the consensus algorithms. In particular we will provide our results considering two different scenarios which are formally described in the following definitions.

Definition III.2 (Randomly persistent comm. network) *A network of N nodes is said to be a* randomly persistent communicating network *if there exists a N-upla $(\beta_1,\dots,\beta_N)$ such that $\beta_i>0$, for all $i\in\{1,\dots,N\}$, and $\sum_{i=1}^N\beta_i=1$, and such that, for all $h\in\mathbb{N}$,*

$$\mathbb{P}\left[\mathcal{A}_{i,h}\right]=\beta_i,$$

where $\mathcal{A}_{i,h}$ is the event

$$\mathcal{A}_{i,h}=\{\textit{node } i \textit{ is the node performing steps 1) and 2) of the a-GL algorithm at iteration } h.\}$$

Definition III.3 (Uniformly persistent comm. network) *A network of N nodes is said to be a* uniformly persistent communicating network *if there exists a positive integer number τ such that, for all $h\in\mathbb{N}$, each node perform steps 1) and 2) of the a-GL algorithm at least once within the iteration-interval $[h,h+\tau)$.*

IV. Convergence analysis in the randomly persistent communicating scenario

The following result characterizes the convergence properties of the a-GL algorithm when the network is a randomly persistent communicating network.

Proposition IV.1 *Consider a* randomly persistent communicating network *of N nodes running the a-GL algorithm over a connected measurement graph $\mathcal{G}$. Assume the weights α_i are such that*

$$0<\alpha_i\le\left(\sum_{j\in\mathcal{N}_i}\frac{1}{R_{ij}}\right)^{-1},\qquad\forall\, i\in V,$$

and assume that $\hat{x}_i$, $i\in\{1,\dots,N\}$, $\hat{x}_j^{(i)}$, $j\in\mathcal{N}_i$, be initialized to any real number. Then the following facts hold true

(i) *the evolution $h\to\hat{\boldsymbol{x}}(h)$ converges almost surely to an optimal solution $\boldsymbol{x}^*\in\chi$, i.e., there exists $\gamma\in\mathbb{R}$ such that*

$$\mathbb{P}\left[\lim_{h\to\infty}\hat{\boldsymbol{x}}(h)=\boldsymbol{x}^*_{opt}+\gamma\mathbf{1}\right]=1,$$

(ii) *the evolution $h\to\hat{\boldsymbol{x}}(h)$ is exponentially convergent in mean-square sense, i.e., there exist $C>0$ and $0\le\rho<1$ such that*

$$\lim_{h\to\infty}\mathbb{E}\left[\|\hat{\boldsymbol{x}}(h)-(\boldsymbol{x}^*_{opt}+\gamma\mathbf{1})\|^2\right]\le C\rho^h\mathbb{E}\left[\|\hat{\boldsymbol{x}}(0)-(\boldsymbol{x}^*_{opt}+\gamma\mathbf{1})\|^2\right].$$

The proofs of the above proposition and of the following one are reported in the technical note [10].

V. Robustness to packet losses and delays in the uniformly persistent communicating scenario

In section III we have introduced the a-GL algorithm under the assumptions that

- the communication channels are reliable, i.e., no packet losses occur; and
- the transmission delays are negligible.

In this section we consider a more realistic scenario where the above two assumptions are relaxed. We are still able to prove that the a-GL algorithm converges to an optimal solution provided that the network is uniformly persistent communicating and the transmission delays and the frequencies of communication failures satisfy mild conditions which we formally describe next.

Assumption V.1 (Bounded packet losses) *There exists a positive integer L such that the number of consecutive communication failures between every pair of neighboring nodes in the graph $\mathcal{G}$ is less than L.*

Assumption V.2 (Bounded delays) *Assume node i broadcasts its estimate to its neighbors during iteration h, and, assume that, the communication link (i,j) does not fail. Then, there exists a positive integer D such that the information $\hat{x}_i(h+1)$ is used by node j to perform its local update not later than iteration $h+D$.*

Loosely speaking Assumption V.1 implies that there can be no more than L consecutive packet losses between any pair of nodes i,j belonging to the communication graph. Differently, Assumption V.2 consider the scenario where the received packets are not used instantaneously, but are subject to some delay not greater than D iterations.
This implies that in general $\hat{x}_i^{(j)}(h)=\hat{x}_j(h'_{ij})$ for some h'_{ij} such that $h-(\tau L+D)\le h'_{ij}\le h$. It turns out that the equation update (4) is, in general, modified as

$$\hat{x}_i(h+1)=p_{ii}\hat{x}_i(h)+\sum_{j\in\mathcal{N}_i}p_{ij}\hat{x}_j(h'_{ij})+u_i.$$

The following proposition characterizes the convergence proprieties in presence of bounded packet losses and bounded delays.

Proposition V.3 *Consider a* uniformly persistent communicating network *of N nodes running the a-GL algorithm over* *a connected measurement graph $\mathcal{G}$. Let Assumptions V.1 and V.2 be satisfied. Assume the weights α_i are such that*

$$0 < \alpha_i < \left(\sum_{j \in \mathcal{N}_i} \frac{1}{R_{ij}} \right)^{-1}, \qquad \forall i \in V,$$

and assume that $\hat{x}_i$, $i \in \{1, \ldots, N\}$, $\hat{x}_j^{(i)}$, $j \in \mathcal{N}_i$, be initialized to any real number. Then the following facts hold true

(i) *the evolution $h \to \hat{\boldsymbol{x}}(h)$ asymptotically converges to an optimal estimate $\boldsymbol{x}^* \in \chi$, i.e., there exists $\gamma \in \mathbb{R}$ such that*

$$\lim_{h \to \infty} \hat{\boldsymbol{x}}(h) = \boldsymbol{x}^*_{opt} + \gamma \mathbf{1};$$

(ii) *the convergence is exponential, namely, there exists $C > 0$ and $0 \leq \rho < 1$ such that*

$$\|\hat{\boldsymbol{x}}(h) - \left(\boldsymbol{x}^*_{opt} + \alpha \mathbf{1}\right)\| \leq C \rho^h \|\hat{\boldsymbol{x}}(0) - \left(\boldsymbol{x}^*_{opt} + \gamma \mathbf{1}\right)\|.$$

Observe that in Proposition V.3 it is assumed that α_i is strictly smaller than $\left(\sum_{j \in \mathcal{N}_i} \frac{1}{R_{ij}}\right)^{-1}$, while the result in Proposition IV.1 holds true also if the equality is satisfied.

VI. Numerical Results

In this section we provide some simulations implementing and comparing the a-GL with three different algorithms: the Randomized Extended Kaczmarz algorithm (REK), the asynchronous Consensus-based Localization algorithm (a-CL) and the Broadcast Coordinate (BC). A brief description of these algorithms can be found in [7].

Example VI.1 In the example we consider a ring graph generated with $N = 50$.Every measurement was corrupted by Gaussian noise with variance $\sigma^2 = 10^{-2}$.
In Figure 1 we plotted the behavior of the error

$$J(h) = \log \left(\|A(\hat{\mathbf{x}}(h) - \mathbf{x}^*)\| \right).$$

Observe that the trajectory of J decreases exponentially.

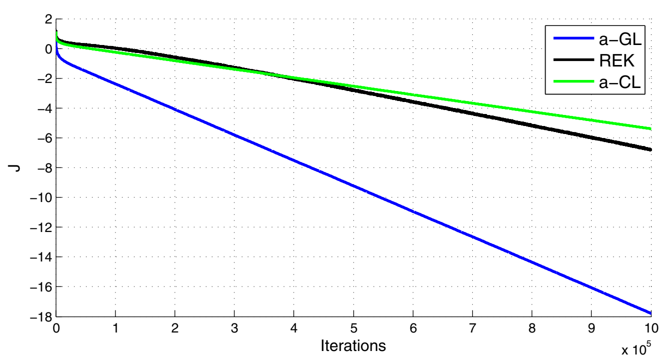

Fig. 1: Comparison of various algorithm in a ring graph.

From the simulation we observe that the a-GL algorithm is the fastest algorithm.

Example VI.2 In this example we assume the same framework of the example VI.1 with the difference that here we are verifying the capability of the a-GL algorithm to converge also if the packets received are delayed. Indeed, from Figure 2 we can see that the algorithm still converge to the optimal solution but with a slower convergence rate.

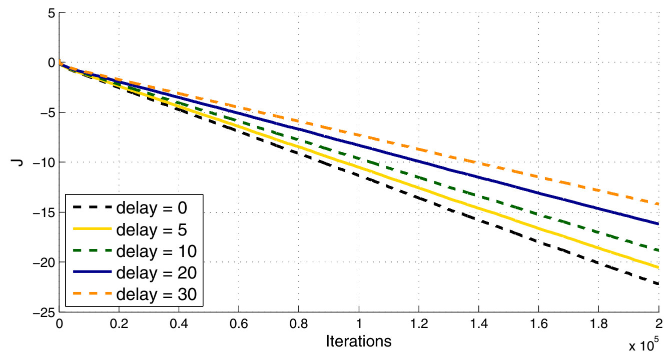

Fig. 2: Comparison of various algorithm in a ring graph.

Example VI.3 In this example we compare the performances of the algorithm proposed in [9], the BC, and the a-GL algorithm. We consider a random geometric graph with $N = 20$ nodes. In Figure 3 we plot the behavior of J respect to the number of sent packets. As we can see the a-GL is much faster then the BC algorithm, so can be considered the most convenient from an energy point of view.

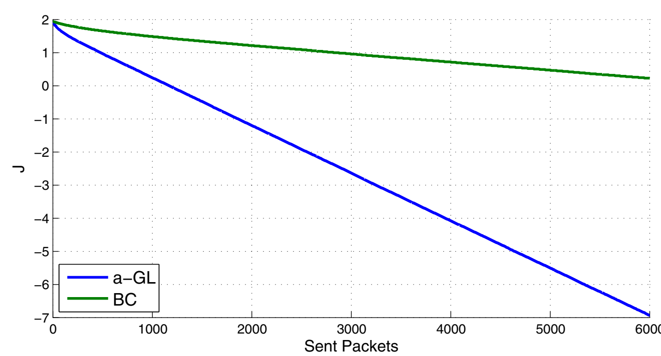

Fig. 3: Comparison of a-GL and BC w.r.t. the number of sent packets.

VII. Conclusions

In this paper we consider the problem of optimally estimating the position of each agent in a network from relative noisy distances. After having formulated the problem in a least-square framework, we proposed a revisited and more efficient version of the algorithm presented in [9]. We proved that the trajectories generated by the algorithm converges to the optimal solution exponentially and that the algorithm is robust against packet losses and bounded delays.

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