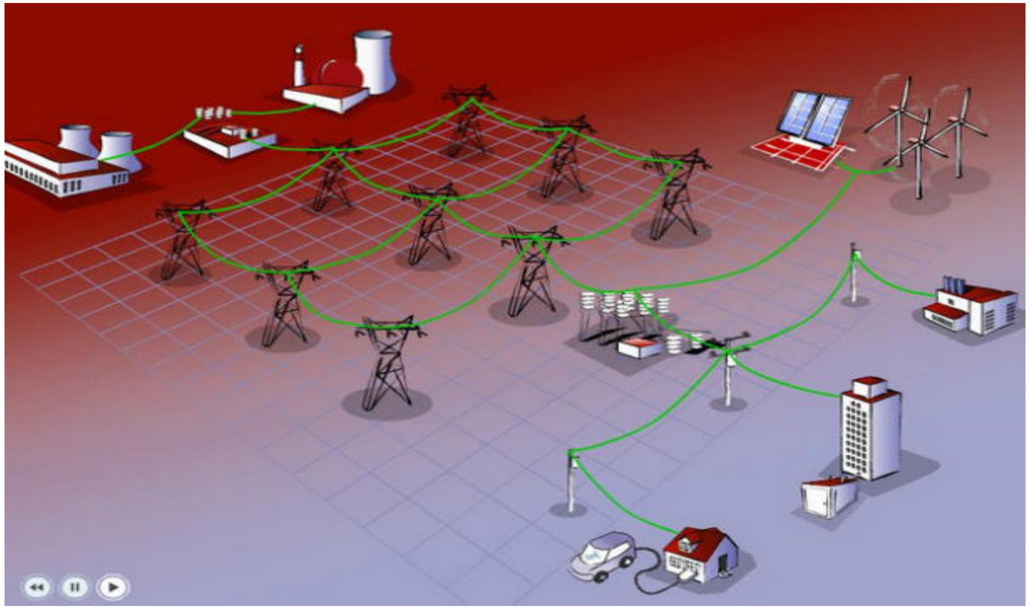
Bayesian Linear State Estimation using Smart Meters and PMUs Measurements in Distribution Grids



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Can we bring PMU to distribution grids ?

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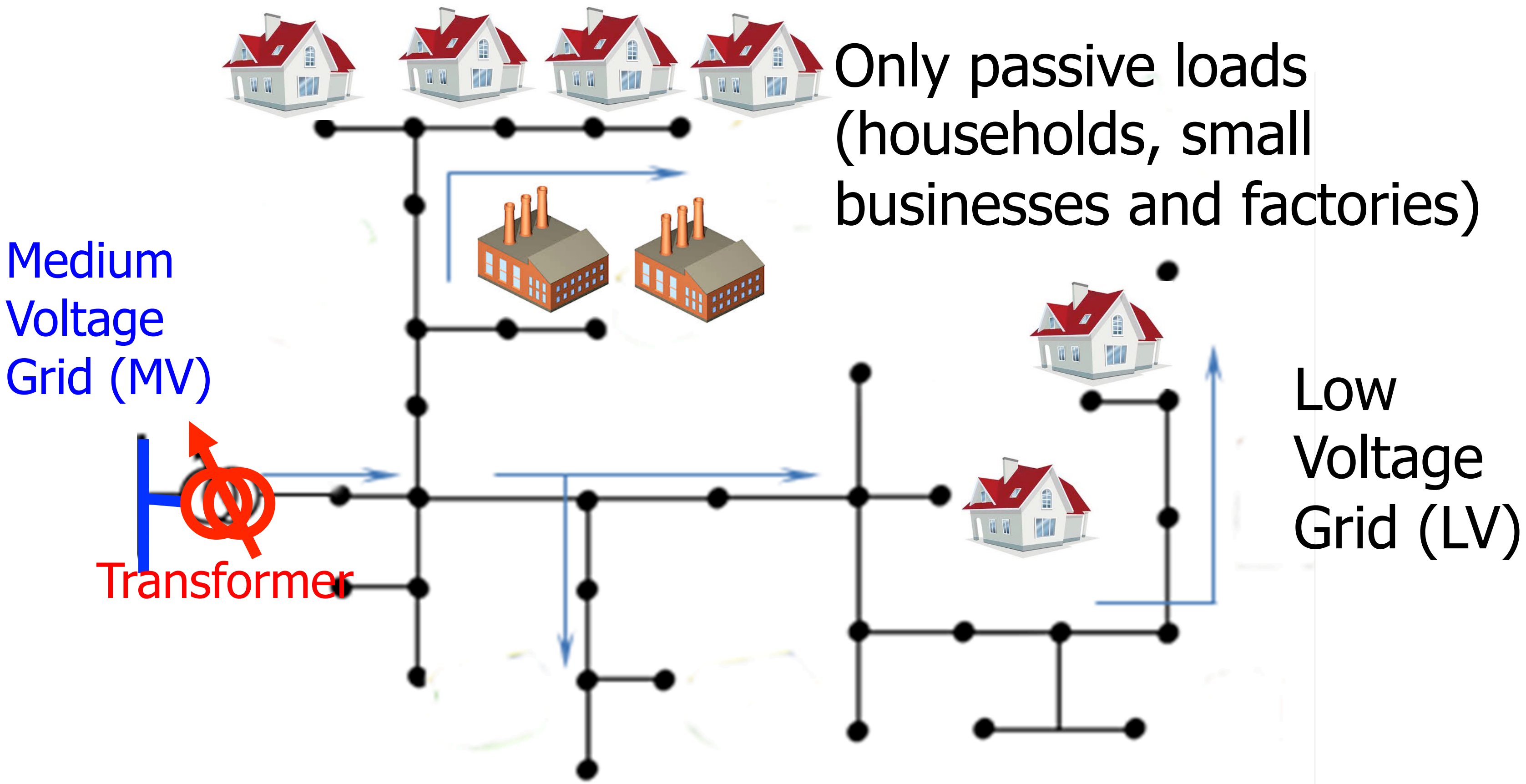
Micro-synchrophasors for distribution systems

- Develop a ultra-high-resolution micro-PMU for measuring voltage angles
- develop a wireless network optimized for power distribution systems
- deploy a hundred of these PMUs at participating utilities
- Investigate diagnostic and control applications

Traditional Distribution Grid



- Passive loads
- Oversized for peak loads
- No need for (real-time) monitoring

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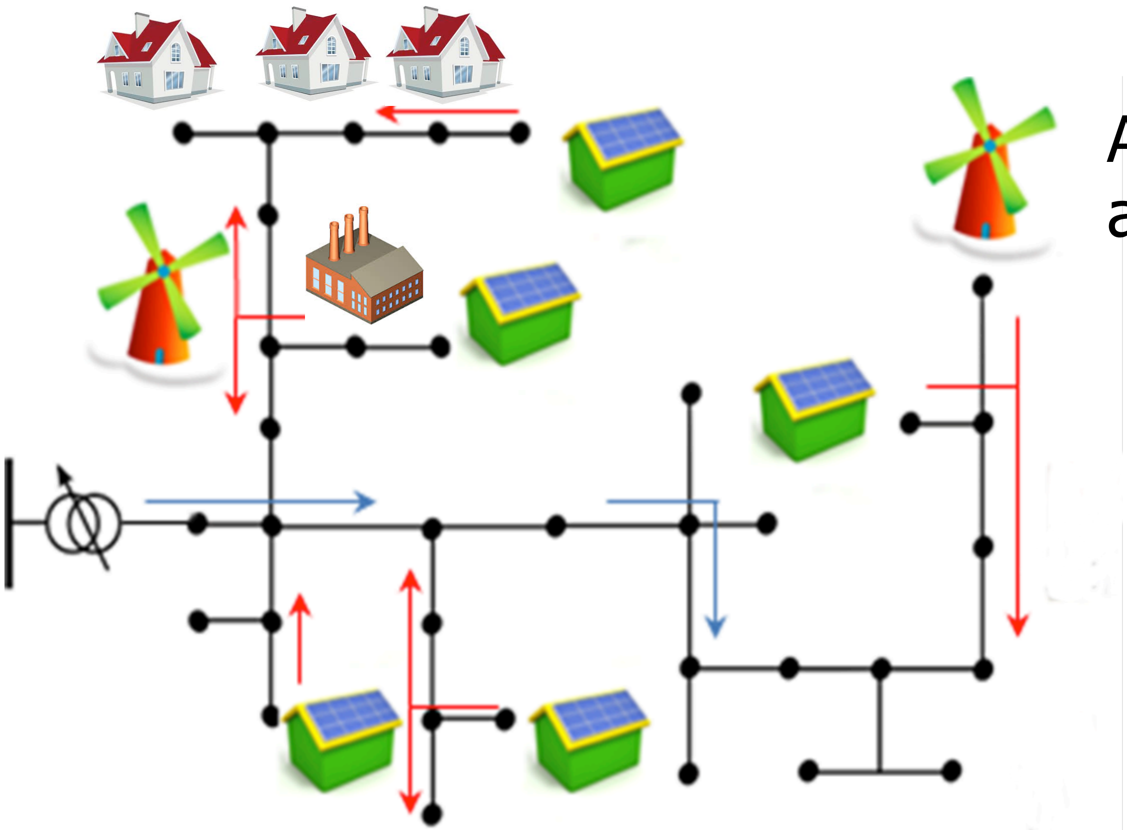
Future Distribution Grid



Active loads (DRES) are being added !!

- DRES might create reverse power flow
- Safety breakers status might not be known
- Rapid power-flow changes

NEED FOR REAL-TIME MONITORING (ESTIMATION)

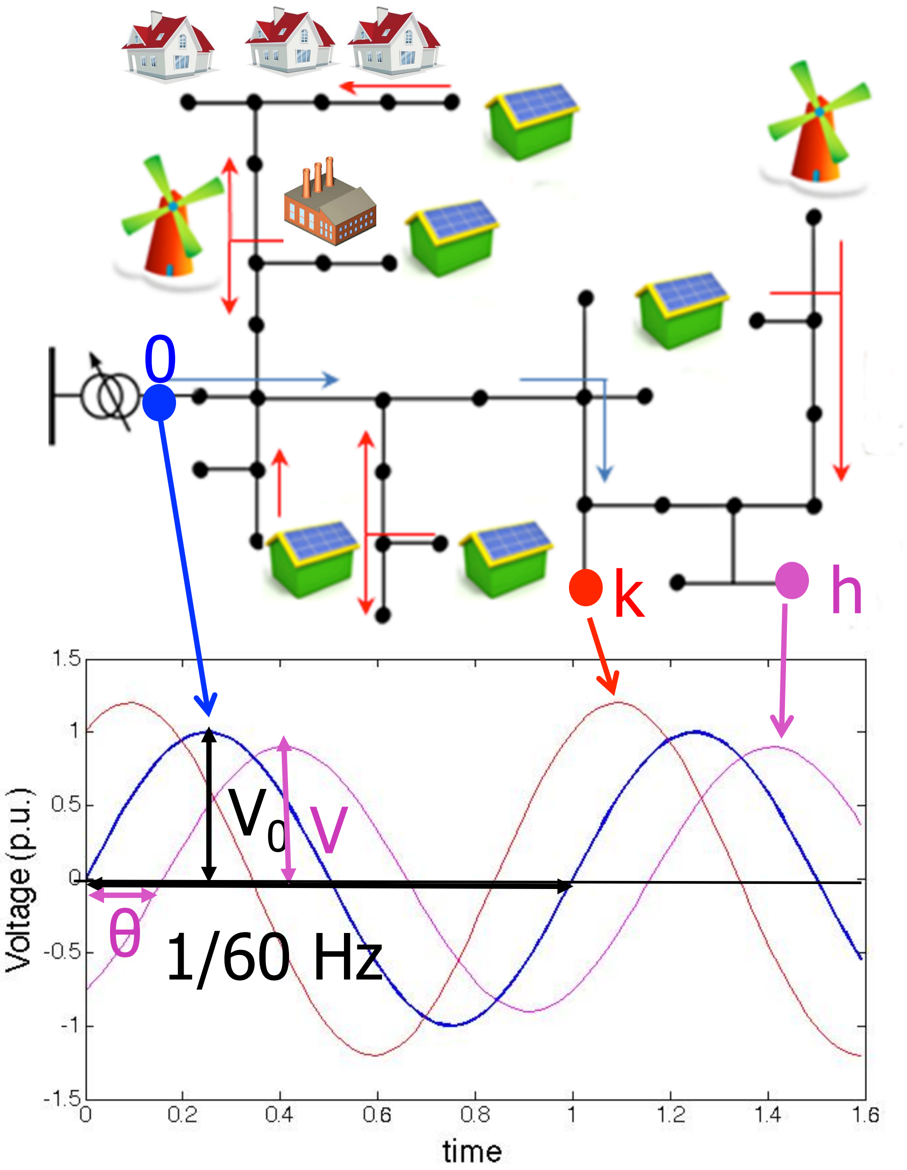
What is state estimation ?

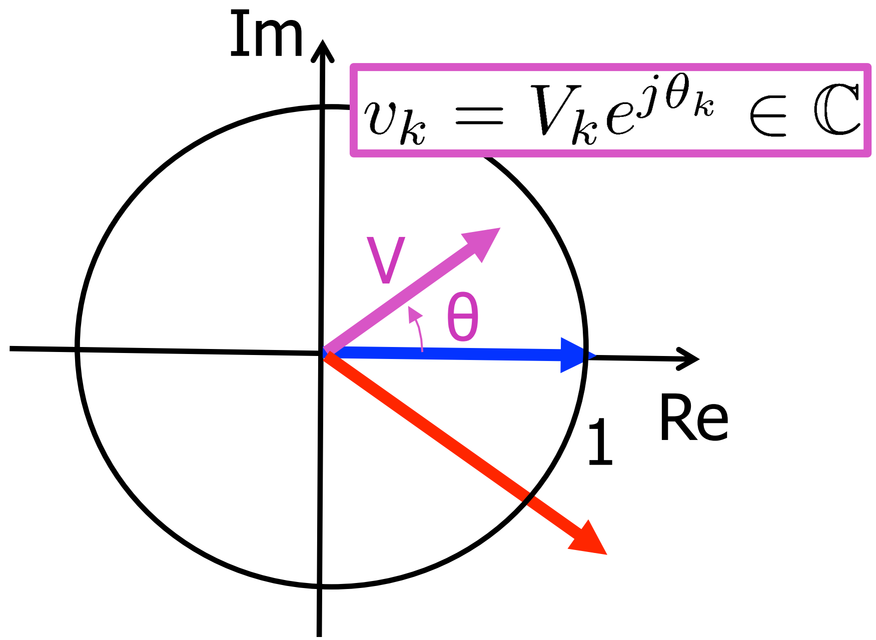
Voltage profile at every node at every point in time: $v_i(t)$

At steady-state and neglecting harmonics:

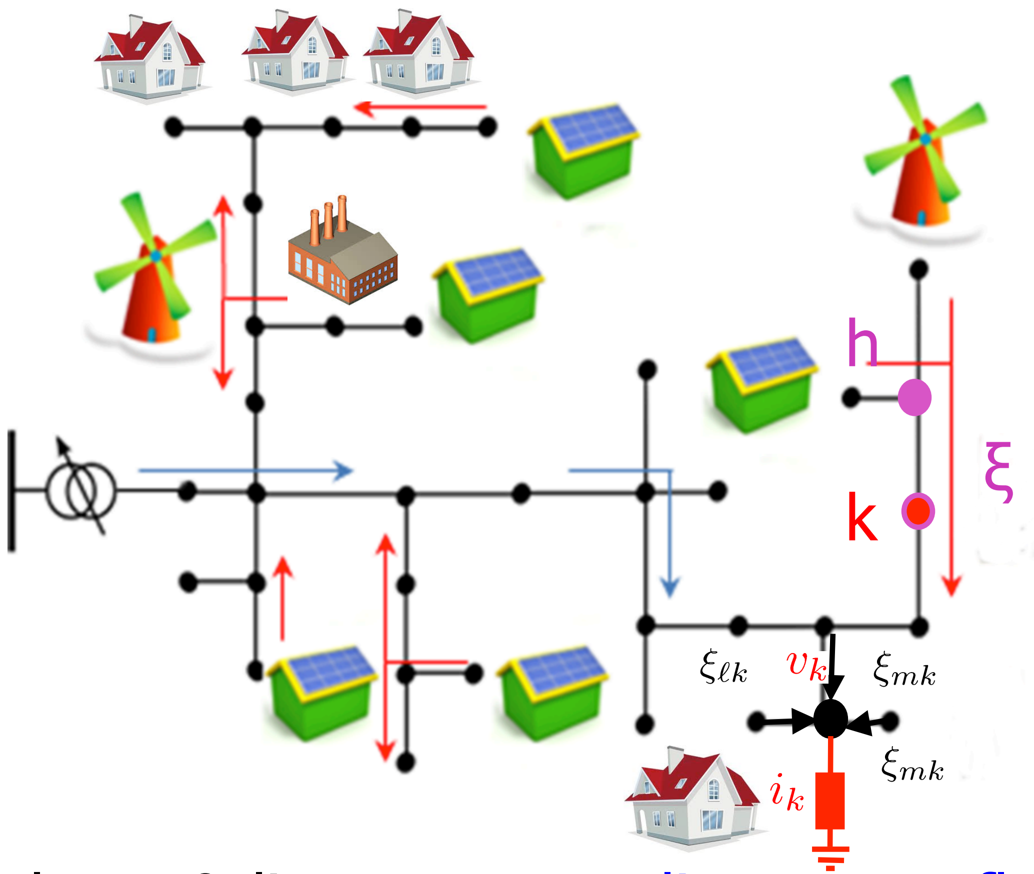
$v_0 = \sin(\omega t)$

$v_h(t) \approx V_h \sin(\omega t + \theta_h)$





Why voltage phasors?



Voltage phasor + grid impendences provide all necessary information

$$z = R + jX \in \mathbb{C}$$

Line currents

$$v_h - v_k = z_{hk}\xi_{hk}$$

Load currents

$$\sum_m \xi_{mk} + i_k = 0$$

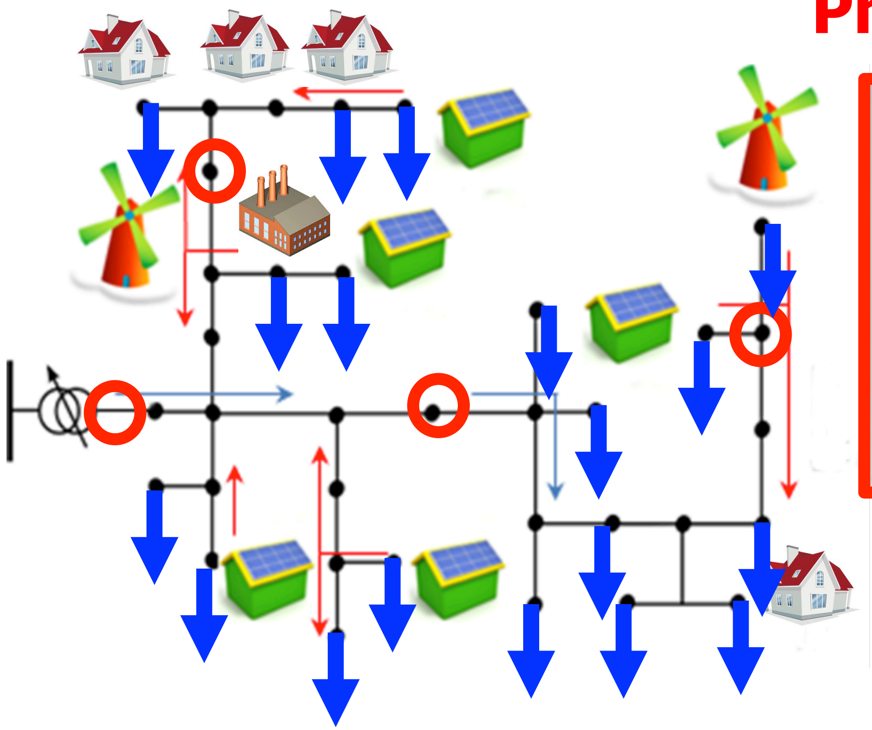
Voltage & line current➔ line power flows $s_{kh} = (v_k - v_h)\overline{\xi}_{hk} \in \mathbb{C}$

Voltage & load current➔ load power flows $s_k = v_k \overline{i}_k \in \mathbb{C}$

Grid state estimation

- Global picture of the grid status
- Enabling Technology :
  - voltage regulation, stability monitoring, contingency analysis, dispatching, fault detection, topology identification
- Hard problem
  - Well studied in transmission grids (Schweppe and Wildes, 1970)
  - Non-linear stochastic problem
  - Lines are mostly resistive in distribution grids
  - Distributiuon grids are often 3-phase unbalanced

Can we measure voltage phasors?



Phasor Measurement Unit (PMU)

- GPS timestamps
- Measure magnitude and phase w.r.t. global time
- Few PMU: costly (5K$)
- Voltage phasor only
- Real-time
- Total Vector Error (TVE): $|v_{true}-v_{meas}|<1\%-0.1\%$

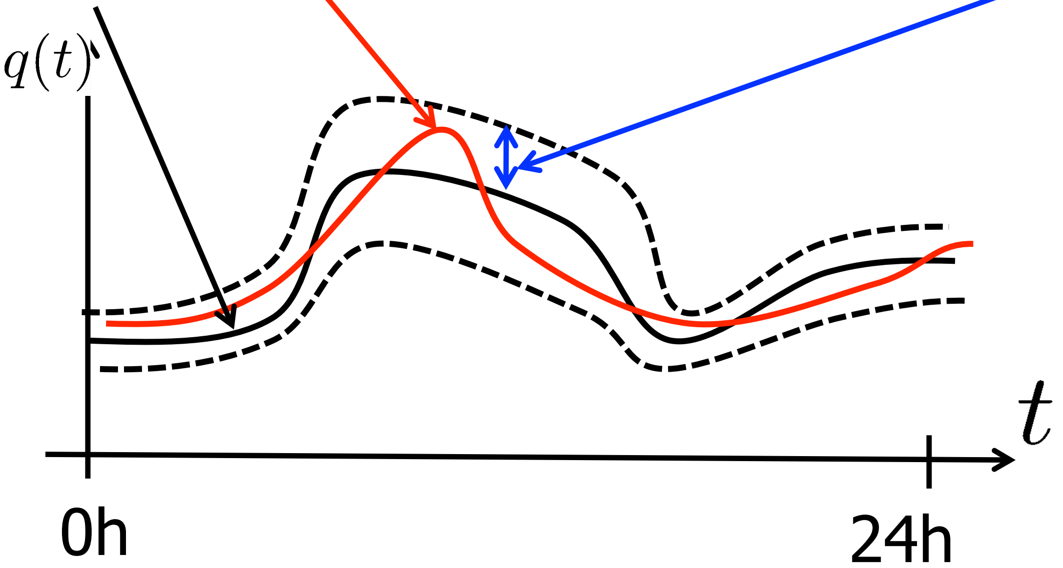
Smart meter

- Measures active (p) and reactive power (q)
- Measures average power over 5-15min
- Data available at “control center” at midnight
- Historical data available **for each household**

Available information: smart meters

SMART METERS:
Based on smart meter historical data, build predictors of $p_k(t)$ and $q_k(t)$ for following day.

$$
\begin{aligned}
P_k(t) &= p_k(t) + w_k^p(t), \quad \mathbb{E}[(w_k^p(t))^2] = \sigma_L^2 P_k^2(t) \\
Q_k(t) &= q_k(t) + w_k^q(t), \quad \mathbb{E}[(w_k^q(t))^2] = \sigma_L^2 Q_k^2(t)
\end{aligned}
$$



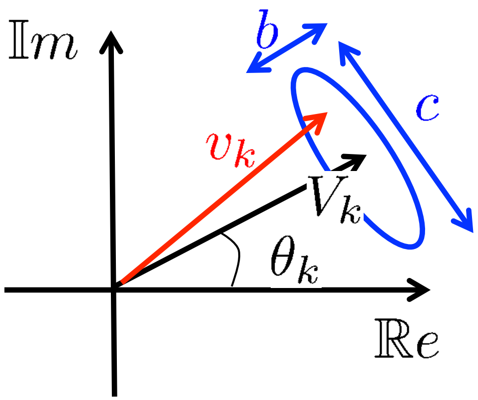
$w_k^p(t), w_k^q(t)$: zero mean prediction error
$p_k(t), q_k(t)$: true power
$P_k(t), Q_k(t)$: predicted power
σ_L: conficence bounds (30-50%) [1]
$p_k(t), q_k(t)$: assumed uncorrelated
$p_k(t), p_h(t)$: prediction errors uncorrelated[1]

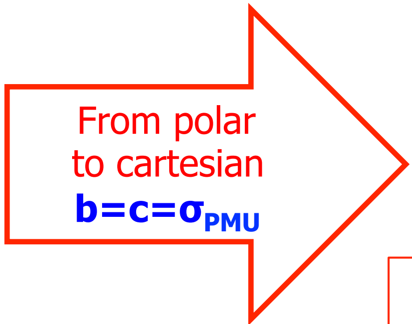
[1] R. Sevlian and R. Rajagopal, "**Short term electricity load forecasting on varying levels of aggregation**," *submitted to IEEE Transactions on Power Systems*

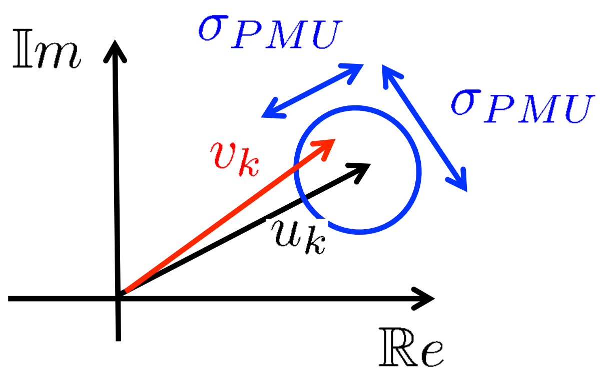
Available information: PMUs

SYNCHRO-PHASOR MEASUREMENT UNIT (PMU):
Multiple samples per AC cycle of voltage(current)
Samples globally synchronized using atomic clock (GPS)
Can compute frequency, magnitude, phase, harmonics, etc..

$$
\begin{aligned}
V_k(t) &= |v_k(t)| + w_k^V(t), \quad \mathbb{E}[(w_k^V(t))^2] = b^2 V_0^2 \\
\theta_k(t) &= \angle v_k(t) + w_k^\theta(t), \quad \mathbb{E}[(w_k^\theta(t))^2] = c^2
\end{aligned}
$$







$v_k(t) \in \mathbb{C}$: true voltage phasor
$V_k(t) \in \mathbb{C}$: measured magnitude
$\theta_k(t)$: measured angle

$$u_k = V_k e^{j\theta_k}$$

$$u_k \approx v_k + V_0 w^{rl} + jV_0 w_k^{im},$$

$$\mathbb{E}[(w_k^{rl})^2] = \mathbb{E}[(w_k^{im})^2] = \sigma_{PMU}^2$$

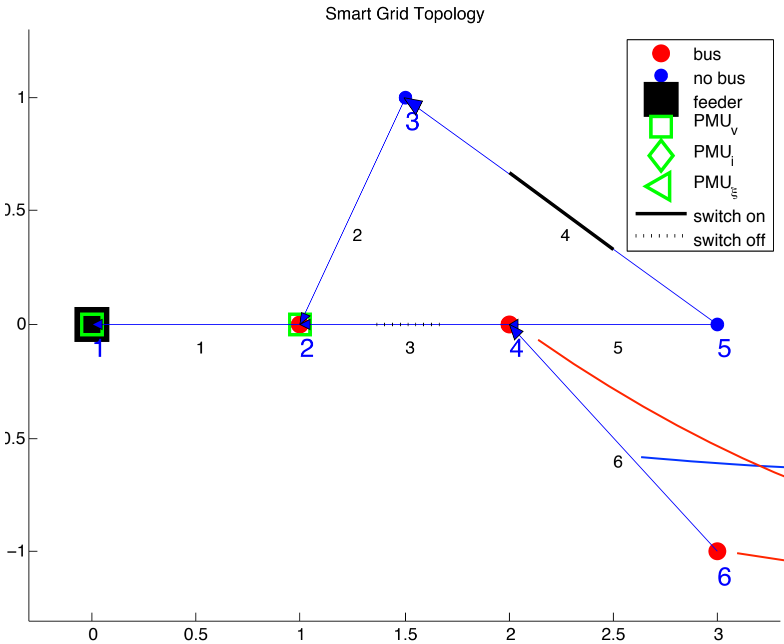
Objectives of this work

- What is the value of PMU in distribution grids ?
- What is the value of Smart Meters ?
- PMU measurement error of PMUs vs P-Q prediction error of smart meters ?
- What is the optimal positioning if only few PMU available ?

Contribution

- Simple Power-Flow solver with P-Q loads
- Linear approximation of PF with P-Q loads
- Bayesian Linear State Estimation
- Same performance but faster then WLS
- Off-line optimal placement of PMUs

Power Grid modeling (single phase)

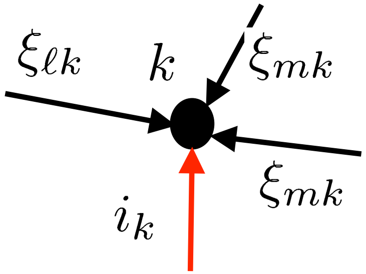


$$Z = diag\{z_1, \ldots, z_E\}$$

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

6 4 6

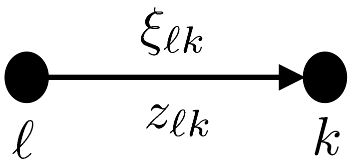
KLC at node k



$$\sum_m \xi_{mk} + i_k = 0$$

$$\boxed{A^T \xi + i = 0}$$

KLV at line (ℓ, k)



$$v_\ell - v_k = z_{\ell k}\xi_{\ell k}$$

$$\boxed{Av + Z\xi = 0}$$

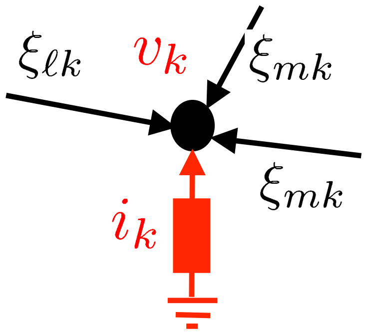
L: admittance matrix

$$L = A^T Z^{-1} A$$

$$\Rightarrow \quad \boxed{Lv = i}$$

Node models

$$f_k(v_k, i_k) = 0$$



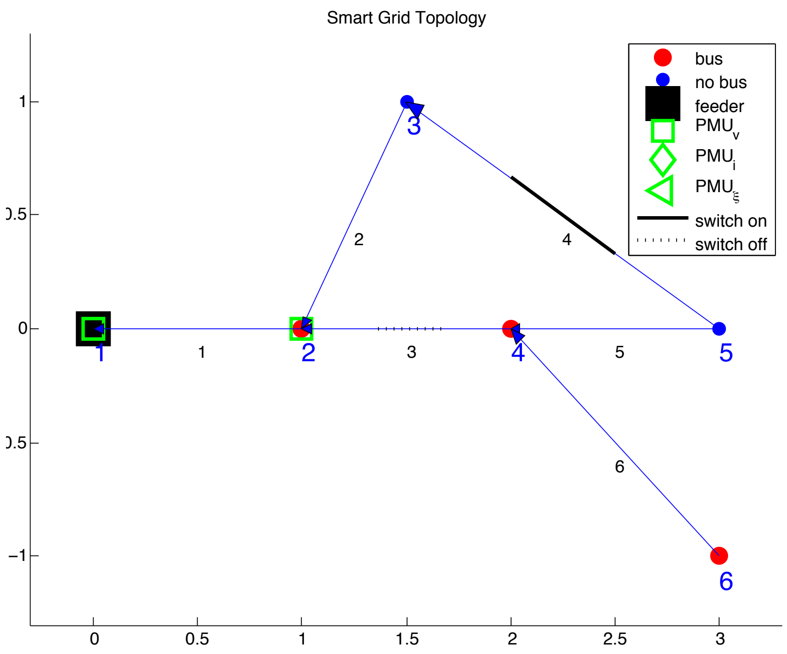
Voltage generator

$$v_k = V_0$$

Current generator

$$i_k = I_k \in \mathbb{C}$$

LINEAR

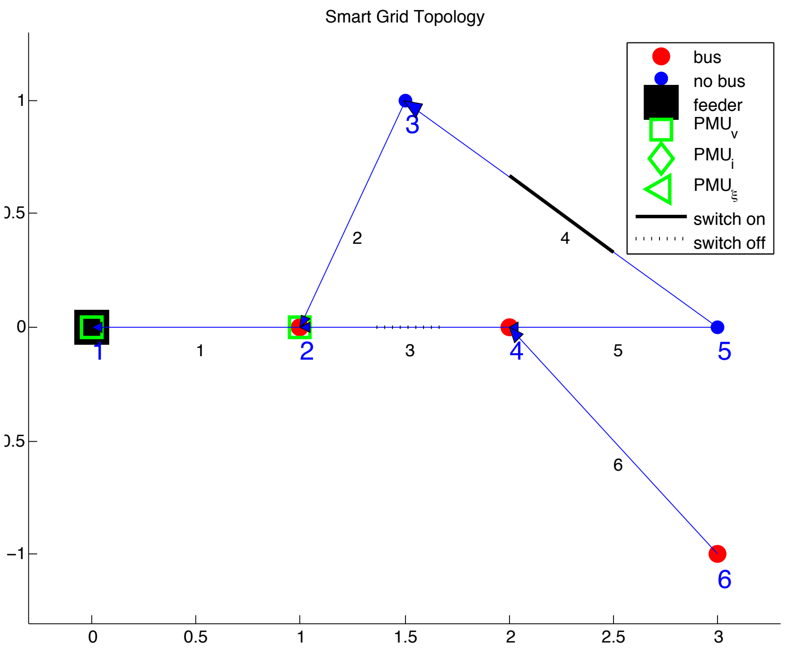


Constant power load (PQ load)

$$v_k \bar{i}_k = p_k + j q_k = s_k \in \mathbb{C}$$

NON LINEAR

Power Flow with PQ loads

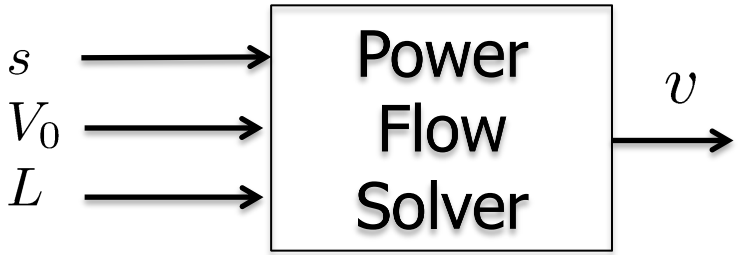


POWER FLOW EQUATIONS:

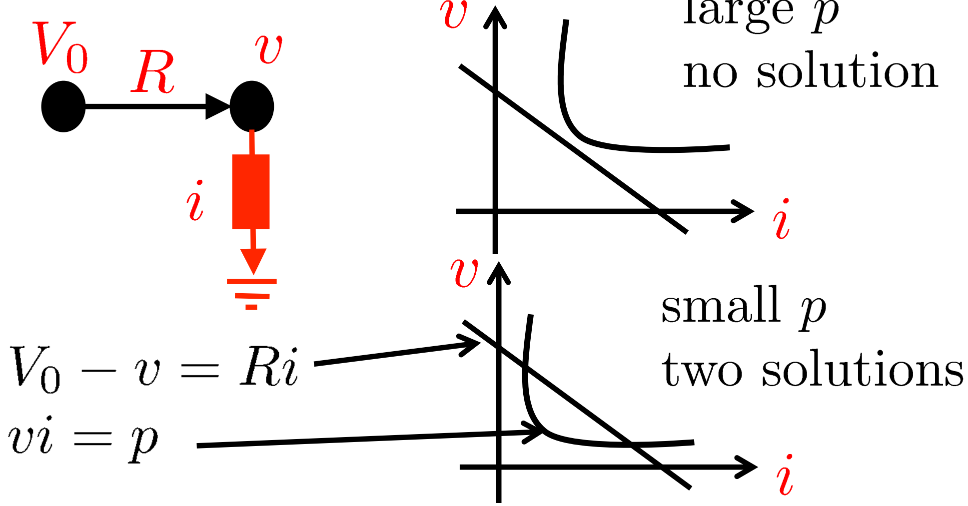
$Lv = i$ KLV+KLC

$v_1 = V_0$ Ideal voltage generator

$v_k \bar{i}_k = s_k, \quad k = 2, \ldots, N$ PQ loads



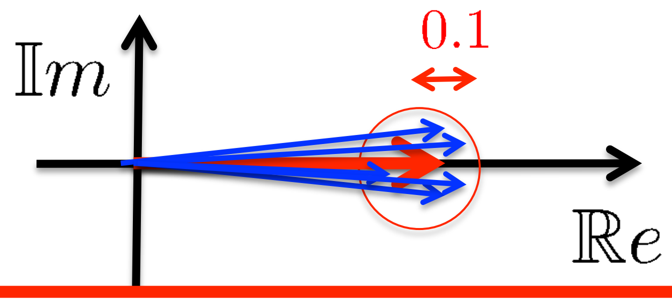
EXAMPLE:



Power Flow equations: A linear approximation

In distribution grids, typically:

$$\frac{|v_k - V_0|}{V_0} \leq 0.02 - 0.1 (2 - 10\%)$$



LINEAR POWER FLOW MODEL

$$i = [i_2 \cdots i_N]^T, \quad v = [v_2 \cdots v_N]^T$$

$$\begin{aligned} i &= \frac{1}{V_0} \bar{s}, \\ Lv &= i \end{aligned}$$

Bayesian Linear State Estimation (BLSE)

LINEAR POWER FLOW MODEL

$i = [i_2 \cdots i_N]^T, \quad v = [v_2 \cdots v_N]^T$

$$i = \frac{1}{V_0}\bar{s},$$

$$v = V_0 \mathbf{1}_N + \frac{1}{V_0} L^{-1} \bar{s}$$

SMART METERS MODEL

$$P_k(t) = p_k(t) + w_k^p(t), \quad \mathbb{E}[(w_k^p(t))^2] = \sigma_L^2 P_k^2(t)$$

$$Q_k(t) = q_k(t) + w_k^q(t), \quad \mathbb{E}[(w_k^q(t))^2] = \sigma_L^2 Q_k^2(t)$$

$$s_k = p_k + jq(k), \qquad k = 2, \ldots, N$$

PMU MODEL

$u_k = V_k e^{j\theta_k},$

$u_k \approx v_k + V_0 w^{rl} + jV_0 w_k^{im},$

$\mathbb{E}[(w_k^{rl})^2] = \mathbb{E}[(w_k^{im})^2] = \sigma_{PMU}^2$

$k \in \mathcal{M} = \{m_1, \ldots, m_M\} \subset \{2, \ldots, N\}$

BLSE: Prior distribution

LINEAR POWER FLOW MODEL

$i = [i_2 \cdots i_N]^T, \quad v = [v_2 \cdots v_N]^T$

$$i = \frac{1}{V_0}\bar{s},$$

$$v = V_0 \mathbf{1}_N + \frac{1}{V_0} L^{-1} \bar{s}$$

+

SMART METERS MODEL

$$P_k(t) = p_k(t) + w_k^p(t),$$

$$Q_k(t) = q_k(t) + w_k^q(t),$$

$$\mathbb{E}[(w_k^p(t))^2] = \sigma_L^2 P_k^2(t)$$

$$\mathbb{E}[(w_k^q(t))^2] = \sigma_L^2 Q_k^2(t)$$

$s_k = p_k + jq(k), \quad k = 2, \ldots, N$

PRIOR DISTRIBUTION ON VOLTAGE PHASORS:

$$v^0 := \mathbb{E}[v] = V_0 \mathbf{1}_N + \frac{1}{V_0} L^{-1} \overline{S}$$

$$\Sigma_0 : = \mathbb{E}[(v - v^0)(v - v^0)^*] = \sigma_L^2 \frac{1}{V_0^2} L^{-1} \Sigma_s (L^{-1})^*$$

$$\Sigma_s = diag\{|S_2|^2, \ldots, |S_N|^2\}$$

$$S = [S_2 \cdots S_N]^T, \quad S_k = p_k + jQ_k$$

BLSE: posterior distribution

PRIOR DISTRIBUTION ON VOLTAGE PHASORS:

$$
\begin{aligned}
v^0 &:= \mathbb{E}[v] = V_0 \mathbf{1}_N + \frac{1}{V_0} L^{-1} \overline{S} \\
\Sigma_0 : &= \mathbb{E}[(v-v^0)(v-v^0)^*] = \sigma_L^2 \frac{1}{V_0^2} L^{-1} \Sigma_s (L^{-1})^* \\
\Sigma_s &= diag\{|S_2|^2, \dots, |S_N|^2\} \\
S &= [S_2 \cdots S_N]^T, \quad S_k = p_k + jQ_k
\end{aligned}
$$

\+

PMU MODEL

$$
\begin{aligned}
& u_k = V_k e^{j\theta_k}, \\
& u_k \approx v_k + V_0 w^{rl} + jV_0 w_k^{im}, \\
& \mathbb{E}[(w_k^{rl})^2] = \mathbb{E}[(w_k^{im})^2] = \sigma_{PMU}^2 \\
& k \in \mathcal{M} = \{m_1, \dots, m_M\} \subset \{2, \dots, N\}
\end{aligned}
$$

POSTERIOR DISTRIBUTION ON VOLTAGE PHASORS:

$$
\widehat{v} := \mathbb{E}[v \,|\, u_{\mathcal{M}}] = v^0 + K(u_{\mathcal{M}} - v^0_{\mathcal{M}})
$$

$$
\Sigma_{\mathcal{M}} = \Sigma_0 - \underbrace{\Sigma_0 C_{\mathcal{M}}^T (C_{\mathcal{M}} \Sigma_0 C_{\mathcal{M}}^T + 2V_0^2 \sigma_{PMU}^2 I_M)^{-1}}_{K} C_{\mathcal{M}} \Sigma_0
$$

$$
C_{\mathcal{M}} = \left.\begin{bmatrix}
0 & \underbrace{1}_{m_1} & 0 & \dots & 0 & 0 \\
0 & 0 & 0 & \underbrace{1}_{m_2} & \dots & 0 \\
\vdots & \vdots & \vdots & & & \vdots \\
0 & 0 & 0 & \dots & \underbrace{1}_{m_2} & 0
\end{bmatrix}\right\} M
$$

Weighted-Least-Squares State Estimator (WLS)

Non-linear measurement model

$$z = h(x) + e$$

$x = [V_1, \ldots, V_N, \theta_1, \ldots, \theta_N]$: voltage magnitude and phase of all nodes

$z = [V_1^{PMU}, \ldots, V_M^{PMU}, \theta_1^{PMU}, \theta_M^{PMU}, P_1, \ldots, P_N, Q_1, \ldots, Q_N]$: PMU magnitude and phase measurements, P-Q forecasts

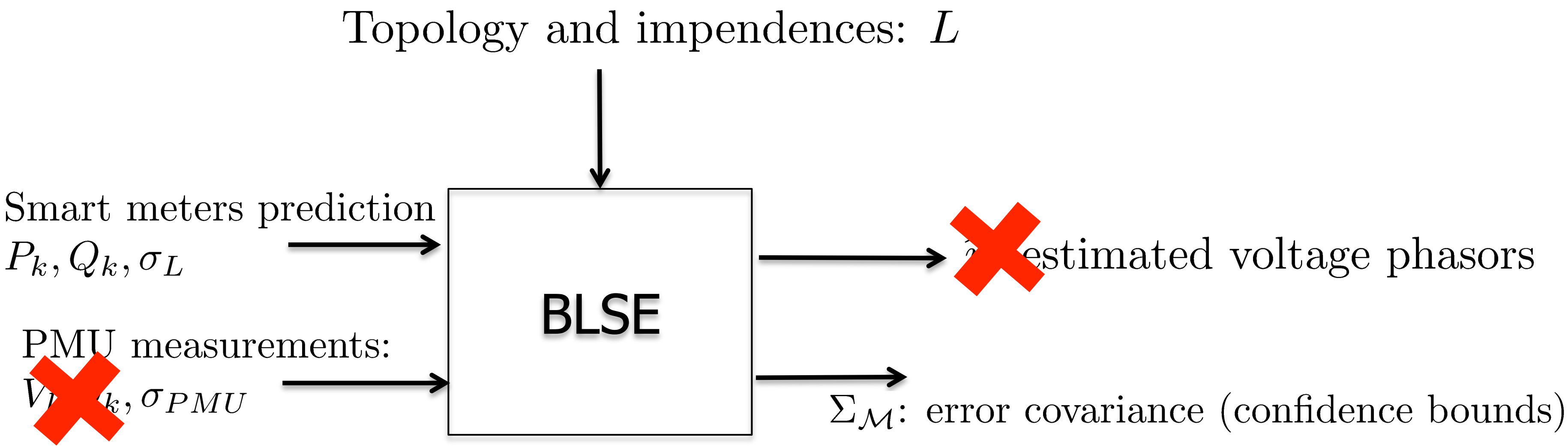
$e = [e_1, \ldots, e_{M+N}]$, $\mathbb{E}[e_i^2] = R_{ii}$: i.i.d. white noise

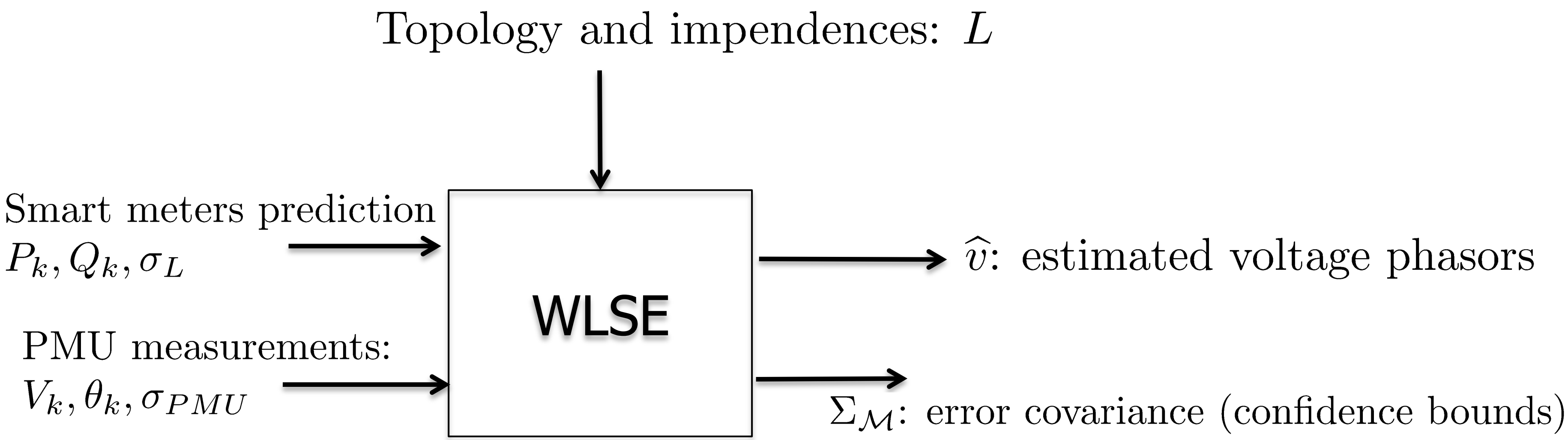
The problem is usually solved by Weighted-Least-Squares method:

$$\min_x = J(x) = \sum_{i=1}^{N+M} \left[z_i - h_i(x)\right]^2 / R_{ii} = \left[z - h(x)\right]^T R^{-1} \left[z - h(x)\right]$$

A. Abur, A. Gomez Exposito, "**Power System State Estimation: Theory and Implementation**," CRC Press, 2004

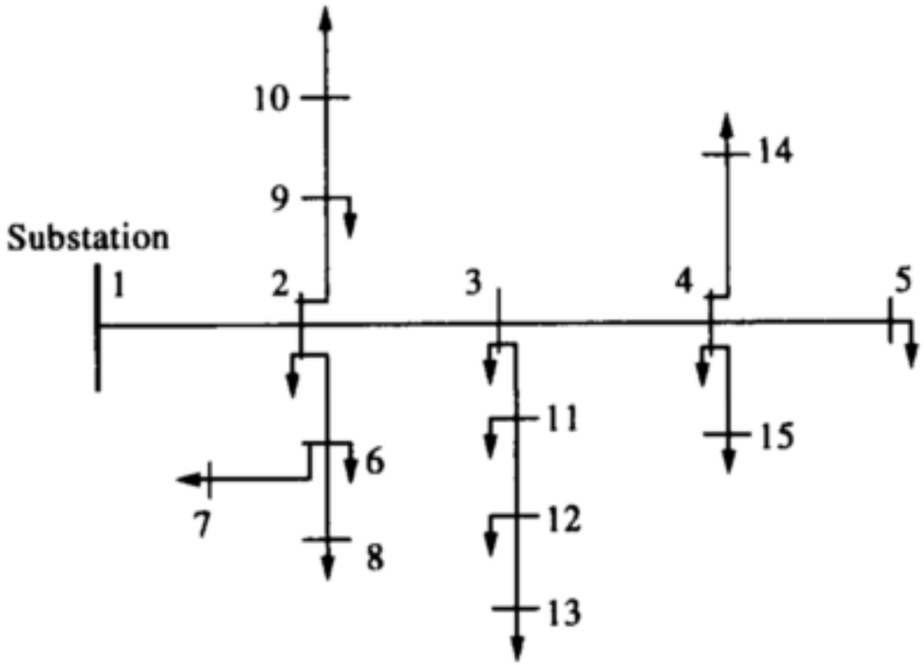
BLSE and WLS: block diagram





Performance Analysis

Example - 15-nodes radial distribution network



Performance metric:

Theoretical (off-line)

average RMSE:

$$ARMSE(\mathcal{M}) := \sqrt{\frac{1}{N}\sum_{k=1}^{N}\mathbb{E}[|v_k - \widehat{v}_k|^2]} = \sqrt{\frac{1}{N}trace(\Sigma_{\mathcal{M}})}$$

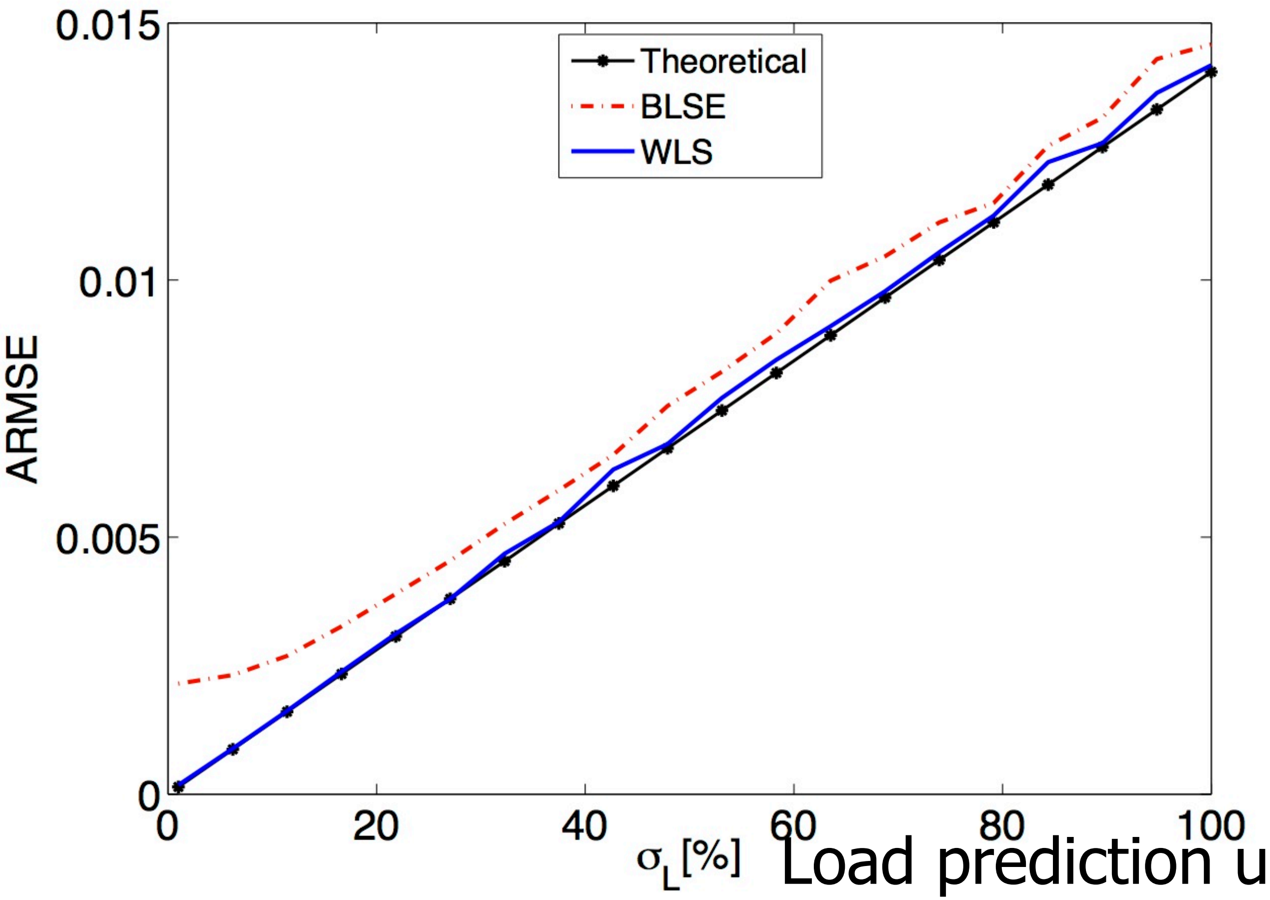
Empirical (Monte Carlo)

average RMSE:

$$\approx \widehat{ARMSE} := \sqrt{\frac{1}{TN}\sum_{t=1}^{T}\sum_{k=1}^{N}\left|\frac{v_k^{[t]} - \hat{v}_k^{[t]}}{V_0}\right|^2}$$

D.Das, D. Kothari, and A. Kalam, **"Simple and efficient method for load flow solution of radial distribution networks,"** *International Journal of Electrical Power & Energy Systems,* vol. 17, no. 5, pp. 335 – 346, 1995.

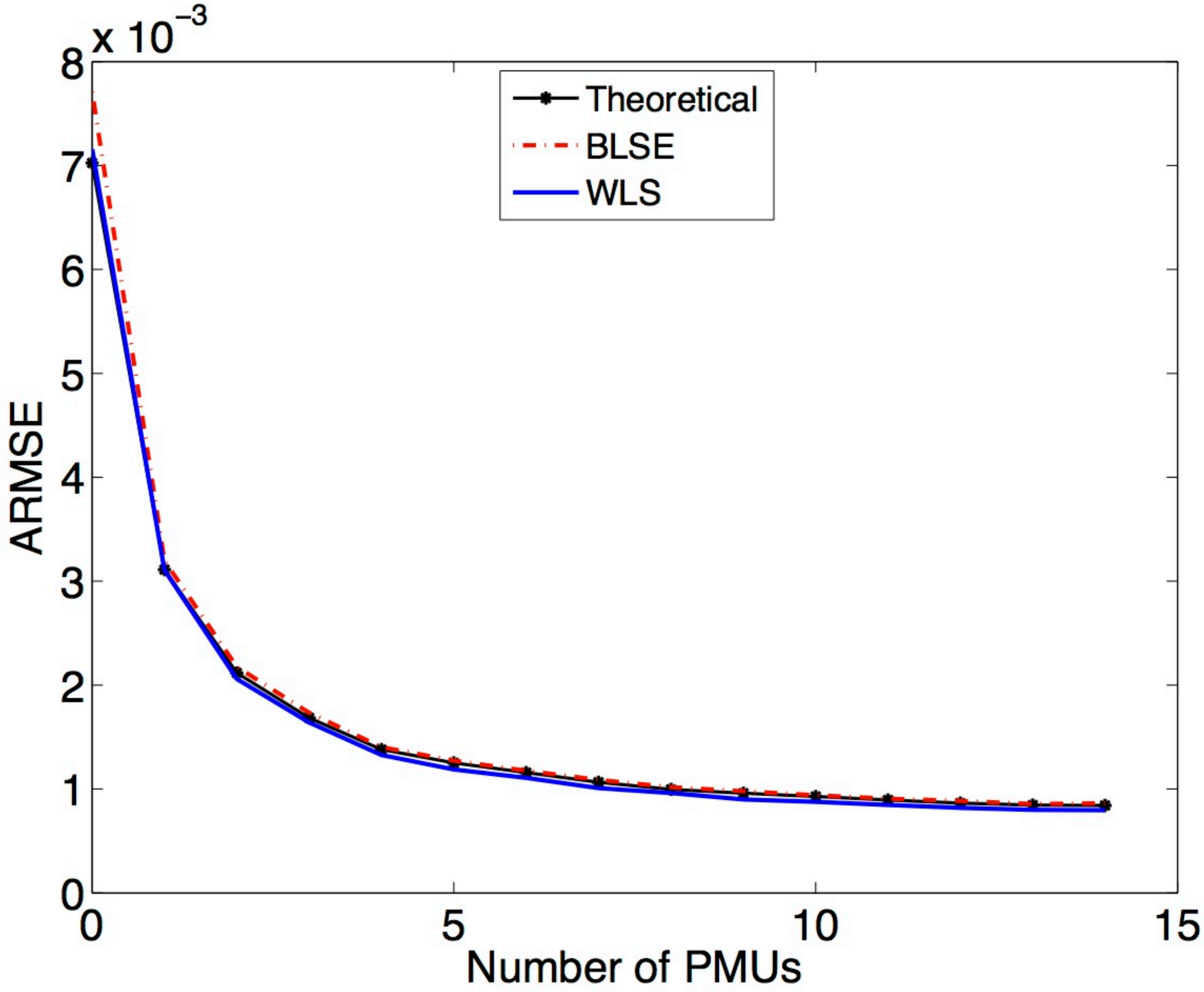
Simulation results: linear vs nonlinear model



Load prediction uncertainty

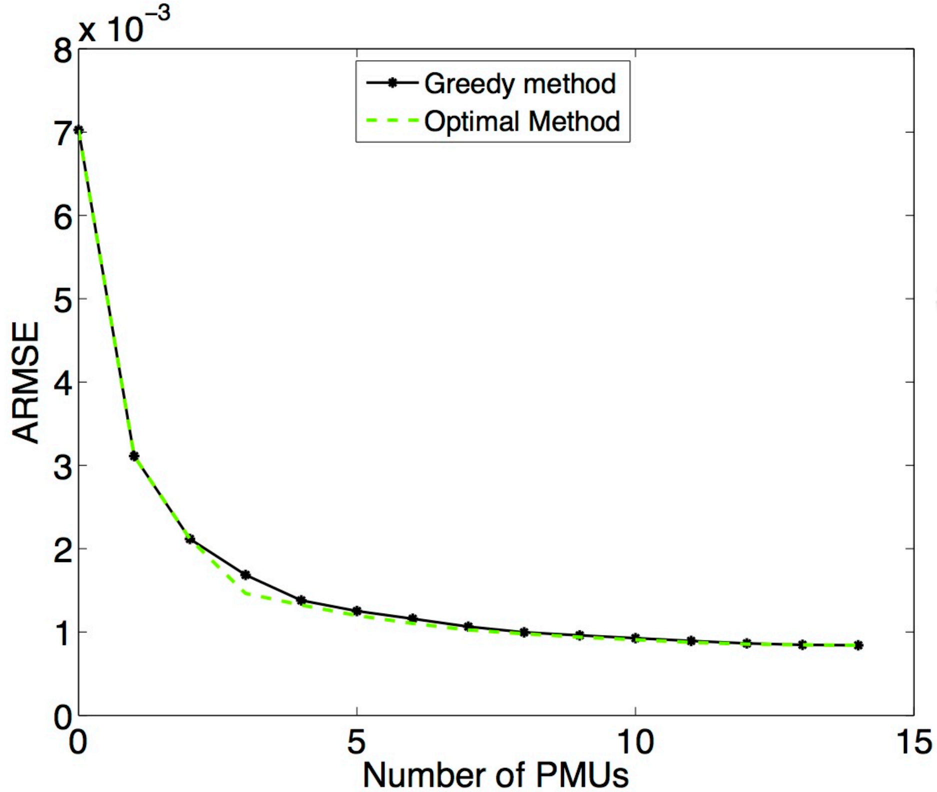
NO PMU

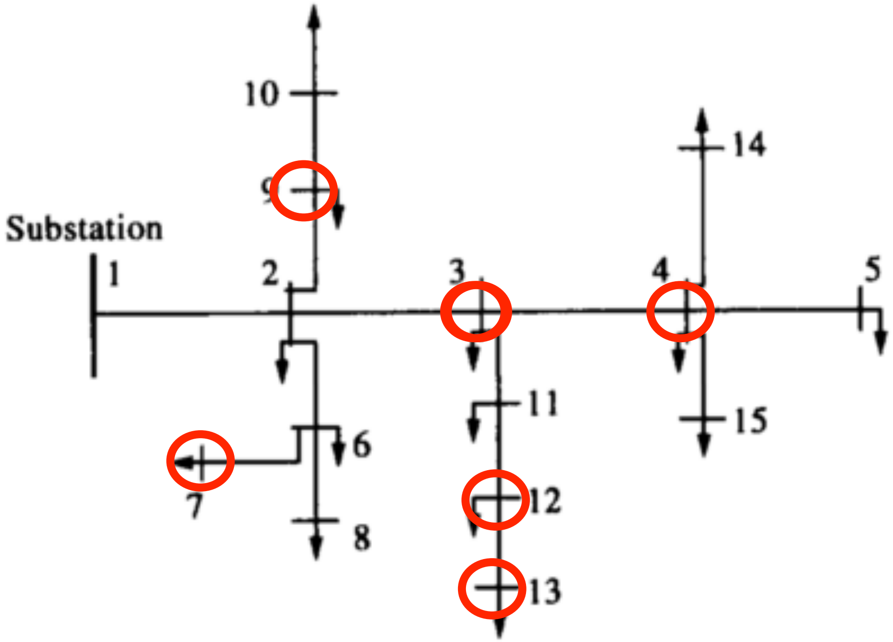
Simulation results: BLSE vs WSE



Simulation result: Greedy vs Optimal placement

ARMSE as a function of number of PMUs with σ_L = 0.1% and σ_{PMU}=0.1% for BLSE

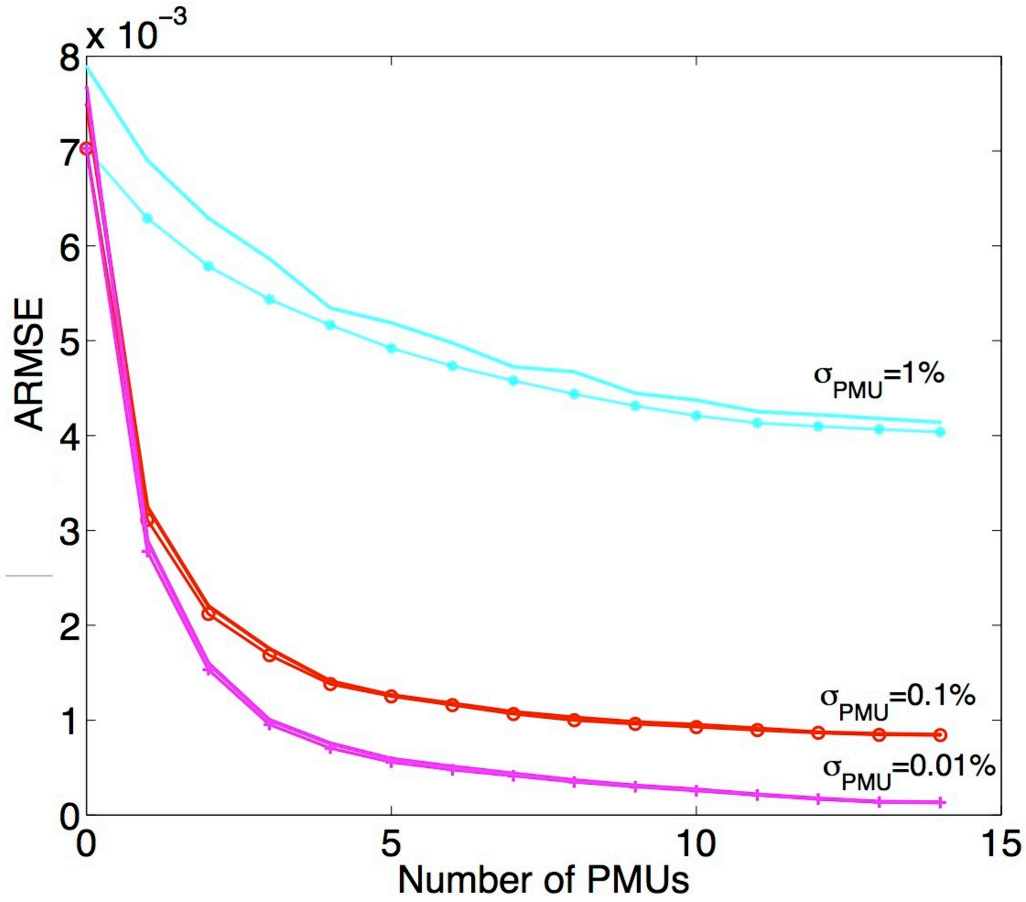




$$\mathcal{M}^{gr}(N) = \{3, 7, 13, 15, 10, 14, 8, 12, 5, 11, 6, 9, 4, 2\}$$

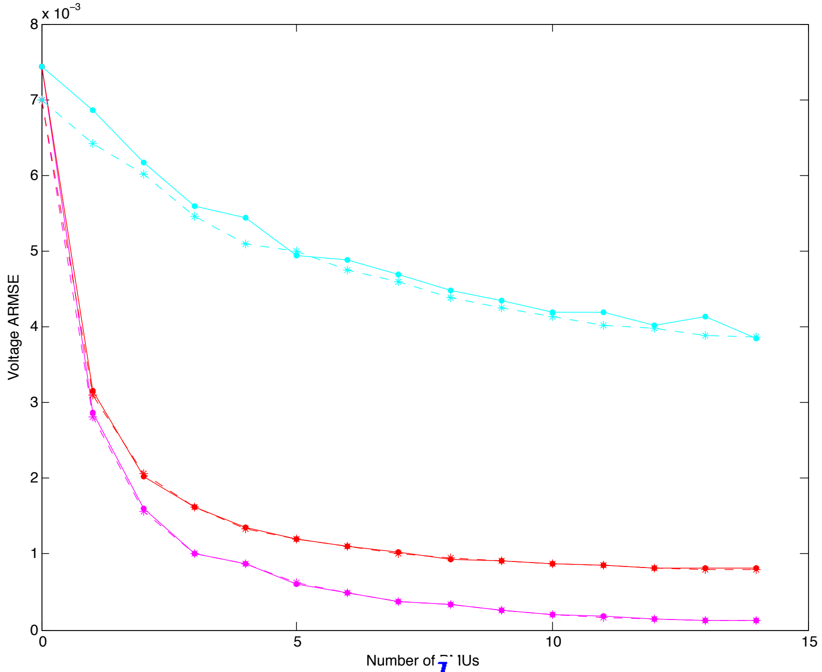
$$\mathcal{M}^{op}(1) = \{3\}$$
$$\mathcal{M}^{op}(2) = \{3, 7\}$$
$$\mathcal{M}^{op}(3) = \{7, 12, 15\}$$
$$\mathcal{M}^{op}(4) = \{7, 10, 12, 15\}$$

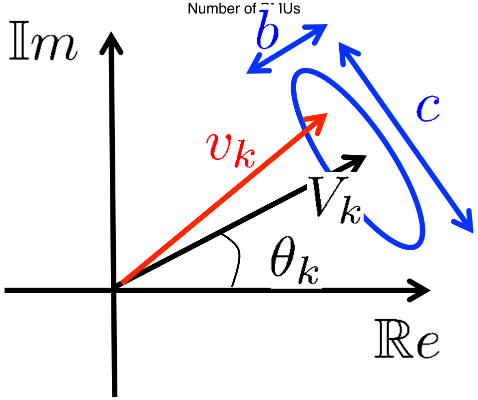
Simulation result: performance vs PMU accuracy



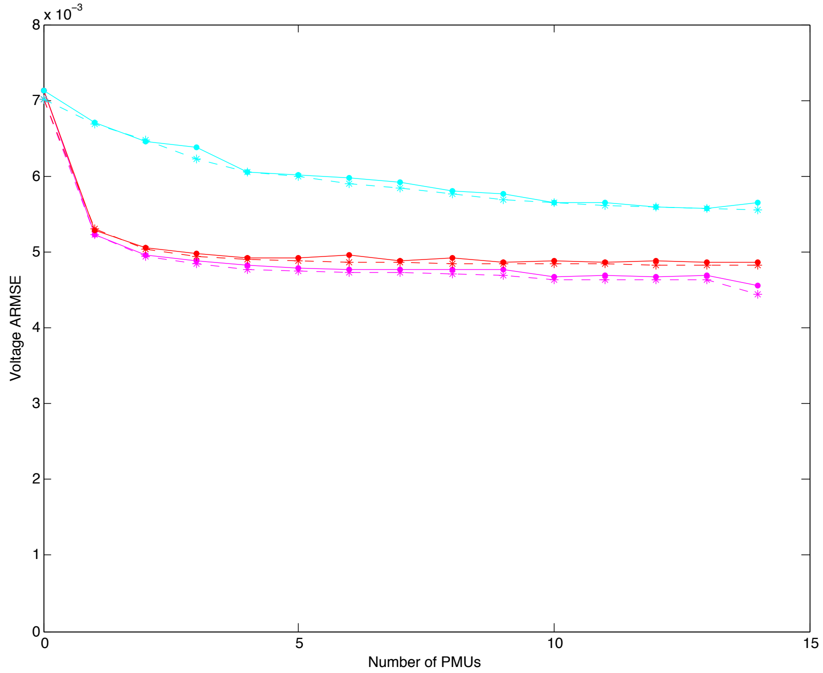
Simulation result: the value of phase measurements

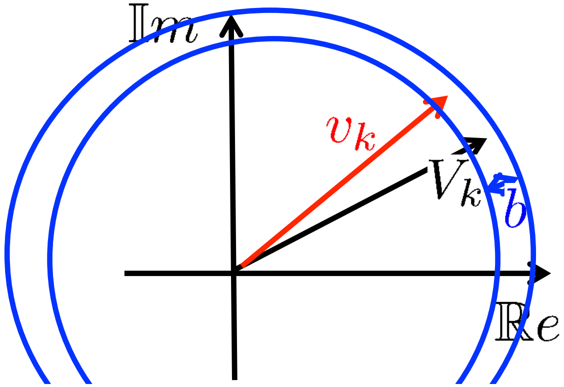
Magnitude and phase (PMU)



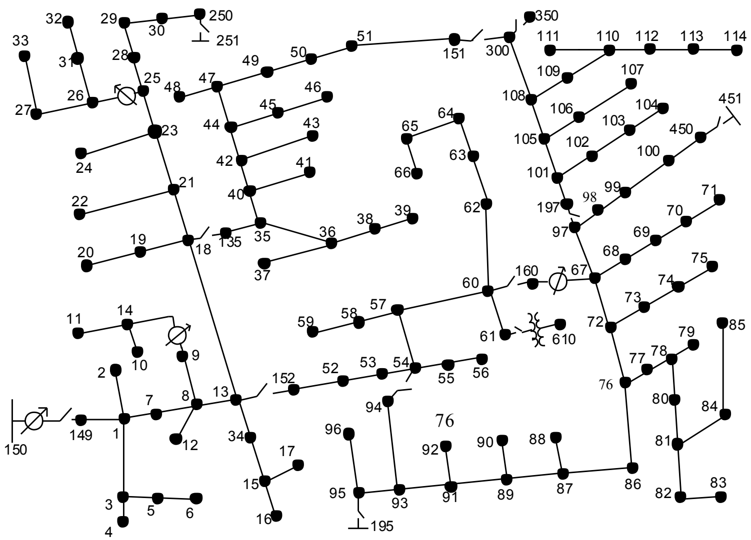


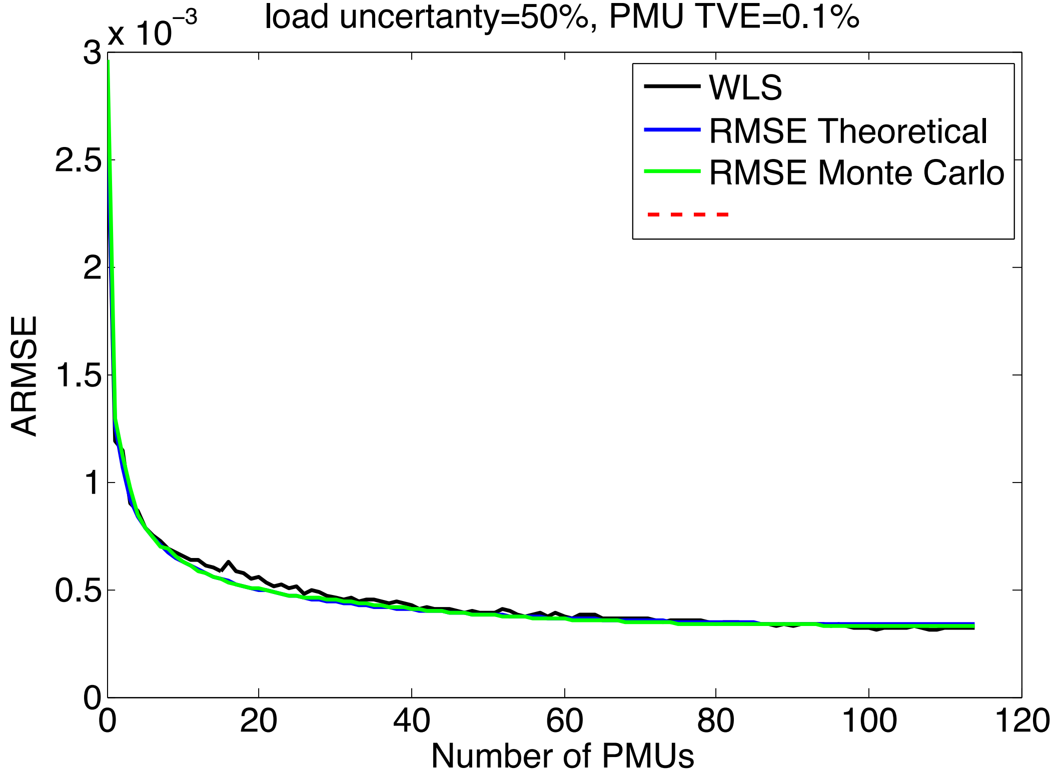
Magnitude only





IEEE 123 node Test Feeder





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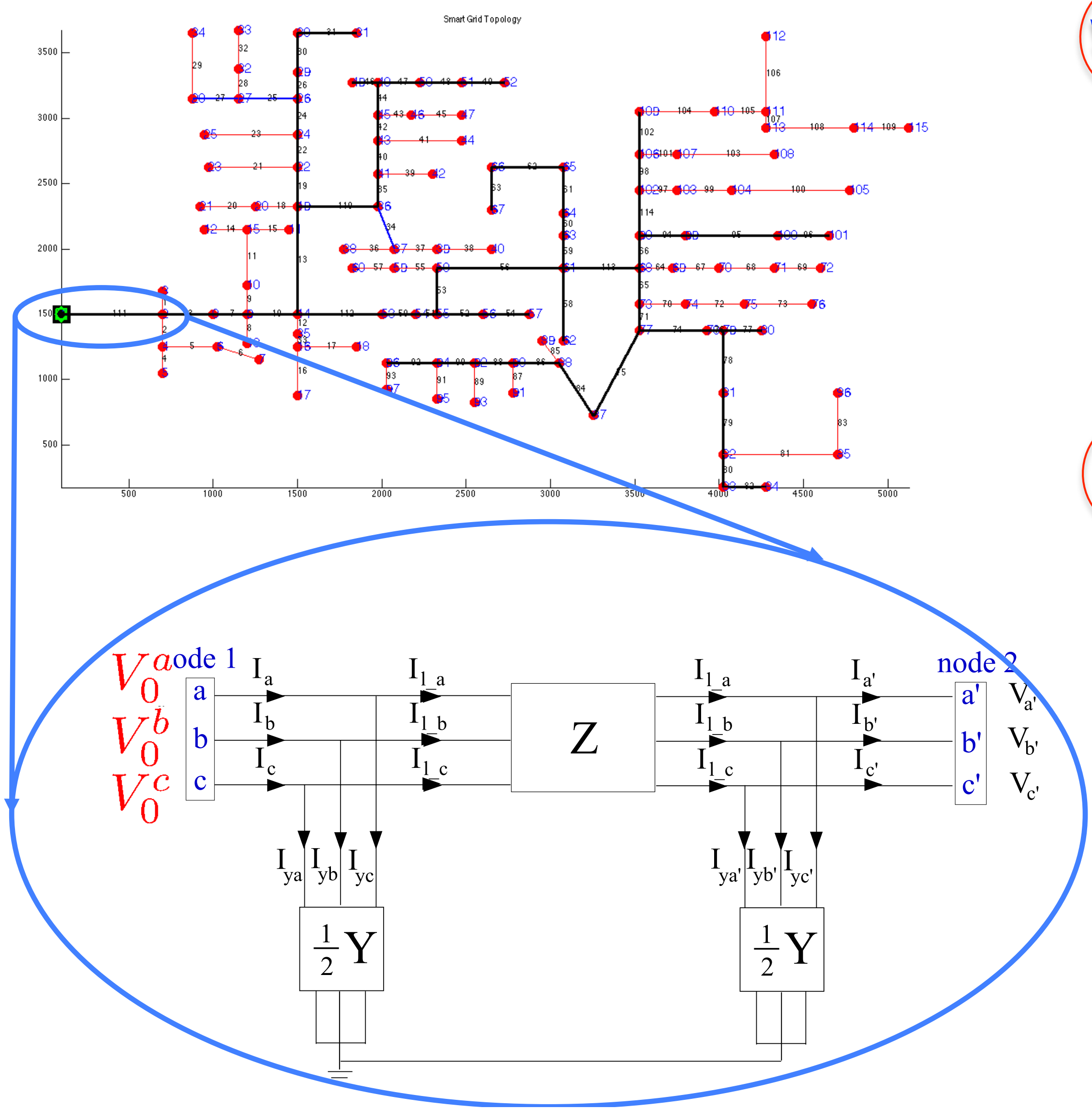
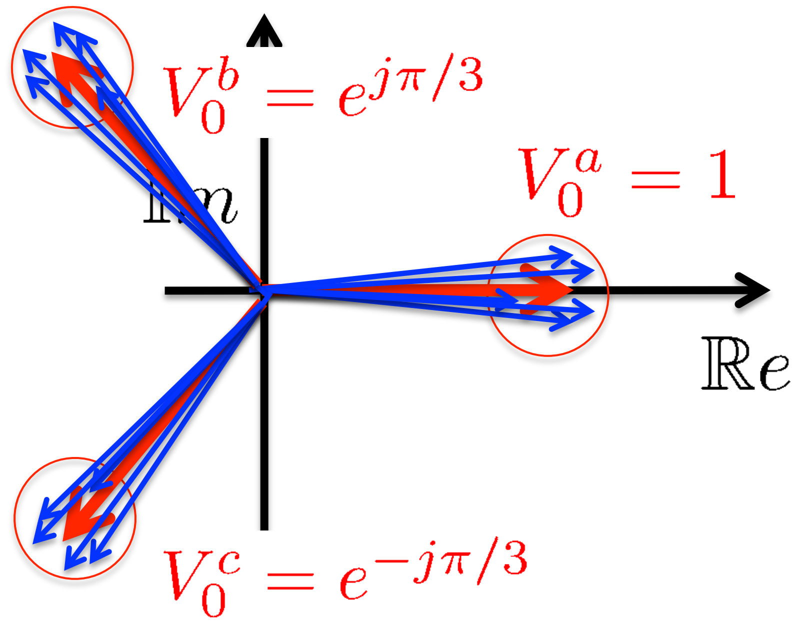
On-going work: 3-phase





$$s_k^a = v_k^a \bar{i}_k^a \approx V_0^a \bar{i}_k^a$$
$$s_k^b = v_k^b \bar{i}_k^b \approx V_0^b \bar{i}_k^b$$
$$s_k^c = v_k^c \bar{i}_k^c \approx V_0^c \bar{i}_k^c$$

M.A.g.I.C. Multi Agent Intelligent Control

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Summary

- BLSE better than WLS for PQ uncertainty >30% and PMU Total Vector Error <1%
- BLSE numerically much superior than WLS (non-iterative method and provides unique solution)
- BLSE allows for off-line computation of estimation confidence bars. Allows off-line:
  - Optimal PMU placement
  - Evaluation of trade-offs between # of PMUs, their accuracy and performance
- PMU with total error vector of 1% might not be sufficient for distribution networks

Questions ?

URL: http://automatica.dei.unipd.it/people/schenato.html