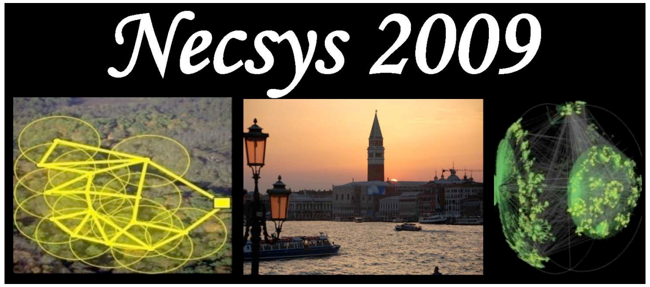
Networked Control Systems subject to packet loss and random delay.

Part II: Random delay and distributed estimation



Luca Schenato
University of Padova

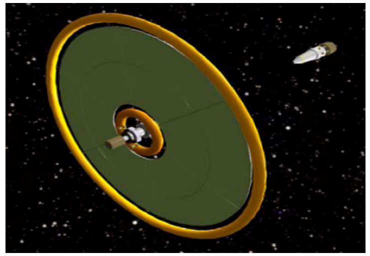
Necsys'09, Tutorial day, 26 September 2009, Venice

Networked Control Systems

Drive-by-wire systems

Swarm robotics

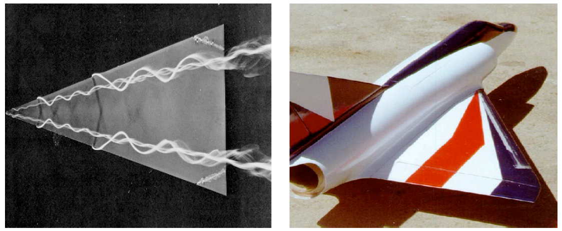
Smart structures: adaptive space telescope



Wireless Sensor Networks

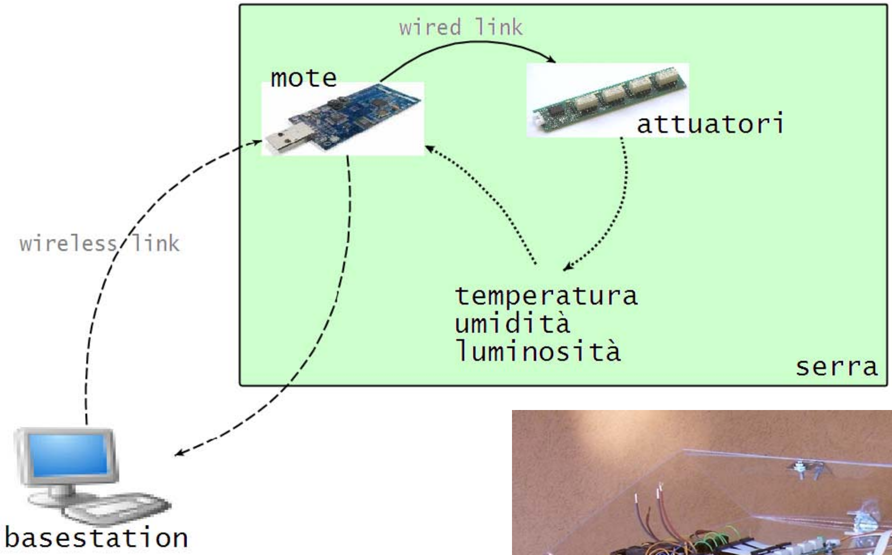
Traffic Control: Internet and transportation

Smart materials: sheets of MEMS sensors and actuators

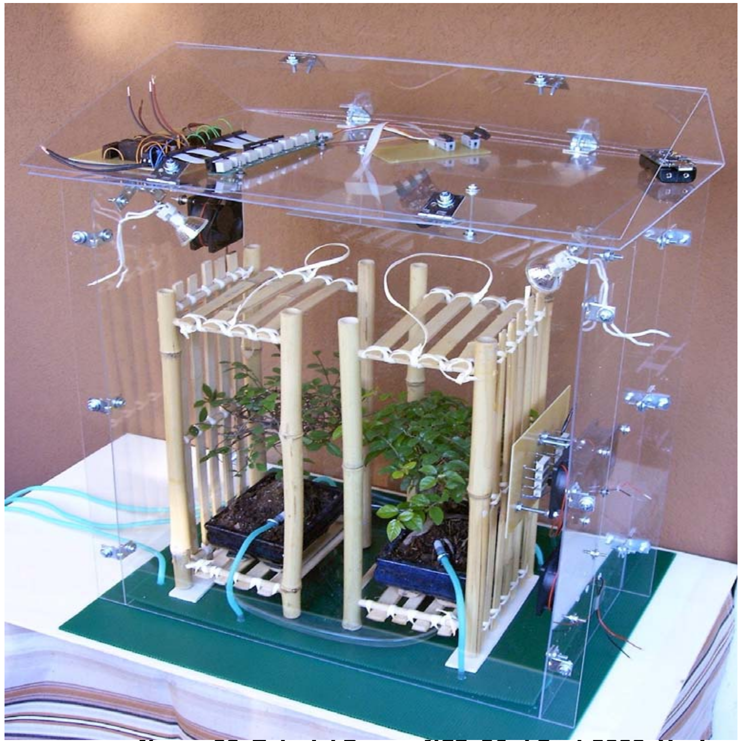


NCSs: physically distributed dynamical systems interconnected by a communication network

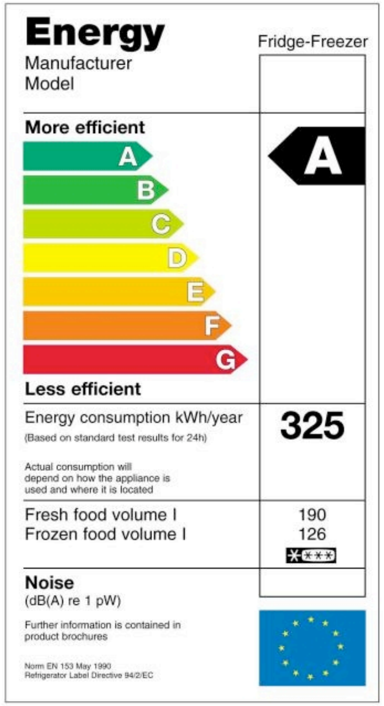
Smart greenhouses and building climate control



- Distributed estimation
- Distributed control
- Control under packet loss & random delay
- Sensor fusion
- Distributed time synchronization



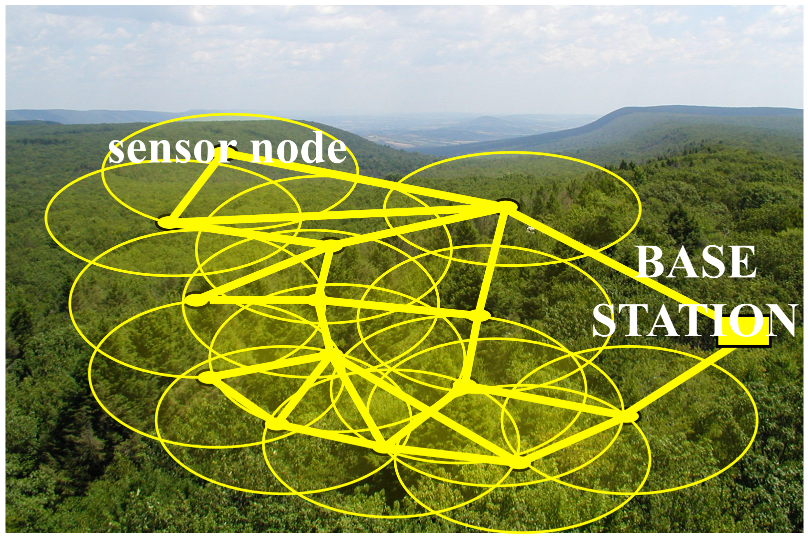
ThermoEfficiency Labeling



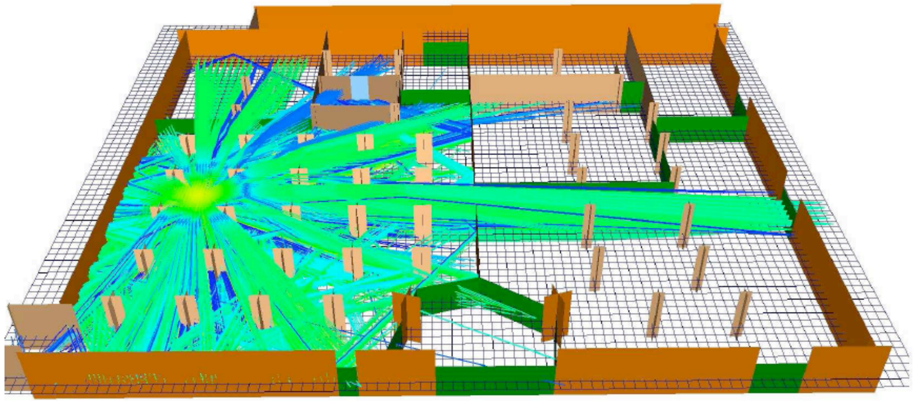
- Building thermodynamics model identification
- Sensor selection for identification
- Optimal sensor placement

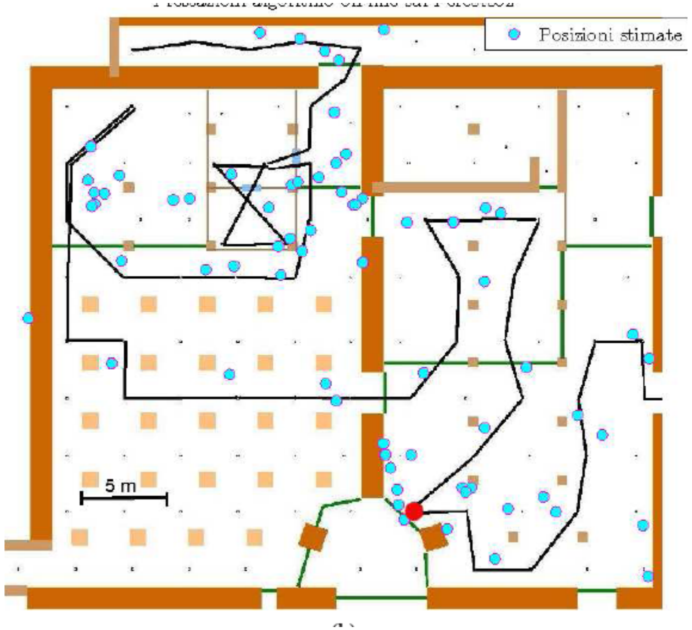
Wireless Sensor Actuator Networks (WSANs)

- Small devices
  - Controller, Memory
  - Wireless radio
  - Sensors & Actuators
  - Batteries
- Inexpensive
- Multi-hop communcation
- Programmable (micro-PC)



Distributed Localization and Tracking with WSNs

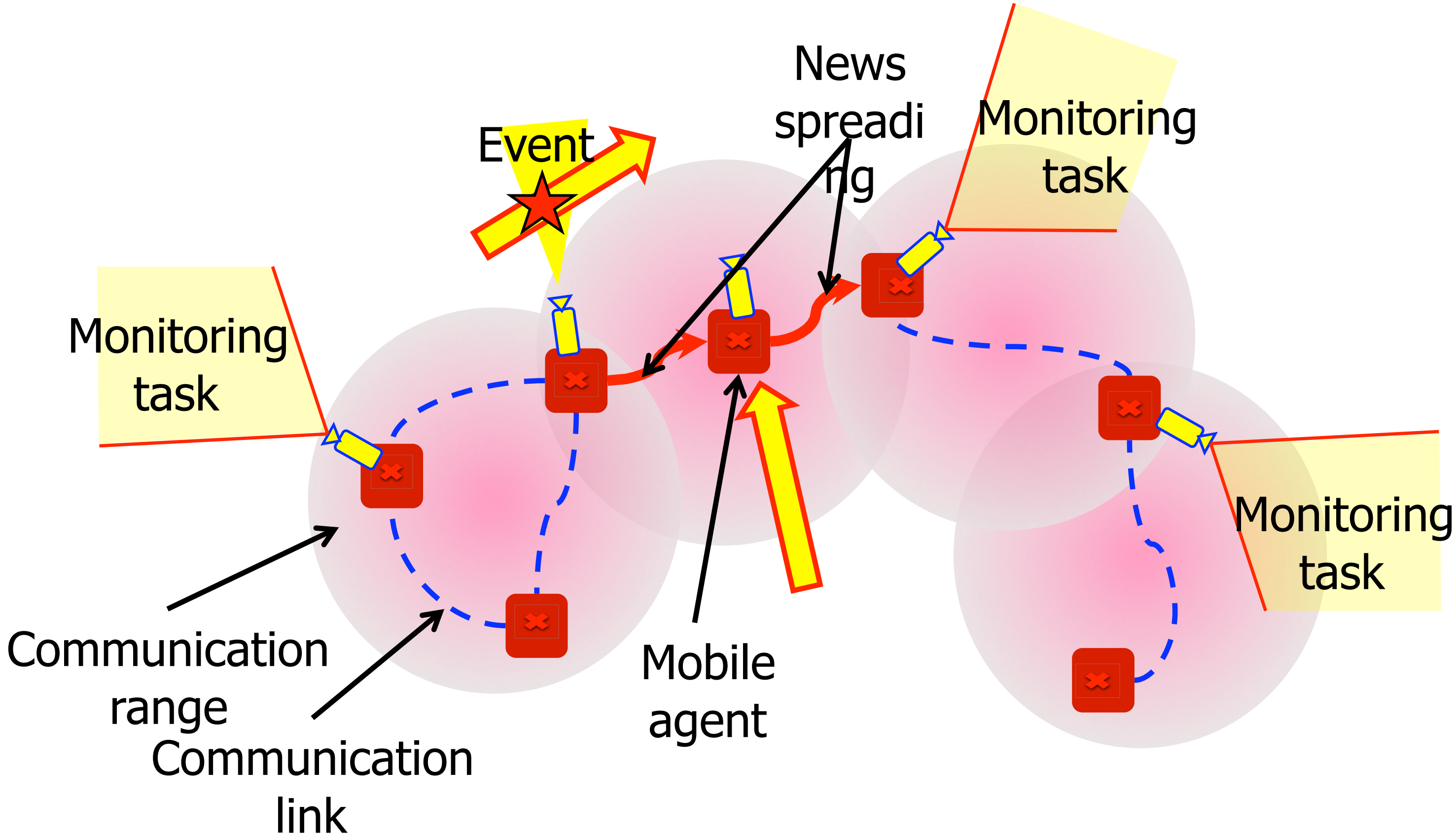




- Indoor radio signal modeling
- Real-time localization
- Distributed tracking
- Coordination

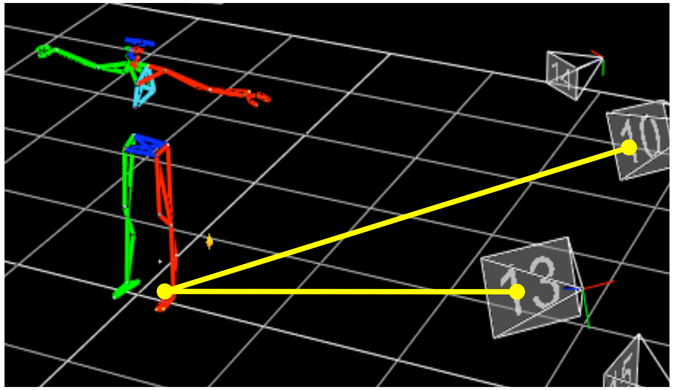
Multi-camera surveillance systems

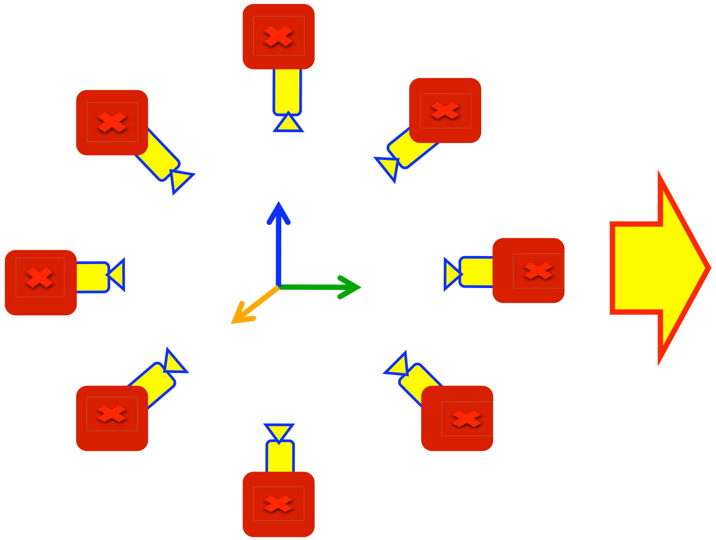
- Rationale
  - The Sensor Actor Network is a multi-agent multi-task finite-resource system



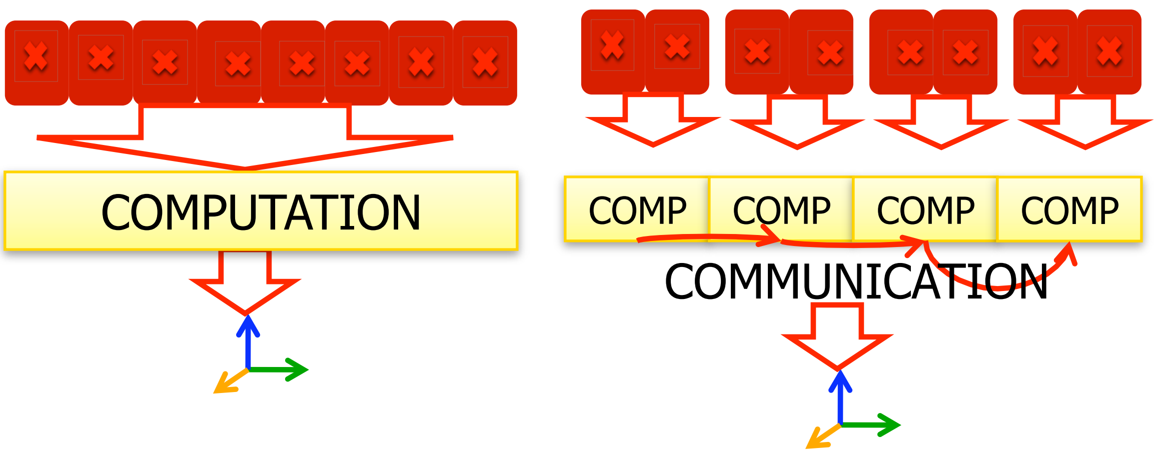
Multi-camera real-time tracking

- Reconstruction Procedure
  - **2D feature point** on the i-th image plane mapped to **ray in 3D space**
  - **3D rays** mapped to **3D feature point**

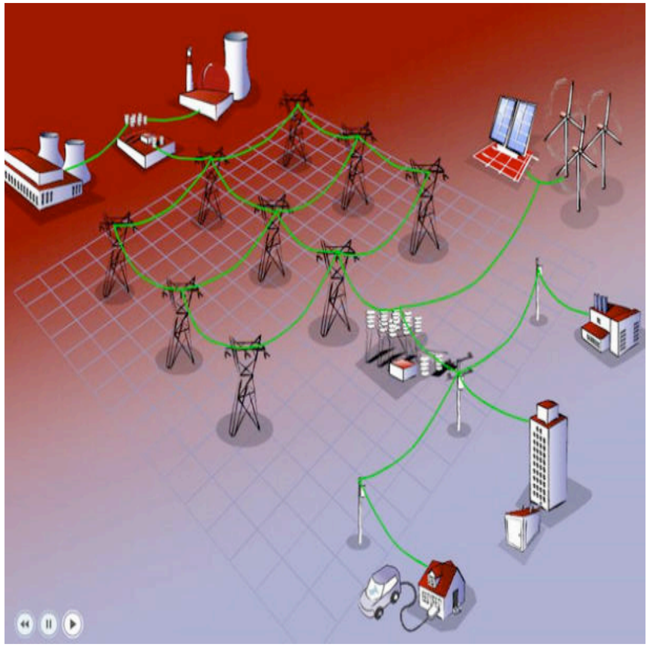




- Centralized or Distributed Strategy?



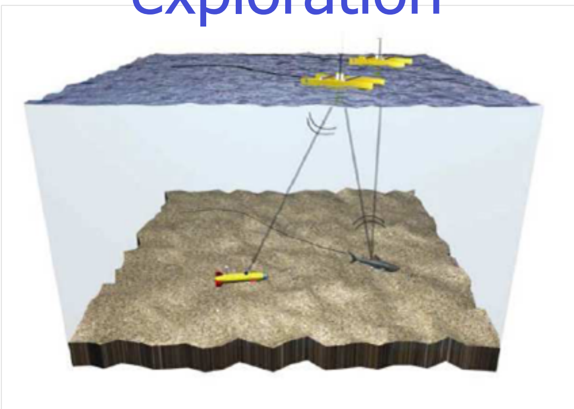
Smart Power Grids

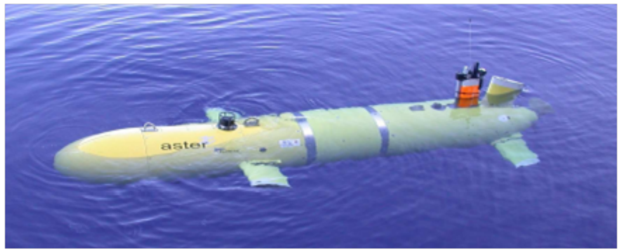


- Foreseeable future
  - Many consumers & producers
  - Cooperation vs greedy behavior
  - Network topology not known and dynamic
  - Need for distributed estimation and control

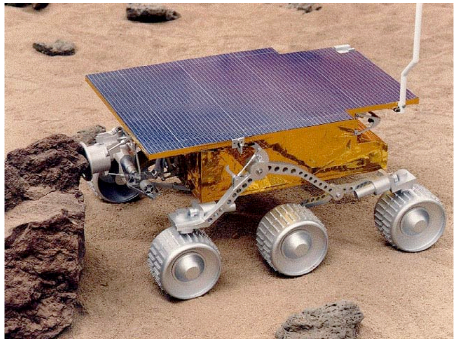
Coordinated robotics for exploration

Underwater exploration





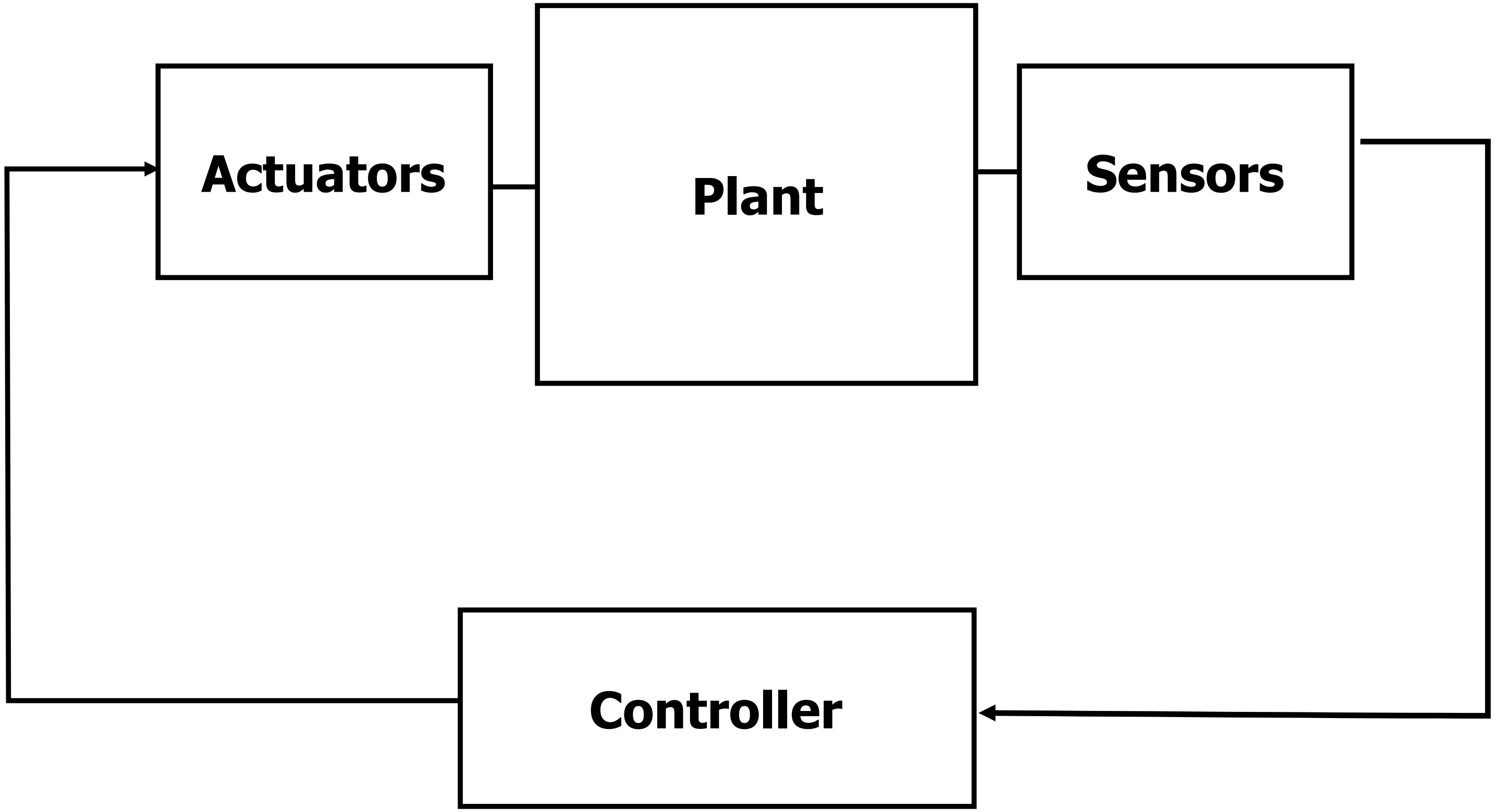
Planetary exploration



Search & rescue missions

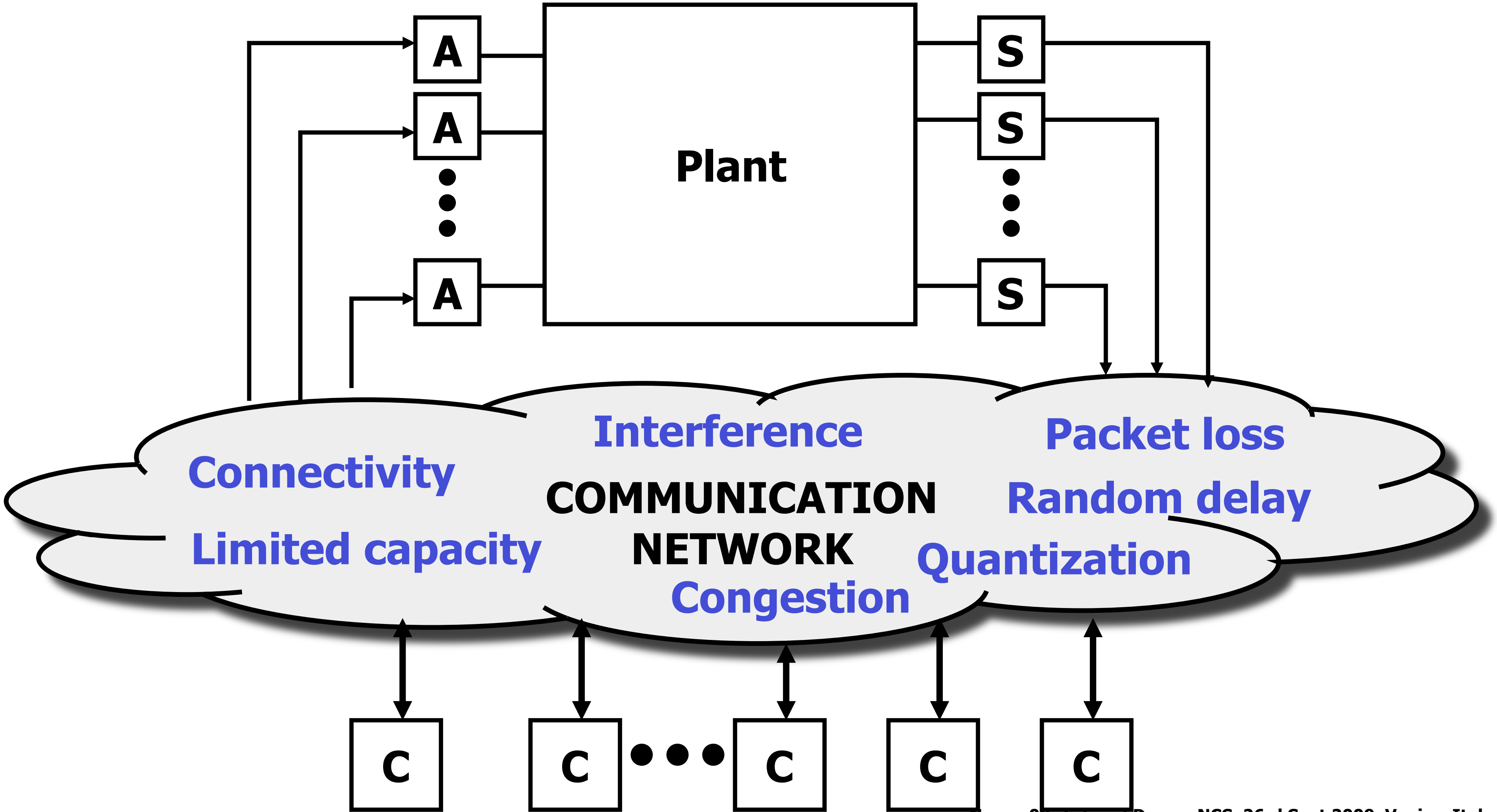
NCSs: what's new for control?

Classical architecture: Centralized structure

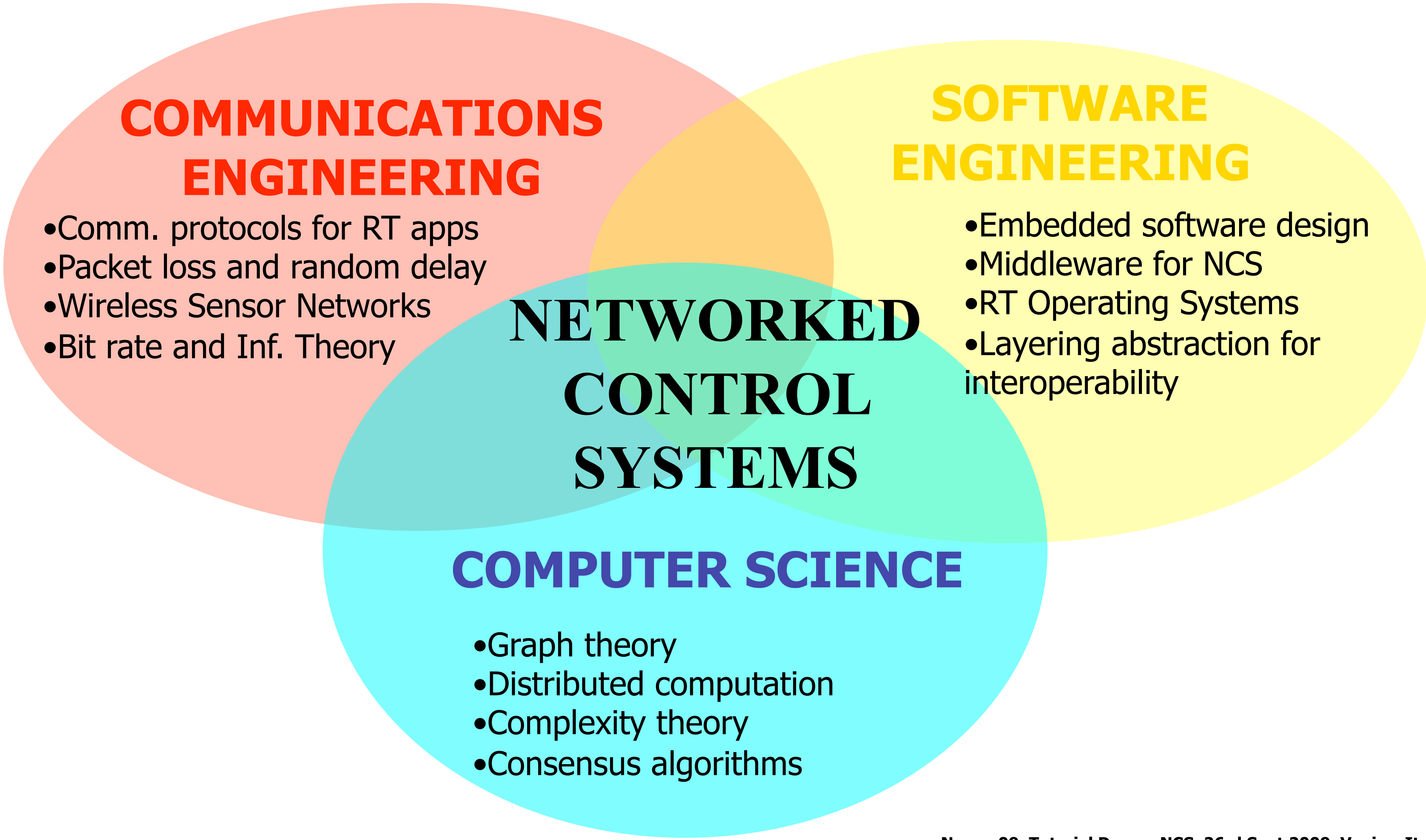


NCSs: what's new for control?

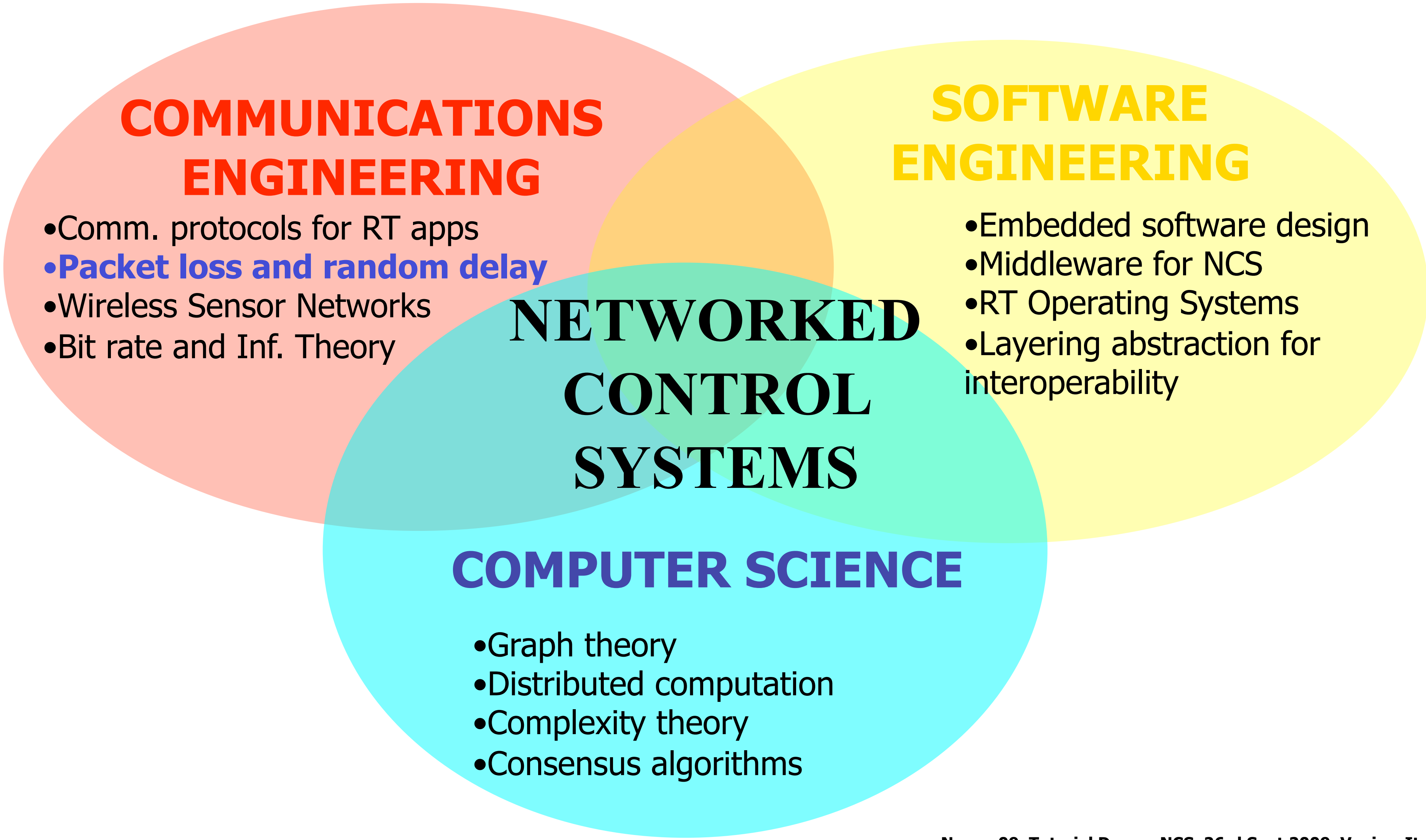
NCSs: Large scale distributed structure



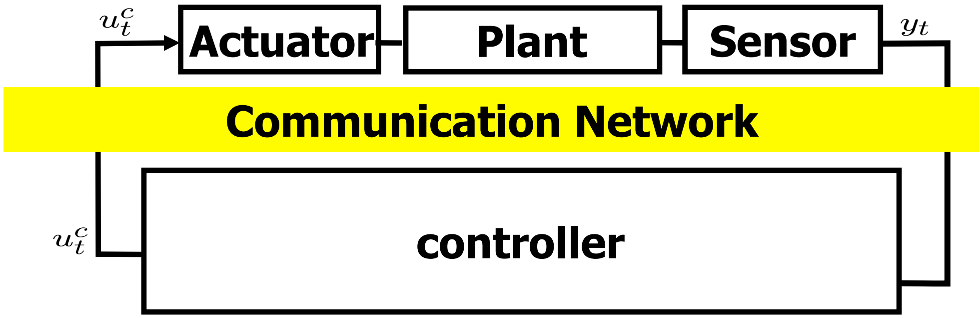
Interdisciplinary research needed



Interdisciplinary research needed

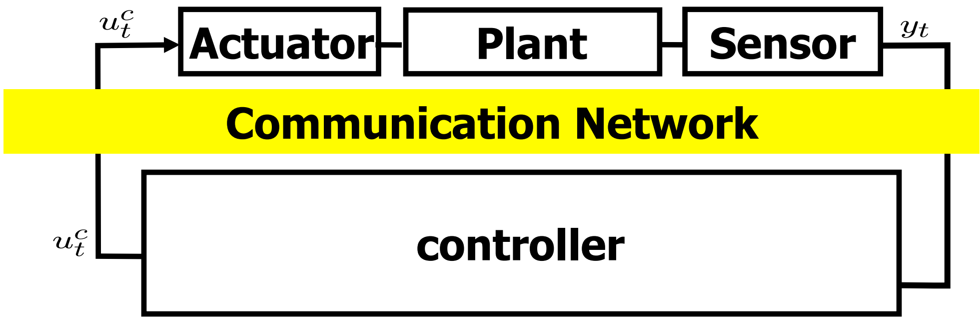


Communication and Control: Modeling with single link



- Problems:
  - Time-varying delay
  - Random packet loss
  - Quantization
- Infinite bandwidth:
  - Deterministic (worst case)
    - Delay and packet loss is time-varying but measurable to receiver
    - Delay and packet loss is NOT known to receiver
  - Stochastic (mean square)
    - Delay and packet loss are random, but measurable and known stats
- Finite bandwidth
  - Quantization
  - Power limited transmission

Communication and Control: Modeling with single link



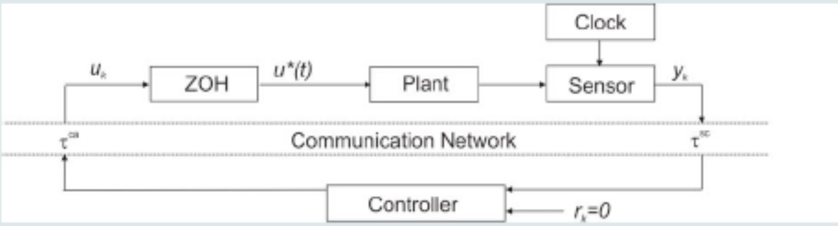
- Problems:
  - Time-varying delay
  - Random packet loss
  - Quantization

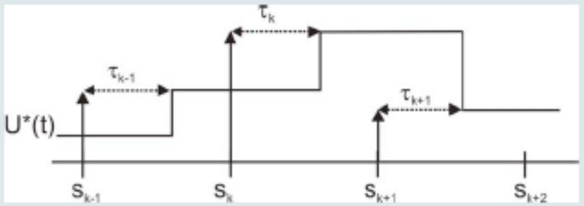
- Infinite bandwidth:
  - Deterministic (worst case)
    - Delay and packet loss is time-varying but measurable to receiver
    - Delay and packet loss is NOT known to receiver
  - Stochastic (mean square)
    - Delay and packet loss are random, but measurable and known stats
- Finite bandwidth
  - Quantization
  - Power limited transmission

Core of this tutorial

Modeling: deterministic with infinite bandwidth

Networked control systems (small delay case: $\tau_k < h$)





Continuous-time plant: $\dot{x}(t) = Ax(t) + Bu^*(t)$

Zero-order hold: $u^*(t) = u_k$, for $t \in [s_k + \tau_k, s_{k+1} + \tau_{k+1})$

Networked control systems: Model

$$x_{k+1} = e^{Ah}x_k + \int_0^{h-\tau_k} e^{As}ds B u_k + \int_{h-\tau_k}^{h} e^{As}ds B u_{k-1}$$

Using the augmented state vector $\xi_k = \begin{pmatrix} x_k \\ u_{k-1} \end{pmatrix}$ we obtain

$$\xi_{k+1} = \begin{pmatrix} x_{k+1} \\ u_k \end{pmatrix} = \underbrace{\begin{pmatrix} e^{Ah} & \int_{h-\tau_k}^{h} e^{As}ds B \\ 0 & 0 \end{pmatrix}}_{=:F(\tau_k)} \begin{pmatrix} x_k \\ u_{k-1} \end{pmatrix} + \underbrace{\begin{pmatrix} \int_0^{h-\tau_k} e^{As}ds B \\ I \end{pmatrix}}_{=:G(\tau_k)} u_k$$

Model with delay:

$$\xi_{k+1} = F(\tau_k)\xi_k + G(\tau_k)u_k$$

time-varing system with parametric uncertaintly

$\tau_k \in [\tau_{min} \tau_{max}]$

Modeling: deterministic with infinite bandwidth

Model with delay:

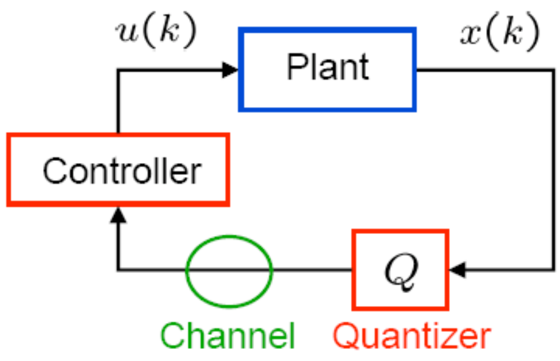
$$\xi_{k+1} = F(\tau_k)\xi_k + G(\tau_k)u_k$$

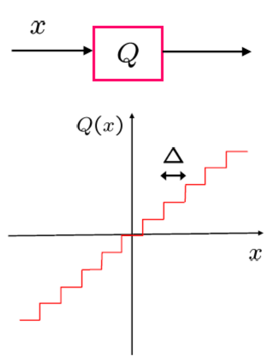
time-varing system with parametric uncertaintly

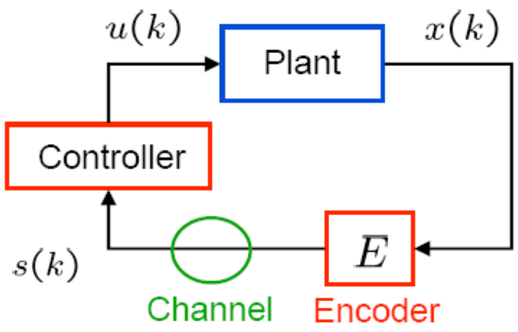
$\tau_k \in [\tau_{\text{min}} \tau_{\text{max}}]$

- If ↲$_\kappa$ is known, then LQG-like approach: optimal time-varying control u_k=K(↲$_\kappa$)$\approx_\kappa$ Nilson (1998)
- If ↲$_\kappa$ is unknown, then robust control approach: worst case analysis with constant control u_k=K$\approx_\kappa$ Zhang (2001), Montestruque (2004), Naghshtabrizi (2006), Cloosterman (2009)
- Most results concern stability and not performance

Modeling of finite bandwidth: rate limited

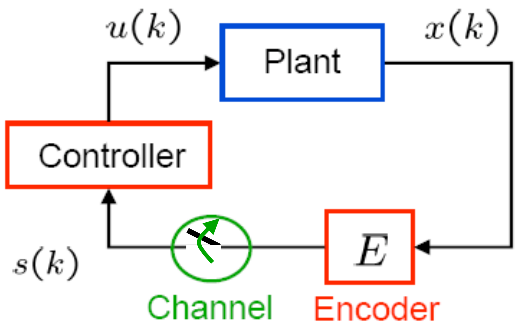






$$s(k) = E_k(x(k), \ldots, x(0), s(k-1), \ldots, s(0))$$

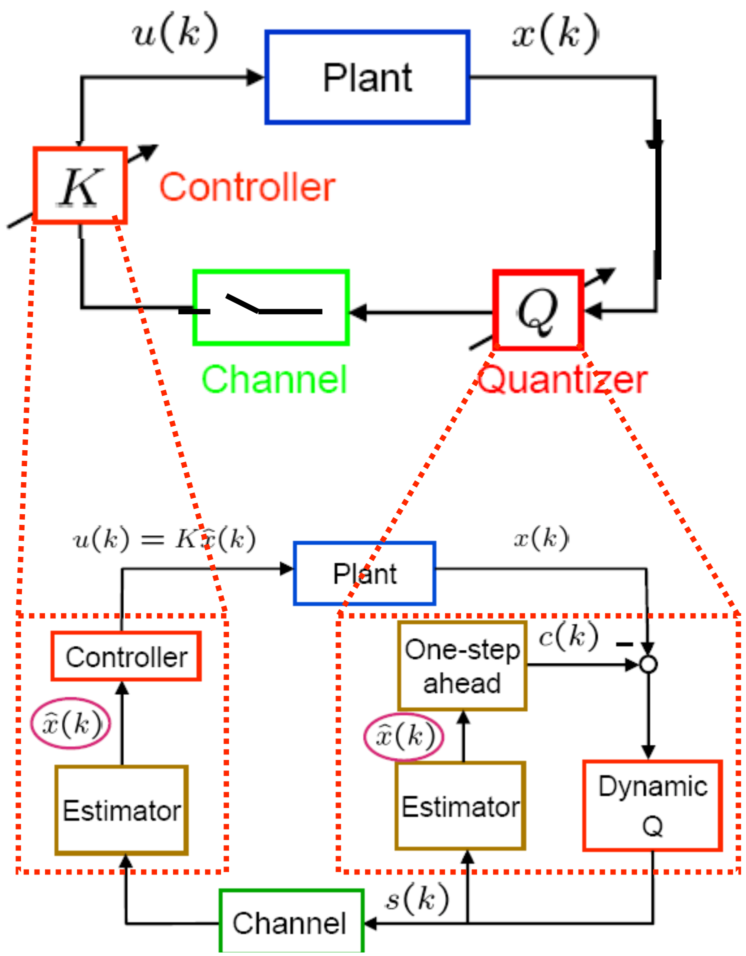
Encoder, i.e. a smart quantizer, can be designed (time-varying)



Packet loss = erasure channel

Necsys 2009

Modeling of finite bandwidth: rate limited

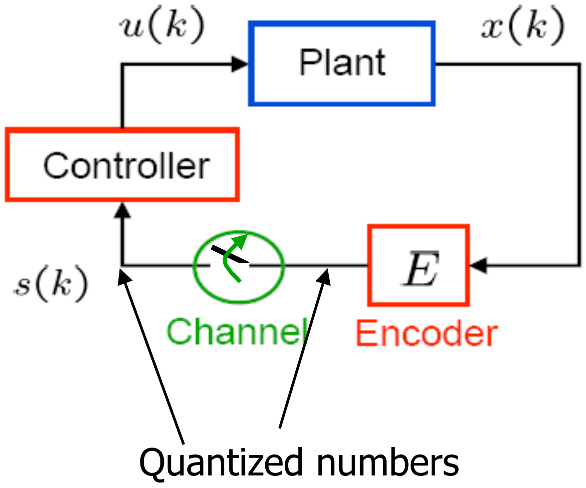


- Problems:
  - Coarseness of quantizer
  - Bit rate
  - Packet loss
- Approach:
  - Design (complex) time-varying encoder/controller
- Main results
  - Bit rate $R > \sum_i \log_2 |\lambda_i^u(A)|$
  - Packet loss → $\alpha < \frac{1}{\prod_i |\lambda_i^u|^2}$
  - Coarseness $|_\chi$ $\rho_c = \frac{\gamma_c + 1}{\gamma_c - 1}$ $\gamma_c = \sqrt{\frac{1 - \alpha}{\frac{1}{\prod_i |\lambda_i^u|^2} - \alpha}}$

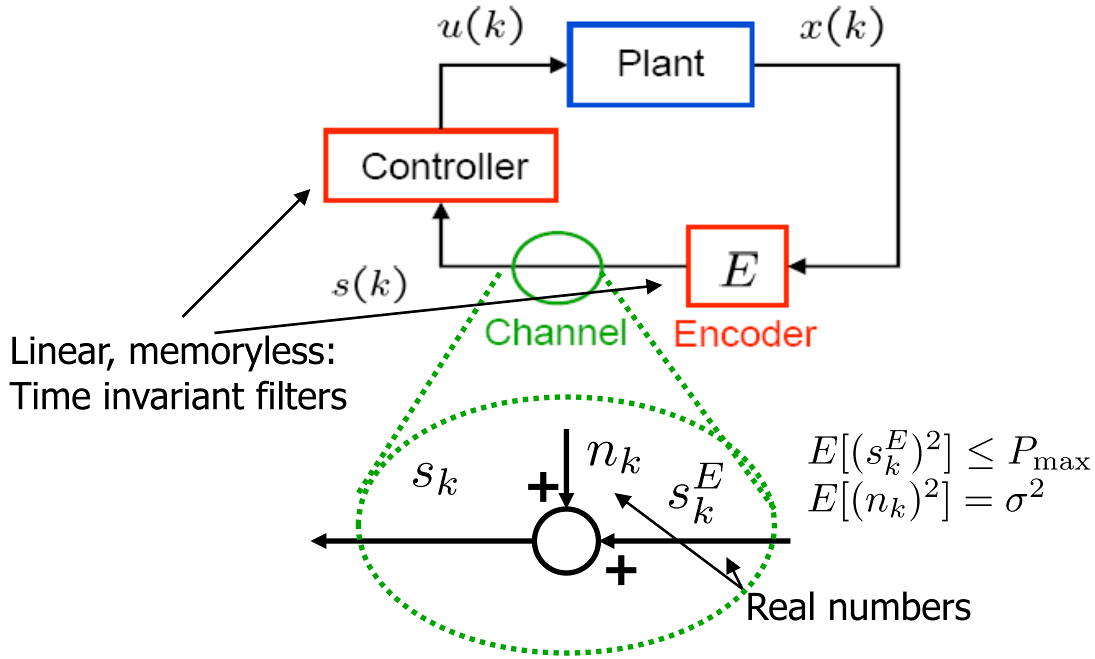
Nair & Evans (2004), Tatikonda et al (2004), Matveev & Savkin (2004), Yuksel & Basar (2006), Ishii et al. (2008), Elia & Mitter (2001), Fu & Xie (2005), Ishii & Francis (2002), Elia (2005)

Modeling of finite bandwidth: signal-to-noise limited

Bit Rate limited



Signal-to-noise limited



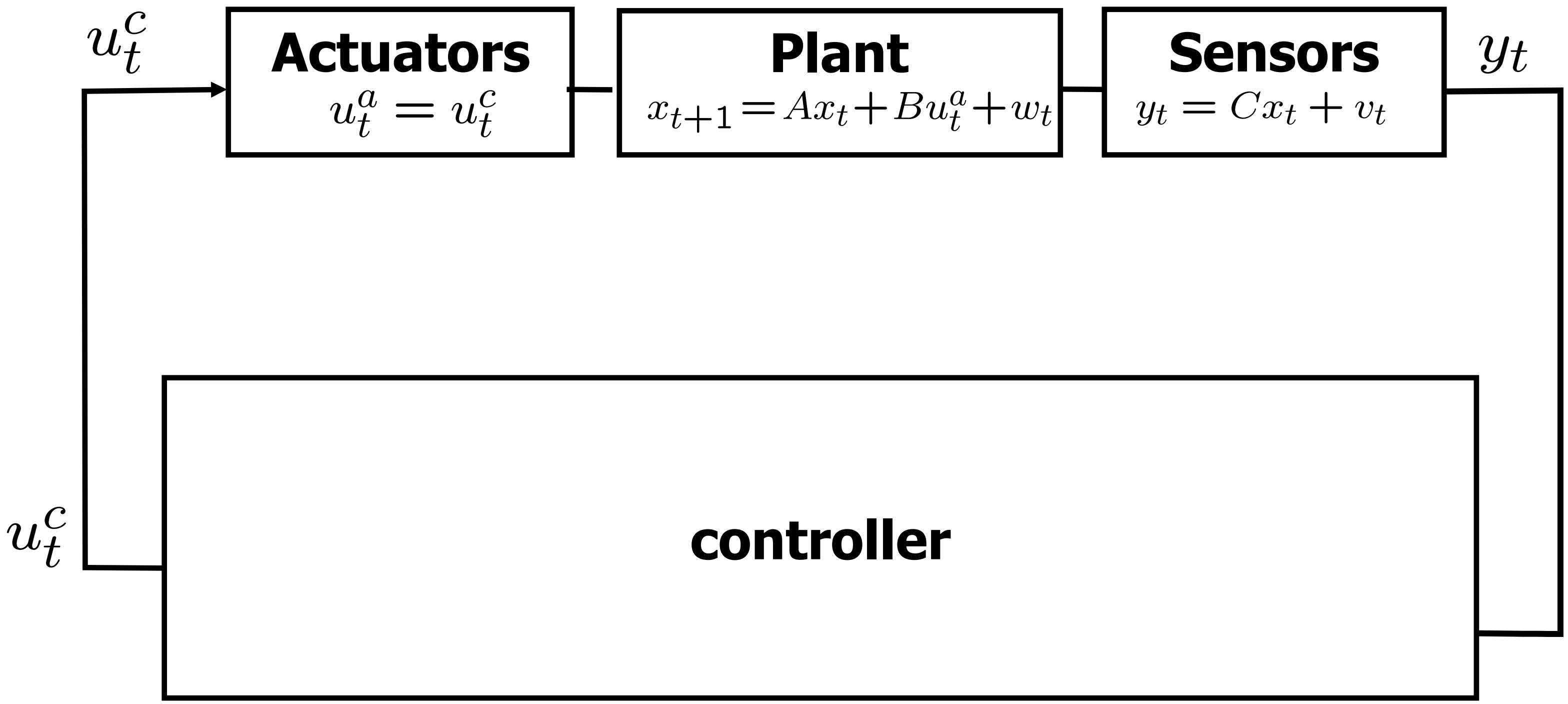
- Takes into account finite bandwidth
- Mathematically clean
- Provide performance bounds

Elia (2004), Martins & Dahleh (2008), Braslavsky at al 2006), Okano et al. (2009)

Communication and Control: Modeling with single link

Modeling	PROS	CONS
Deterministic + infinite bandwidth	▪ easy to implement ▪ good for delay	▪ worst case packet loss ▪ no performance bounds
Stochastic + infinite bandwidth	▪ performance bounds ▪ good for packet loss	▪ time synch required
Rate limited (quantization)	▪ more realistic ▪ links with info theory	▪ hard to implement ▪ no performance bounds
Signal-to-noise-ratio (SNR) limited	▪ more realistic ▪ clean results	▪ coder/decoder to be designed

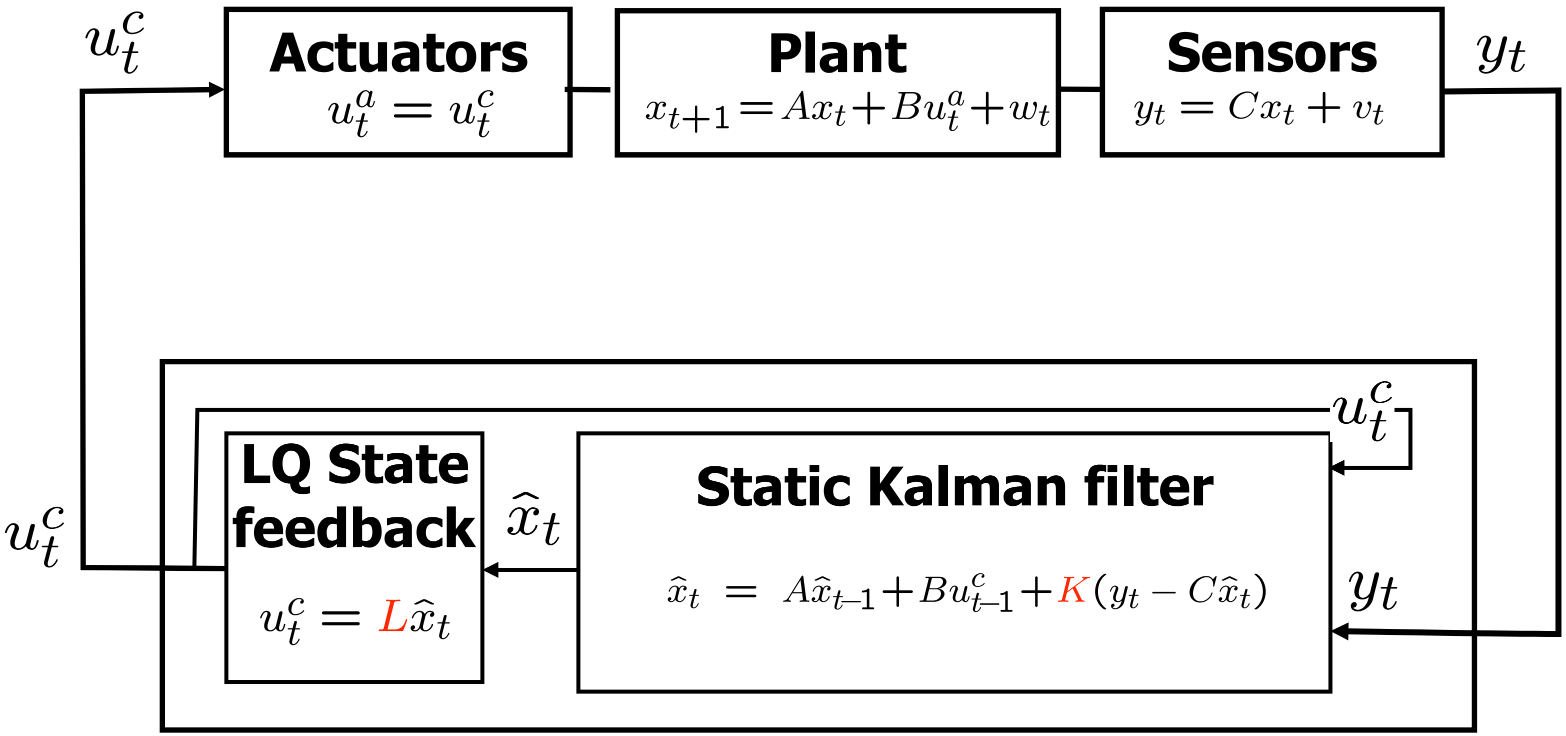
Optimal LQG



$$\min_{u_1^c,\ldots u_T^c} J = \sum_{t=1}^{T} E[x_t^T W x_t + u_t^T U u_t], \quad T \to \infty$$

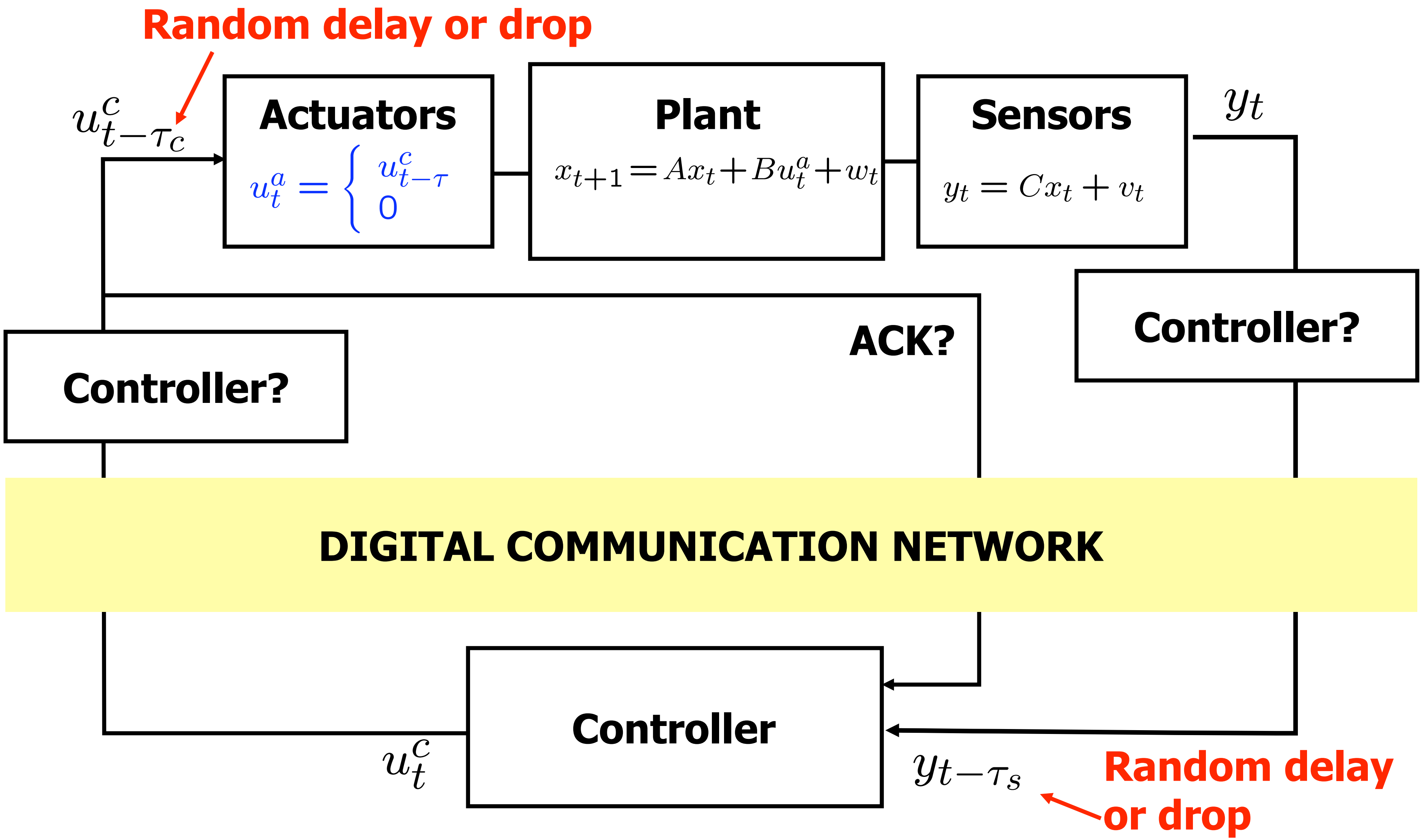
Sensors and actuators are co-located, i.e. no delay nor loss

Optimal LQG

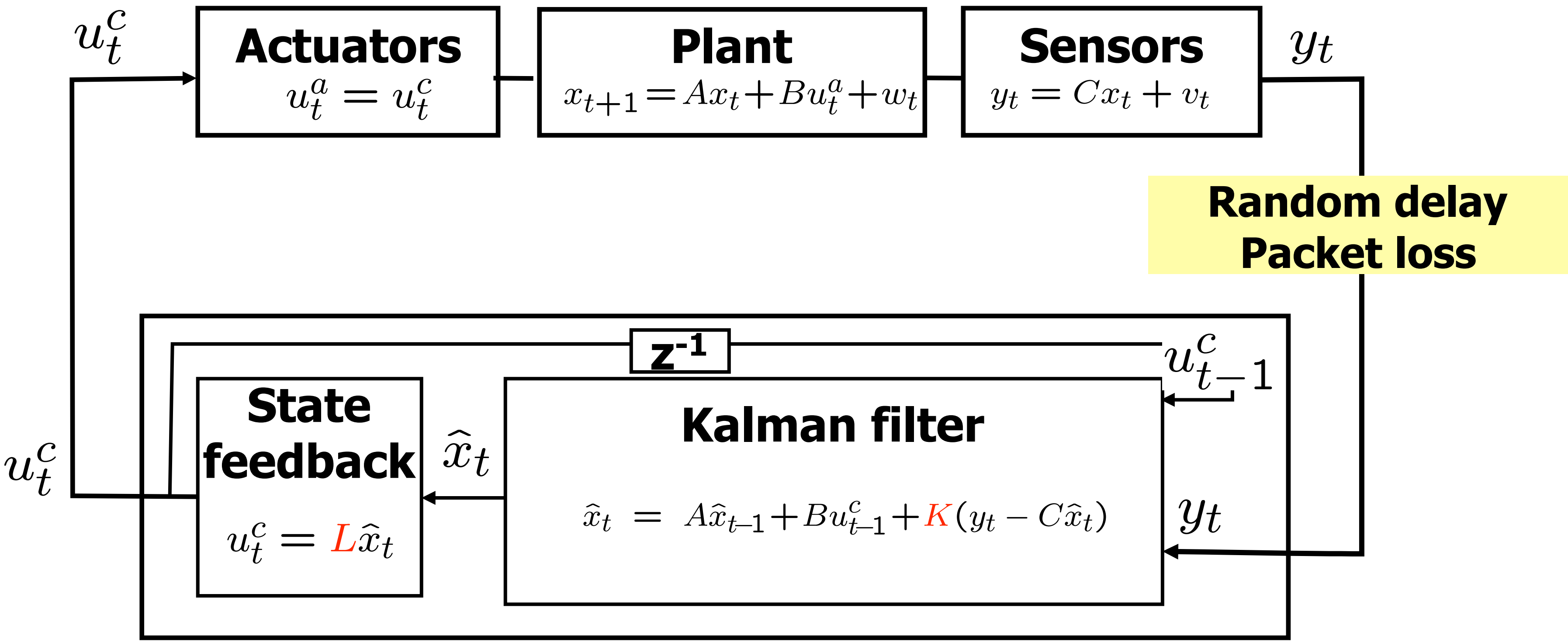


1. **Separation principle holds: Optimal controller = Optimal estimator design + Optimal state feedback design**
2. **Closed Loop system always stable (under standard reach./det. hypotheses)**
3. **Gains K,L are constant solution of Algebraic Riccati Equations**

Optimal LQG control over DCN



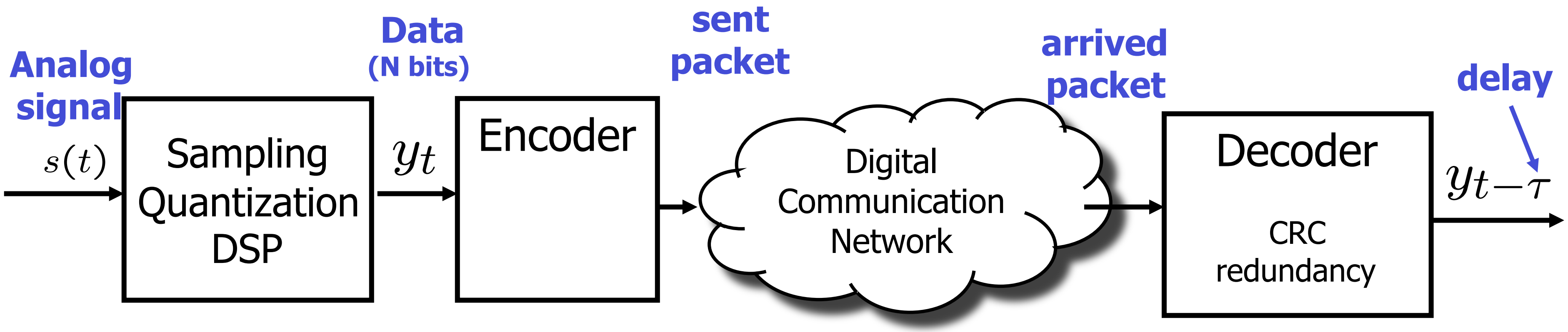
Some consideration on the separation principle

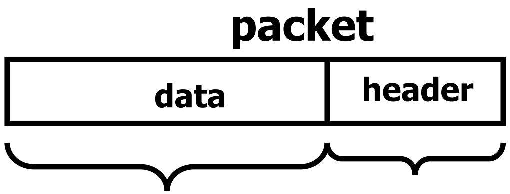


$$\hat{x}_t = E[x_t | y_t, y_{t-1}, .., y_0, u_{t-1}^a, .., u_1^a]$$

$$\text{if } (u_{t-1}^a, ... u_1^a) \text{ known} \Longrightarrow e_t = x_t - \hat{x}_t = f(y_t, \cancel{y_{t-1}}, .., \cancel{y_1}, y_0)$$

Modeling of Digital Communication Network (DCN)





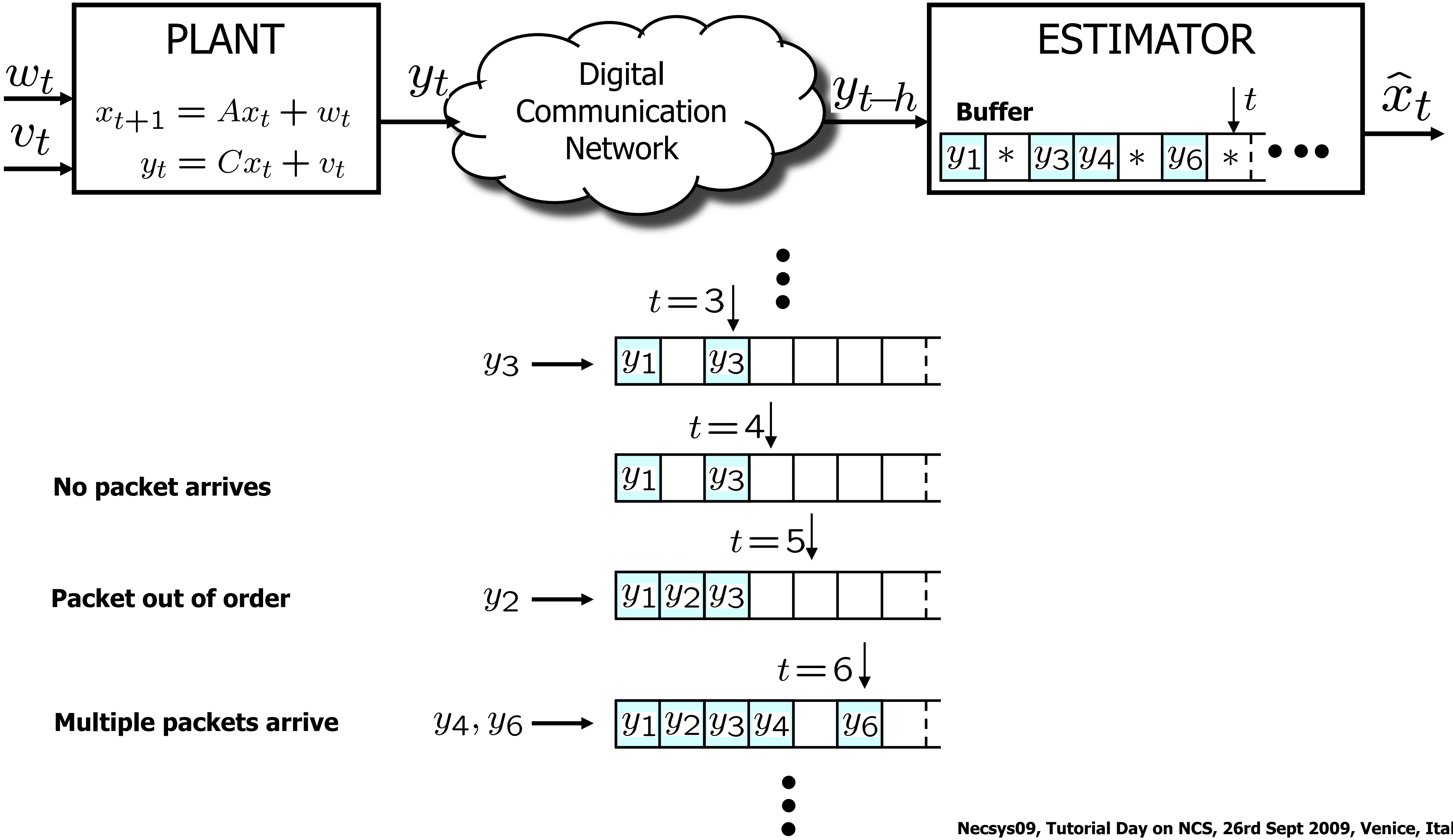
	data	header
ATM	384 bits	40 bits
Ethernet	>368 bits	112 bits
Bluetooth	>499 bits	~100 bits
Zigbee	<1000 bits	128 bits

Assumptions:

(1) Quantization noise<<sensor noise
(2) Packet-rate limited (≠ bit-rate)
(3) No transmission noise (data corrupted=dropped packet)
(4) Packets are time-stamped

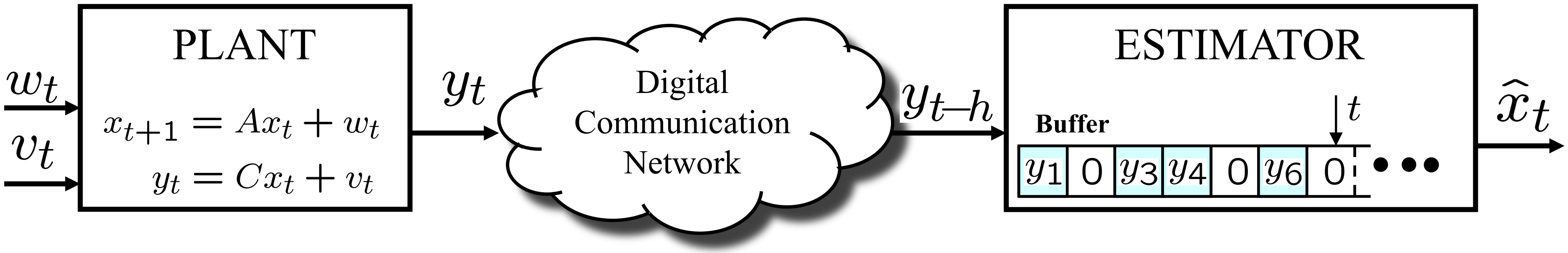
→ **Random delay & Packet loss at receiver**

Estimation modeling



Minimum variance estimation

$$\hat{x}_t = \mathbb{E}[x_t | \{y_k\} \text{ available at estimator at time } t]$$



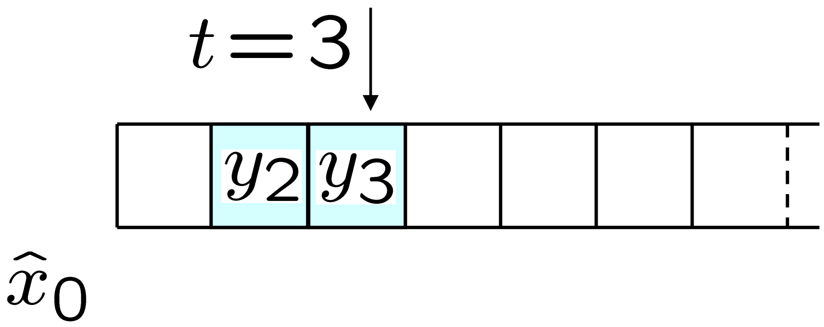
$$\gamma_k^t = \begin{cases} 1 & \textit{if } y_k \text{ arrived before or at time } t, \ t \geq k \\ 0 & \textit{otherwise} \end{cases}$$

$$\tilde{y}_k = \gamma_k^t (Cx_k + v_k) = C_k^t x_k + u^t$$

Kalman time-varying linear system

$$\hat{x}_t = \mathbb{E}[x_t \,|\, \tilde{y}_1, \ldots, \tilde{y}_t, \gamma_1^t, \ldots, \gamma_t^t]$$

Minimum variance estimation



$\gamma = 0$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} \\ P^+ &= APA^T + Q \end{aligned}$$

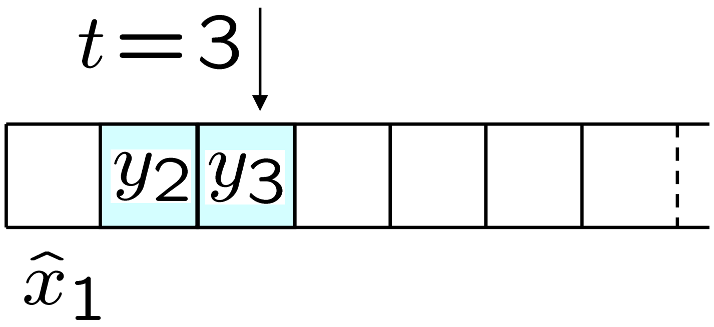
Lyapunov Equation (unstable)

$\gamma = 1$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x}) \\ P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \end{aligned}$$

Riccati Equation (stable)

Minimum variance estimation



$$\gamma = 0$$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} \\ P^+ &= APA^T + Q \end{aligned}$$

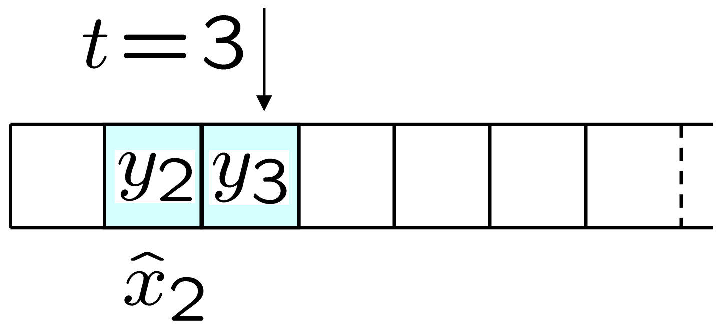
Lyapunov Equation (unstable)

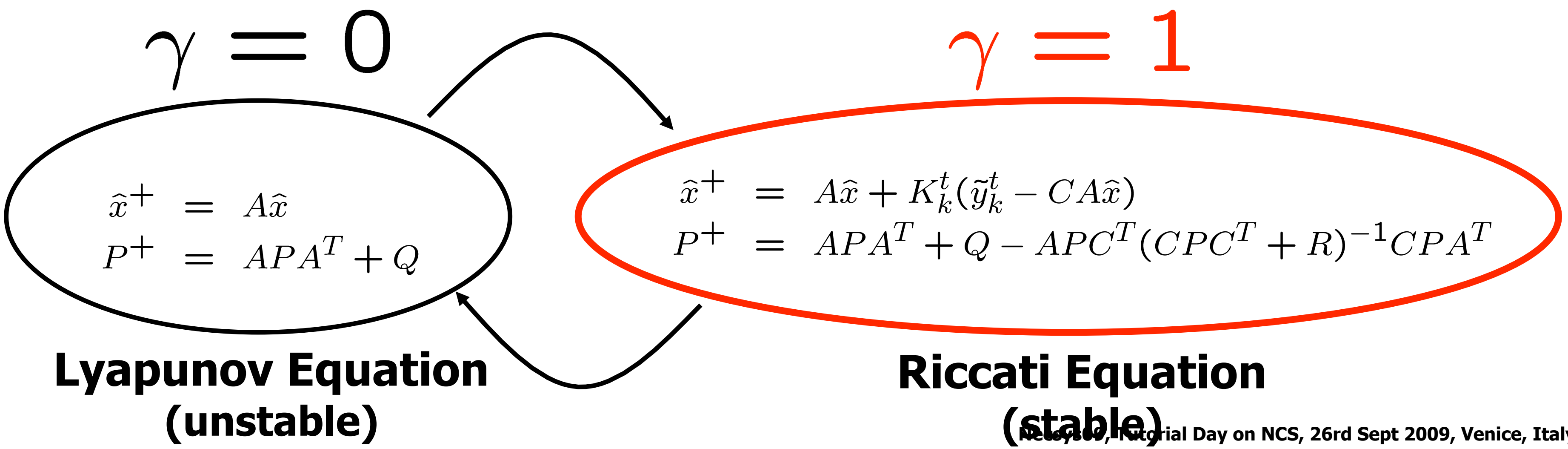
$$\gamma = 1$$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x}) \\ P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \end{aligned}$$

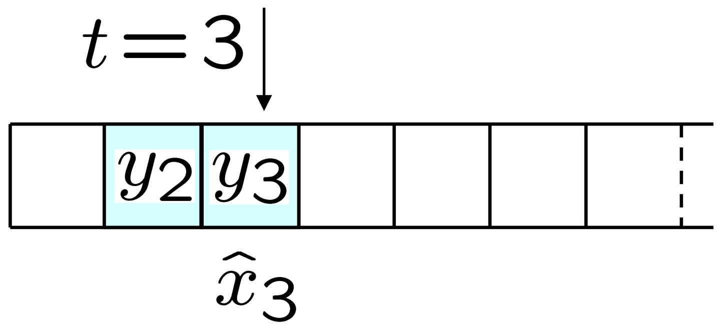
Riccati Equation (stable)

Minimum variance estimation





Minimum variance estimation



$\gamma = 0$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} \\ P^+ &= APA^T + Q \end{aligned}$$

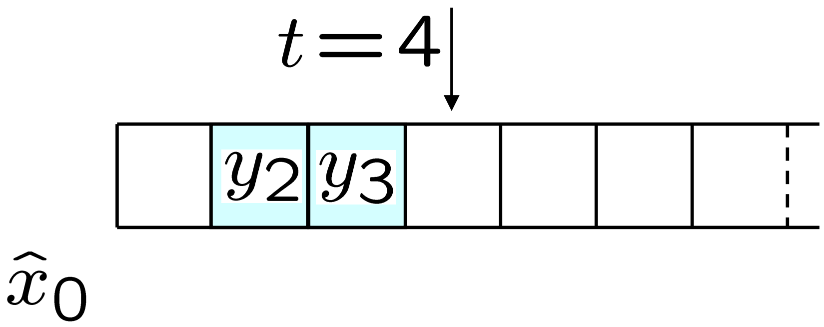
Lyapunov Equation (unstable)

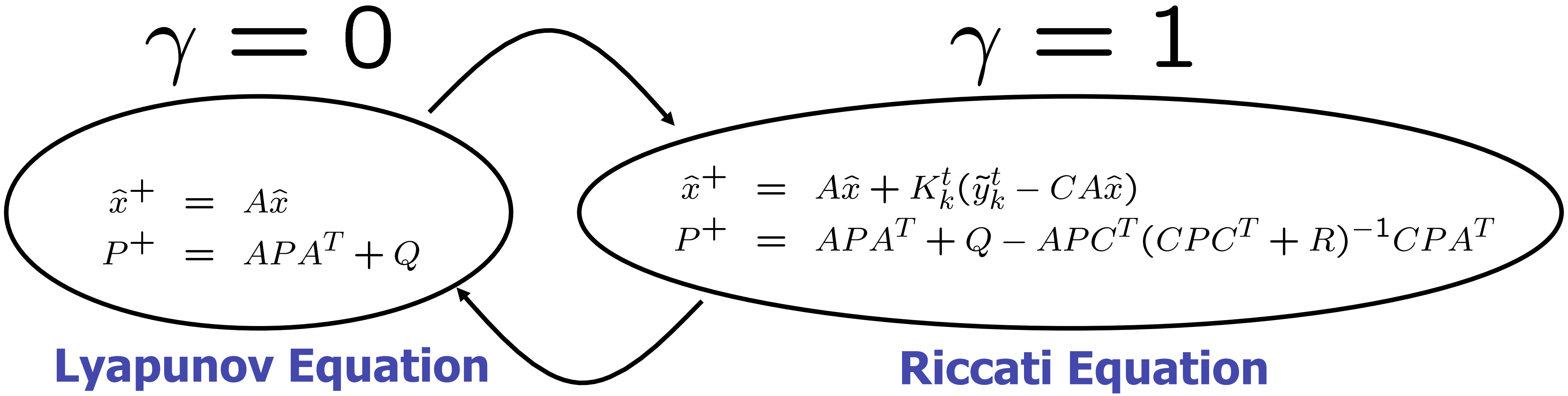
$\gamma = 1$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x}) \\ P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \end{aligned}$$

Riccati Equation (stable)

Minimum variance estimation

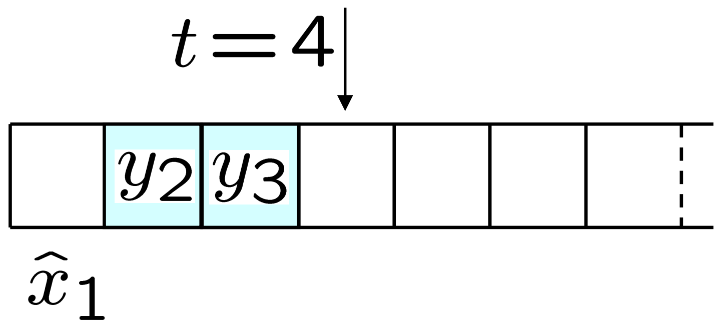




Lyapunov Equation (unstable)

Riccati Equation (stable)

Minimum variance estimation



$\gamma = 0$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} \\ P^+ &= APA^T + Q \end{aligned}$$

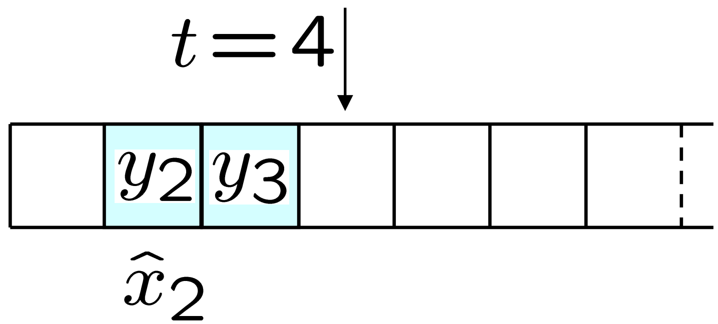
Lyapunov Equation (unstable)

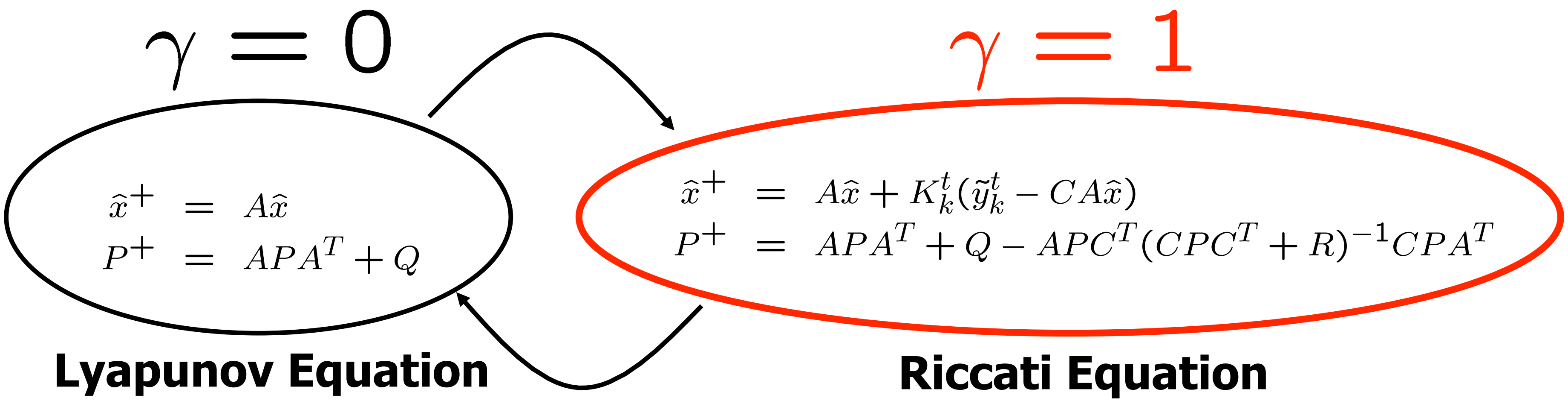
$\gamma = 1$

$$\begin{aligned} \hat{x}^+ &= A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x}) \\ P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \end{aligned}$$

Riccati Equation (stable)

Minimum variance estimation

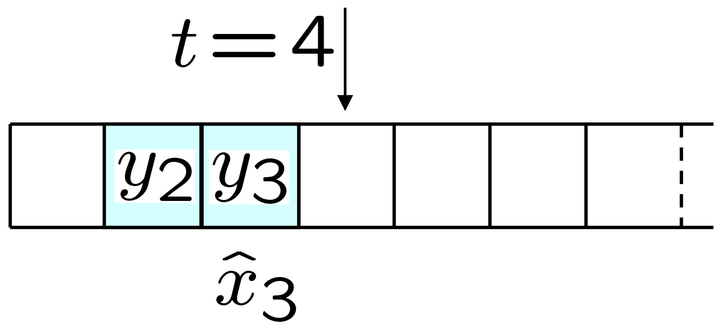


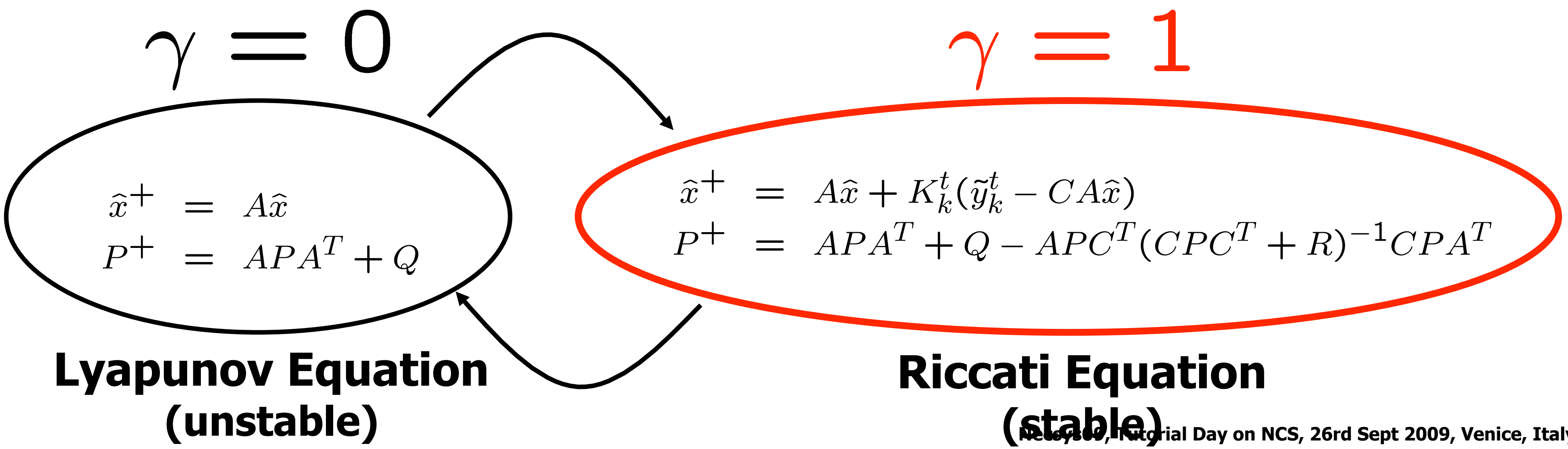


Lyapunov Equation (unstable)

Riccati Equation (stable)

Minimum variance estimation

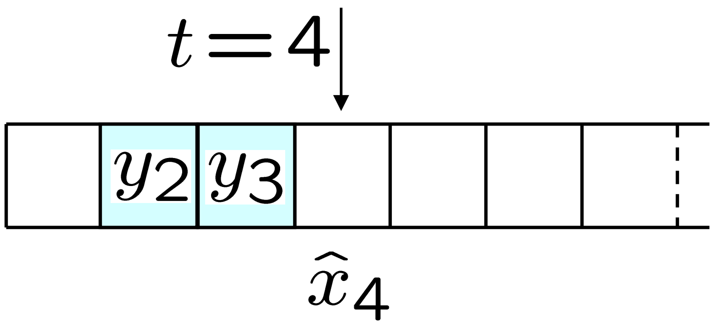


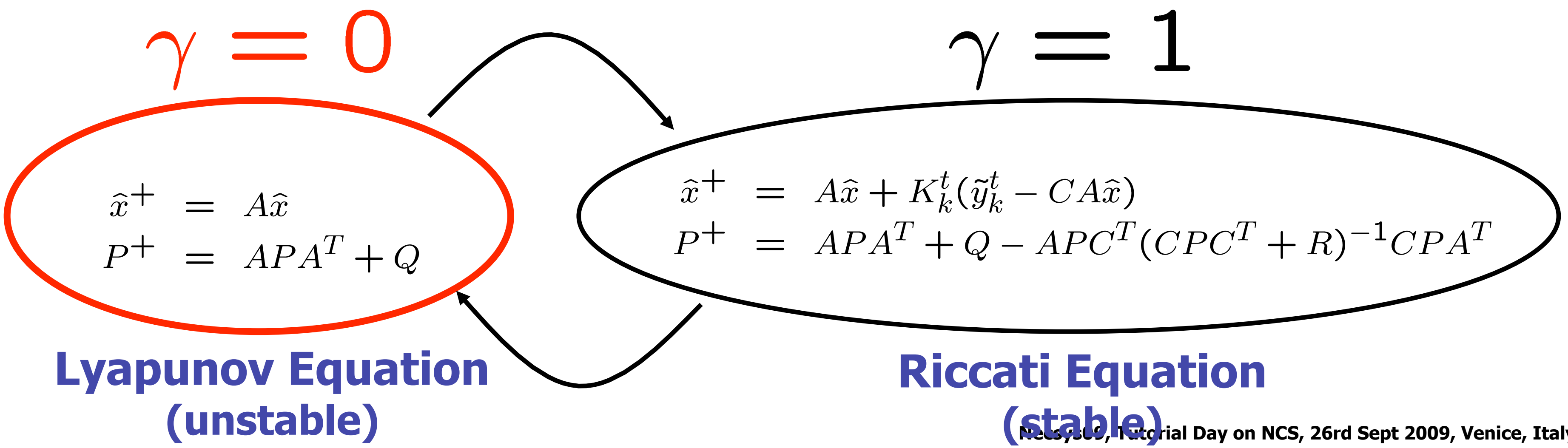


$$\begin{aligned} \hat{x}^+ &= A\hat{x} \\ P^+ &= APA^T + Q \end{aligned}$$

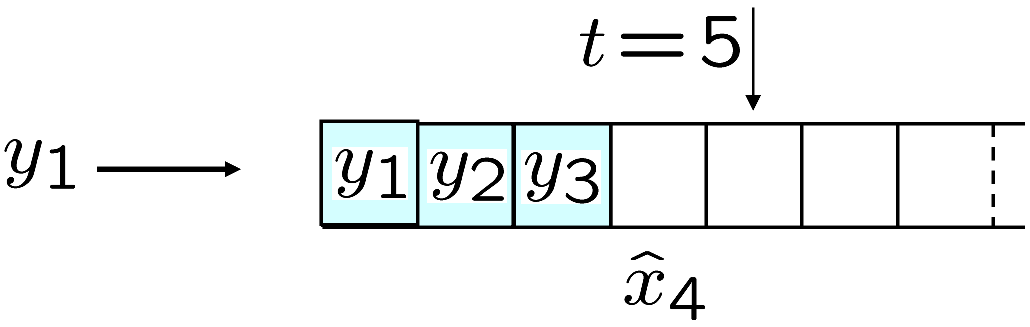
$$\begin{aligned} \hat{x}^+ &= A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x}) \\ P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T \end{aligned}$$

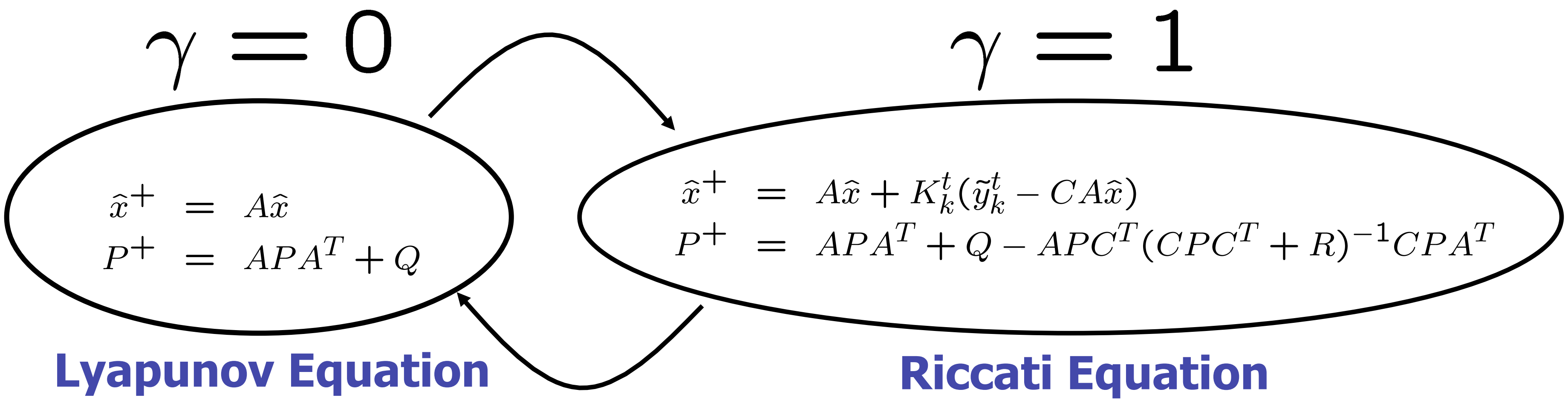
Minimum variance estimation





Minimum variance estimation





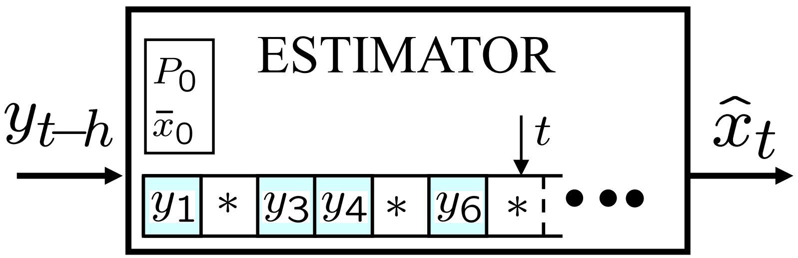
$$\hat{x}^+ = A\hat{x}$$
$$P^+ = APA^T + Q$$

$$\hat{x}^+ = A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x})$$
$$P^+ = APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T$$

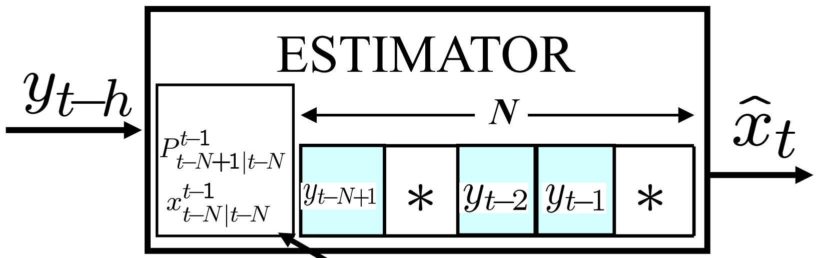
Lyapunov Equation (unstable)

Riccati Equation (stable)

Properties of Optimal Estimator



- Optimal for any arrival process
- Stochastic time-varying gain $K_t = K(\gamma_1, .., \gamma_t)$
- Stochastic error covariance $P_t = P(\gamma_1, .., \gamma_t)$
- Possibly infinite memory buffer
- Inversion of up to *t* matrices at any time *t*

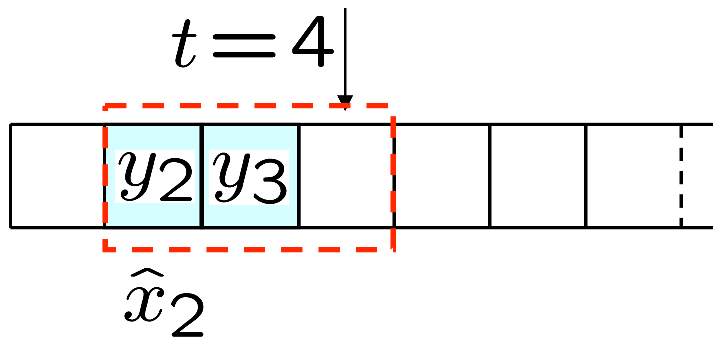


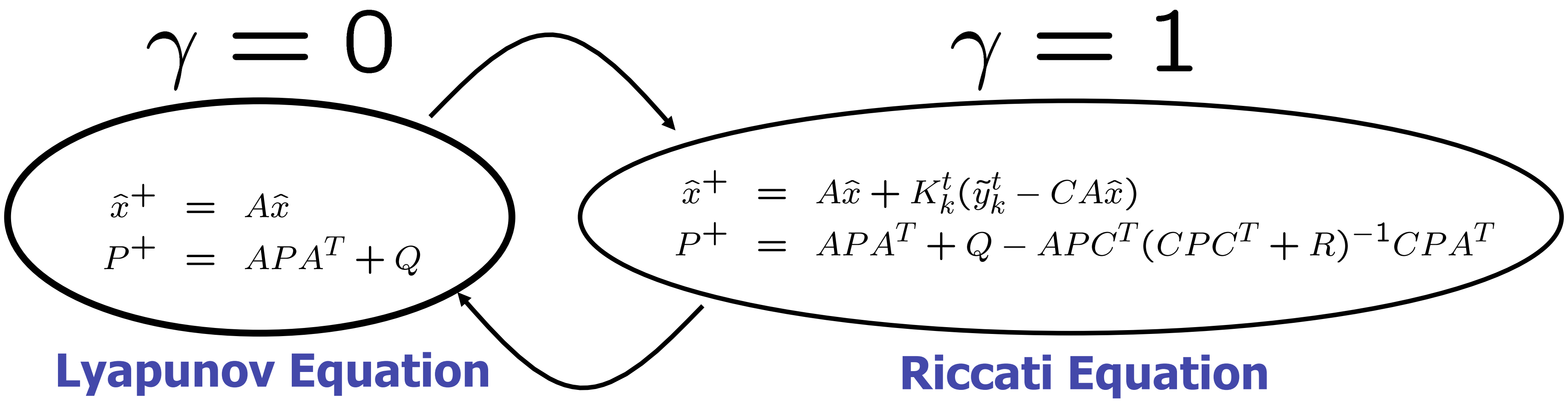
$$\gamma_k^t = \text{cost}, \quad t \geq k + \tau_{max}$$
$$\tau_{max} = N, \quad \text{delay}$$

$$\hat{x}^{t-1}_{t-N|t-N} \triangleq \hat{x}$$
$$P^{t-1}_{t-N+1|t-N} \triangleq P$$

$$\hat{x}^+ = A\hat{x} + \gamma^t_{t-N} PC^T(CPC^T + R)^{-1}(\tilde{y}^t_{t-N} - CA\hat{x}),$$
$$P^+ = APA^T + Q - \gamma^t_{t-N} APC^T(CPC^T + R)^{-1}CPA^T$$

Minimum variance estimation





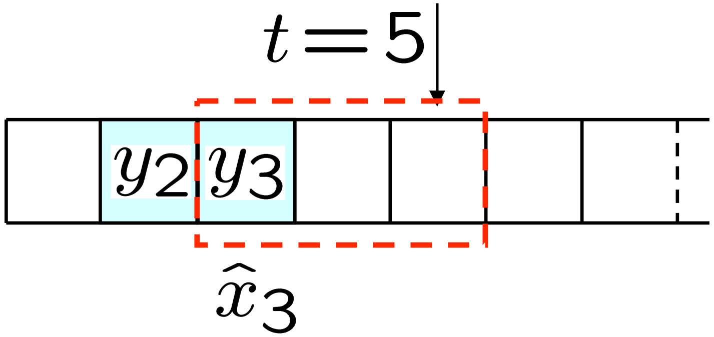
$$\begin{aligned}\widehat{x}^{+} &= A\widehat{x} \\ P^{+} &= APA^T + Q\end{aligned}$$

Lyapunov Equation (unstable)

$$\begin{aligned}\widehat{x}^{+} &= A\widehat{x} + K_k^t(\tilde{y}_k^t - CA\widehat{x}) \\ P^{+} &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T\end{aligned}$$

Riccati Equation (stable)

Minimum variance estimation



$\gamma = 0$

$$\begin{aligned}\hat{x}^+ &= A\hat{x}\\ P^+ &= APA^T + Q\end{aligned}$$

Lyapunov Equation (unstable)

$\gamma = 1$

$$\begin{aligned}\hat{x}^+ &= A\hat{x} + K_k^t(\tilde{y}_k^t - CA\hat{x})\\ P^+ &= APA^T + Q - APC^T(CPC^T + R)^{-1}CPA^T\end{aligned}$$

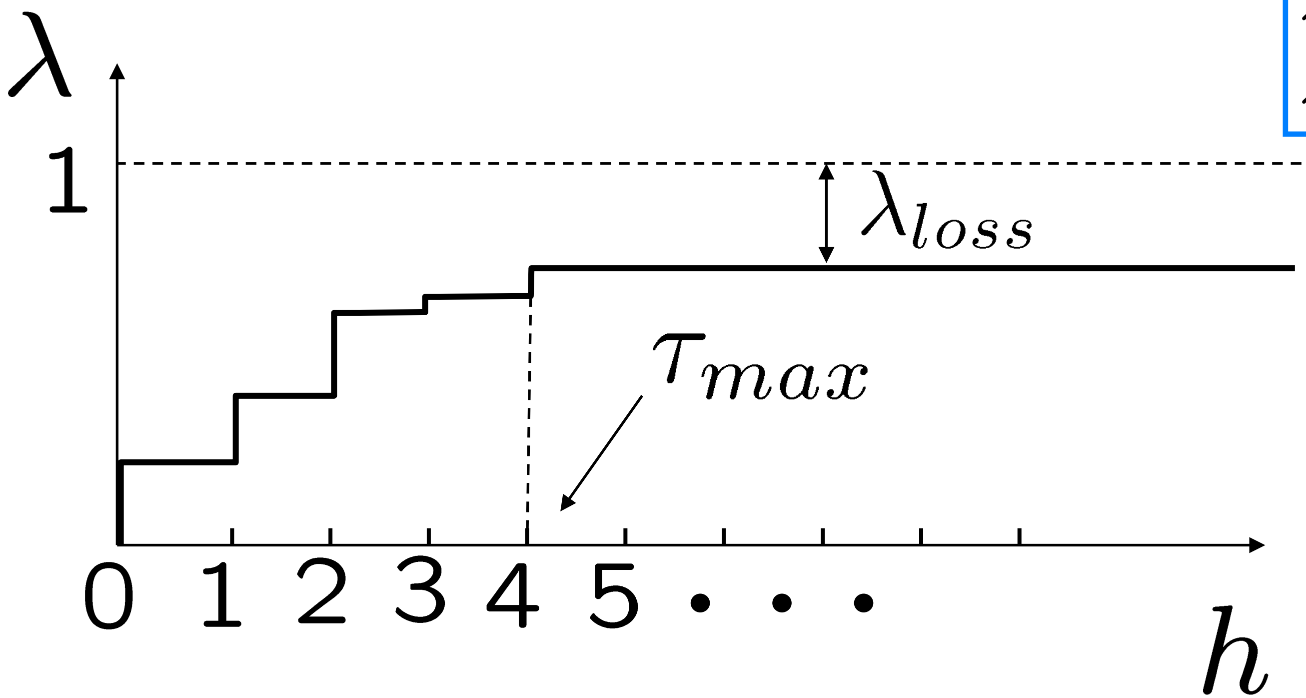
Riccati Equation (stable)

What about stability and performance?

Additional assumption on arrival sequence necessary:
i.i.d. arrival with stationary distribution

τ_k : delay of packet y_k, $\tau_k = \infty$ if y_k never arrives

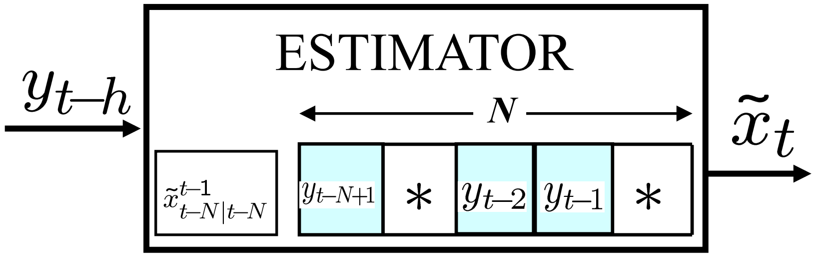
$$\lambda_h \triangleq \mathbb{P}[\tau_k \leq h],$$
$$\lambda_{loss} \triangleq \mathbb{P}[\tau_k = \infty]$$



Optimal estimation with constant gains and buffer finite memory

$\{K_h\}_{h=0}^{N-1}$, N static gains

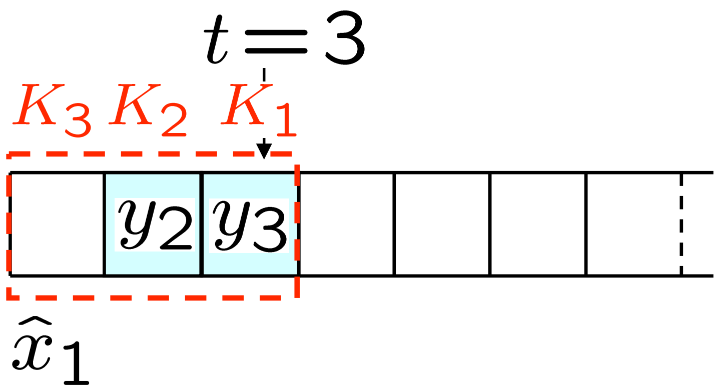
$$\tilde{x}^+ = A\tilde{x} + \gamma_{t-h}^{t} K_h(\tilde{y}_{t-h}^{t} - CA\tilde{x}), \quad h = N-1, \ldots, 0$$

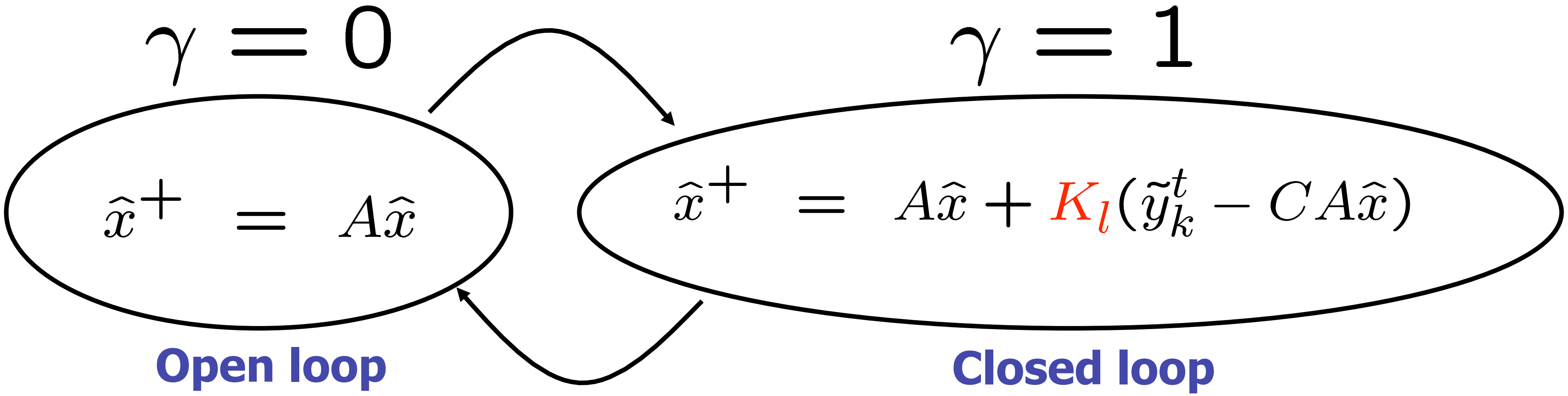


- Does not require any matrix inversion
- Simple to implement
- Upper bound for optimal estimator: $P_t \leq \tilde{P}_{t|t} \Longrightarrow \mathbb{E}_\gamma[P_{t|t}] \leq \mathbb{E}_\gamma[\tilde{P}_{t|t}] = \overline{P}_{t|t}$
- N is design parameter

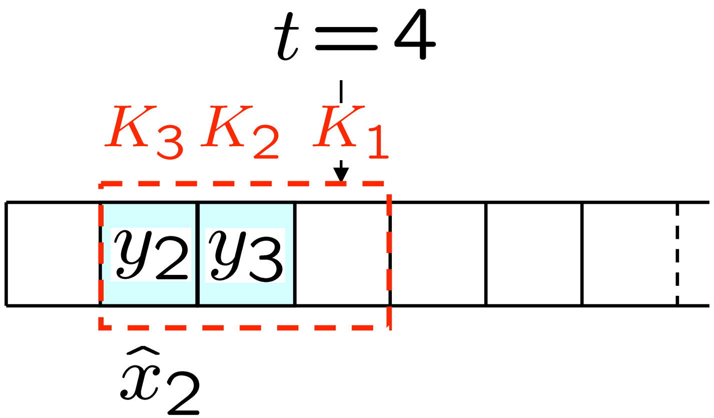
GOAL: compute $\overline{P}_{t|t}$

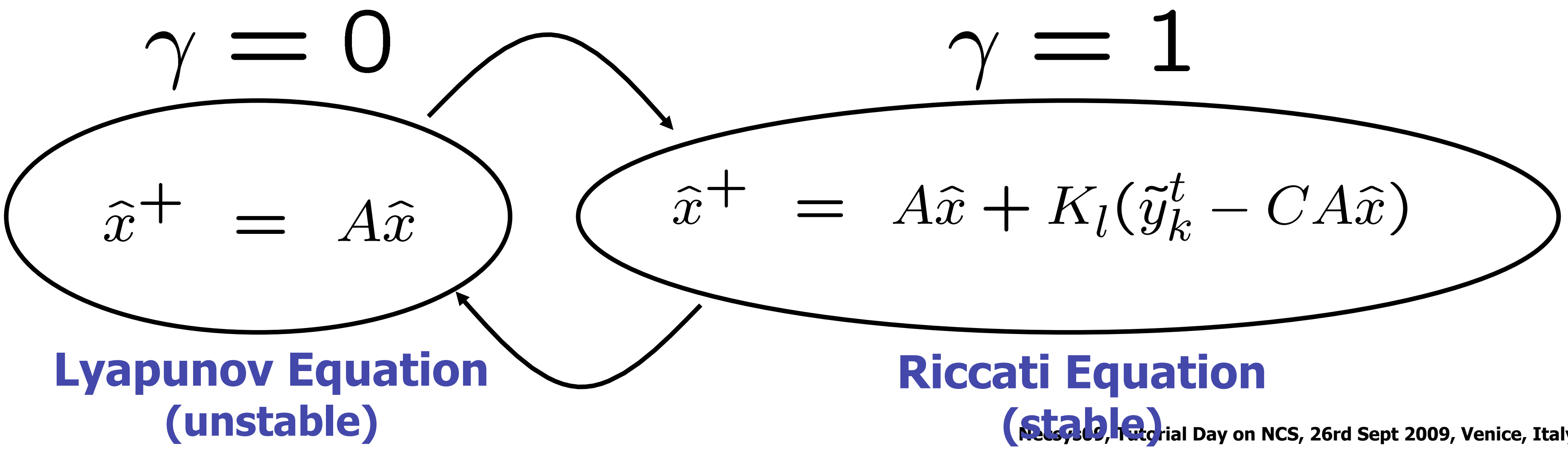
Suboptimal minimum variance estimation





Suboptimal minimum variance estimation





Necsys 2009

Steady state estimation error

Fixed gains:

$$\mathcal{L}_\lambda(K,P) = \lambda A(I-KC)P(I-KC)^T A^T + (1-\lambda)APA^T + Q + \lambda AKRK^T A^T$$

$$\begin{aligned} \overline{P} &= \mathcal{L}_{\lambda_{N-1}}(K_{N-1}, \overline{P}) \\ \overline{P}^+ &= \mathcal{L}_{\lambda_k}(K_k, \overline{P}), \qquad k = N-2, \ldots, 0 \end{aligned}$$

$$\lim_{t\to\infty} \overline{P}_{t|t} = \overline{P}$$

Optimal fixed gains:

$$\Phi_\lambda(P) = APA^T + Q - \lambda\, APC^T(CPC^T + R)^{-1} CPA^T$$

Modified Algebraic Riccati Equation (MARE) (Φ_1(P)=ARE)

$$\min_{K_0,\ldots,K_{N-1}} \overline{P}$$

$$\begin{aligned} \overline{P}_{N-1} &= \Phi_{\lambda_{N-1}}(\overline{P}_{N-1}) \\ \overline{P}_k &= \Phi_{\lambda_k}(\overline{P}_{k+1}), \qquad k = N-2, \ldots, 0 \\ K_k &= \overline{P}_k C^T (C\overline{P}_k C^T + R)^{-1} \end{aligned}$$

(off-line computation)

Stability issues

Static estimator is stable iff there exists $P \geq 0$ such that:

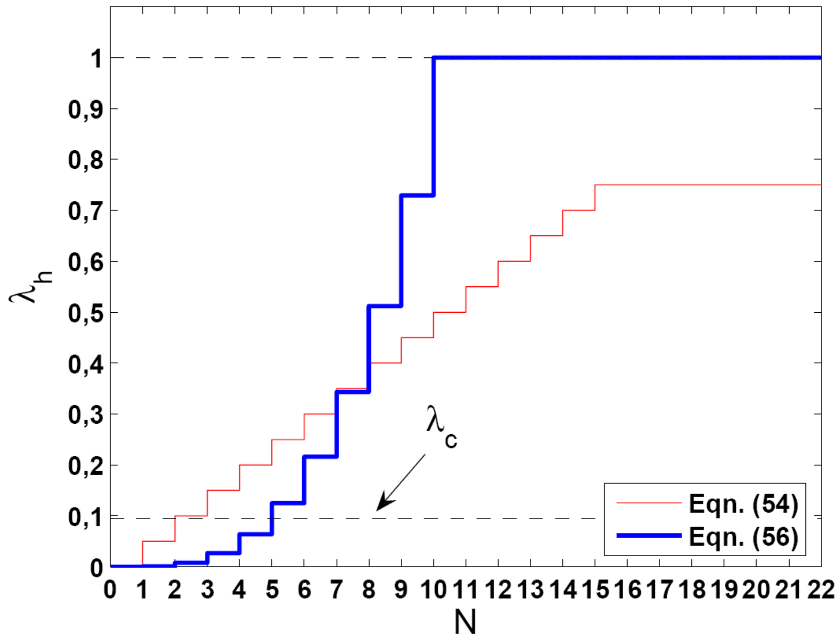
$$P = APA^T + Q - (1-\lambda)\, APC^T(CPC^T + R)^{-1}CPA^T$$

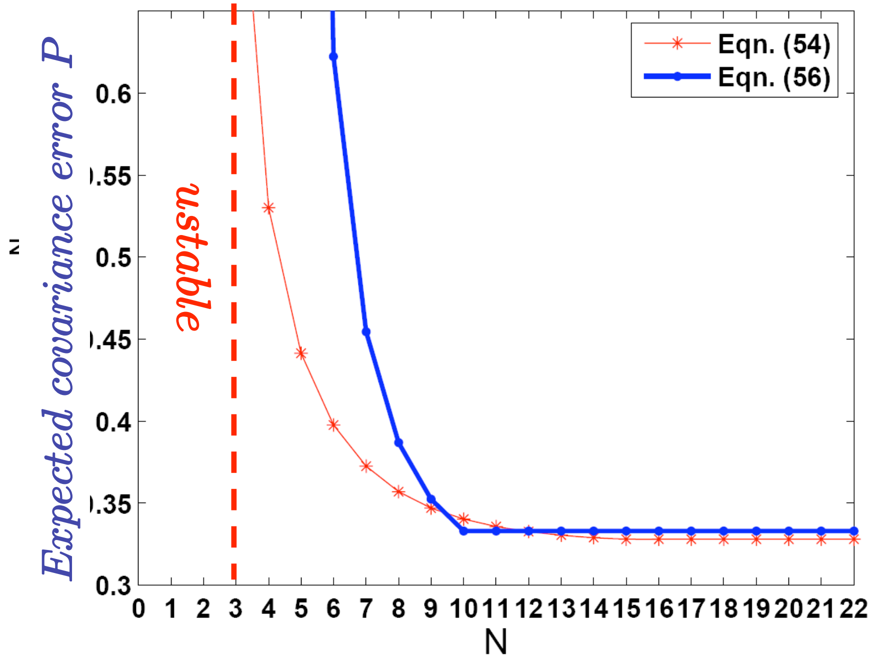
- If $\lambda = 0$ then standard ARE
- Modified Riccati Algebraic Equation known since [Nahi TIF'69]
- If A is unstable then there exist critical probability: if $\lambda < \lambda_c$ stable, if $\lambda > \lambda_c$ unstable
- Upper bound $\lambda_c \leq \frac{1}{\max |\text{eig}(A)|^2}$. Equality if C invertible [Katayama TAC"76]
- Lower bounf $\lambda_c \geq \frac{1}{\prod_{unstable} |\text{eig}(A)|^2}$. Equality if $\text{rank}(C) = 1$ [Elia TAC'01, SCL'05]
- Closed form expression for λ_c not known for general (A, C)

Numerical example (I)

Discrete time linearized inverted pendulum:

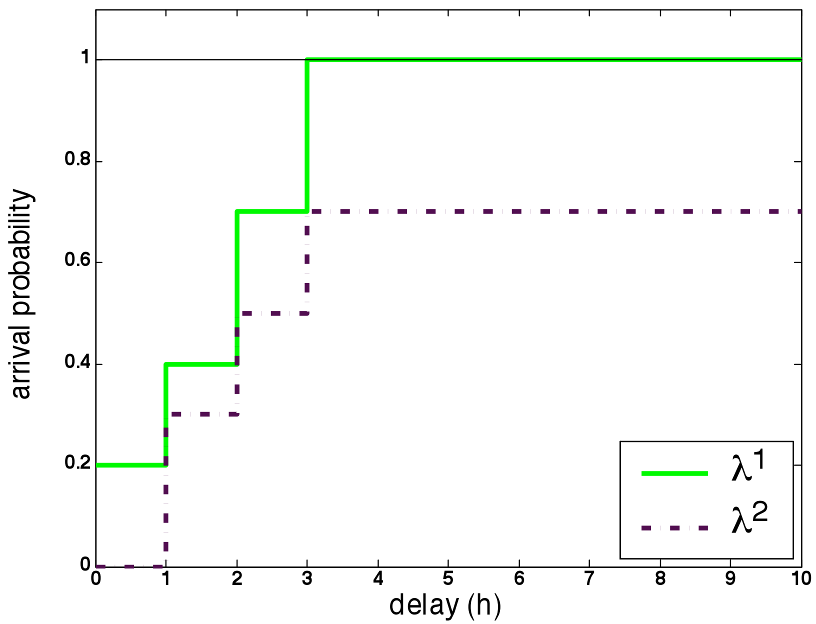
$$A = \begin{bmatrix} 1.01 & 0.05 \\ 0.05 & 1.01 \end{bmatrix}, \quad C = [\,1 \quad 0\,], \quad R = 1, \quad Q = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 1 \end{bmatrix}$$

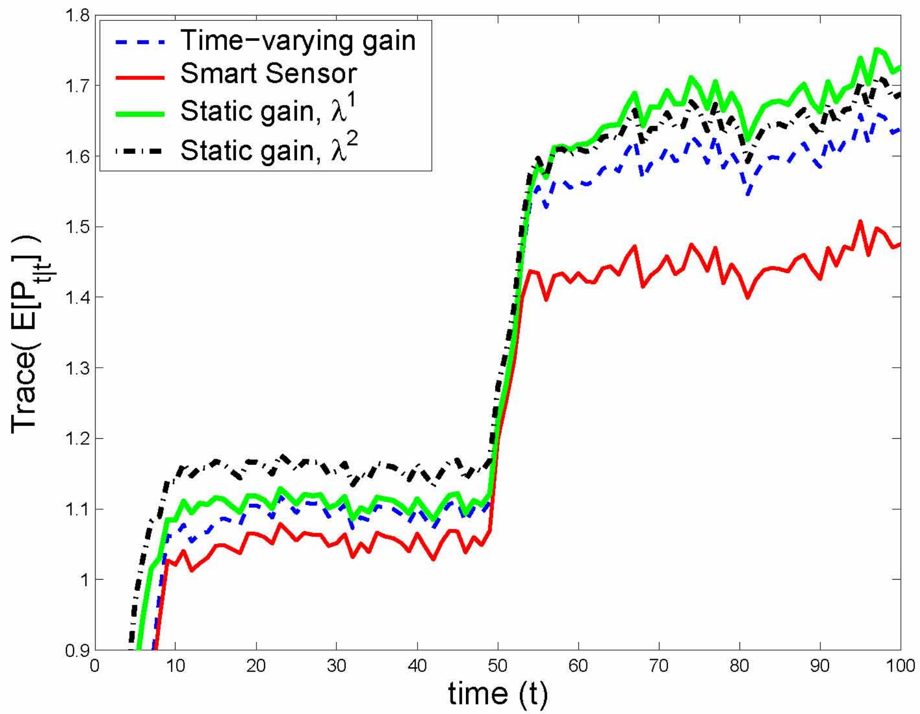




Numerical example (II)

Time-varying arrival probability distribution

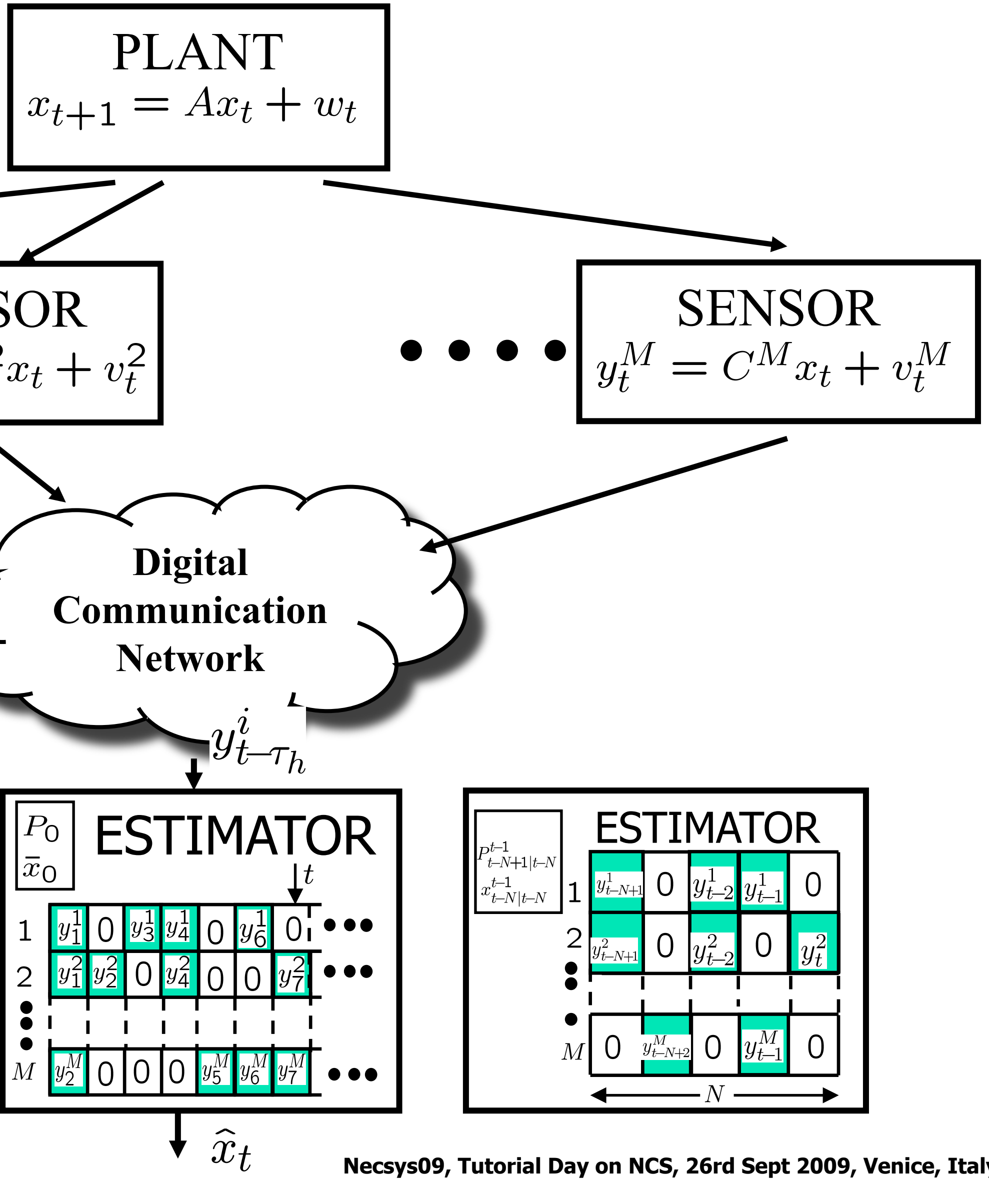


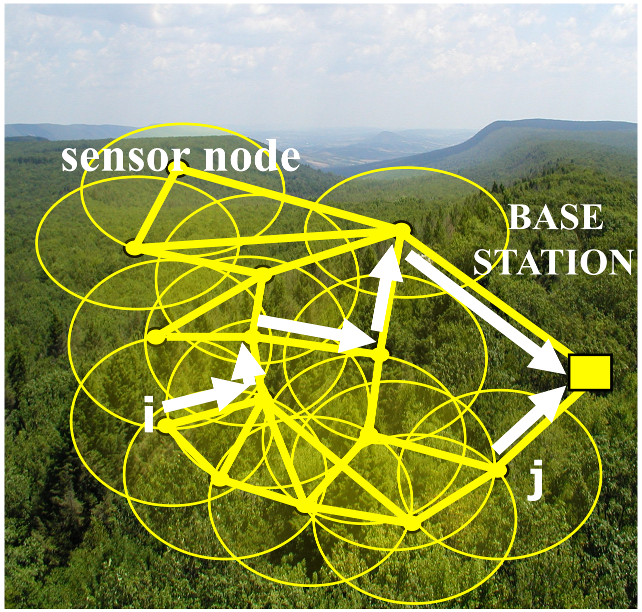


$$\lambda^1 \qquad 0 \leq t \leq 50$$

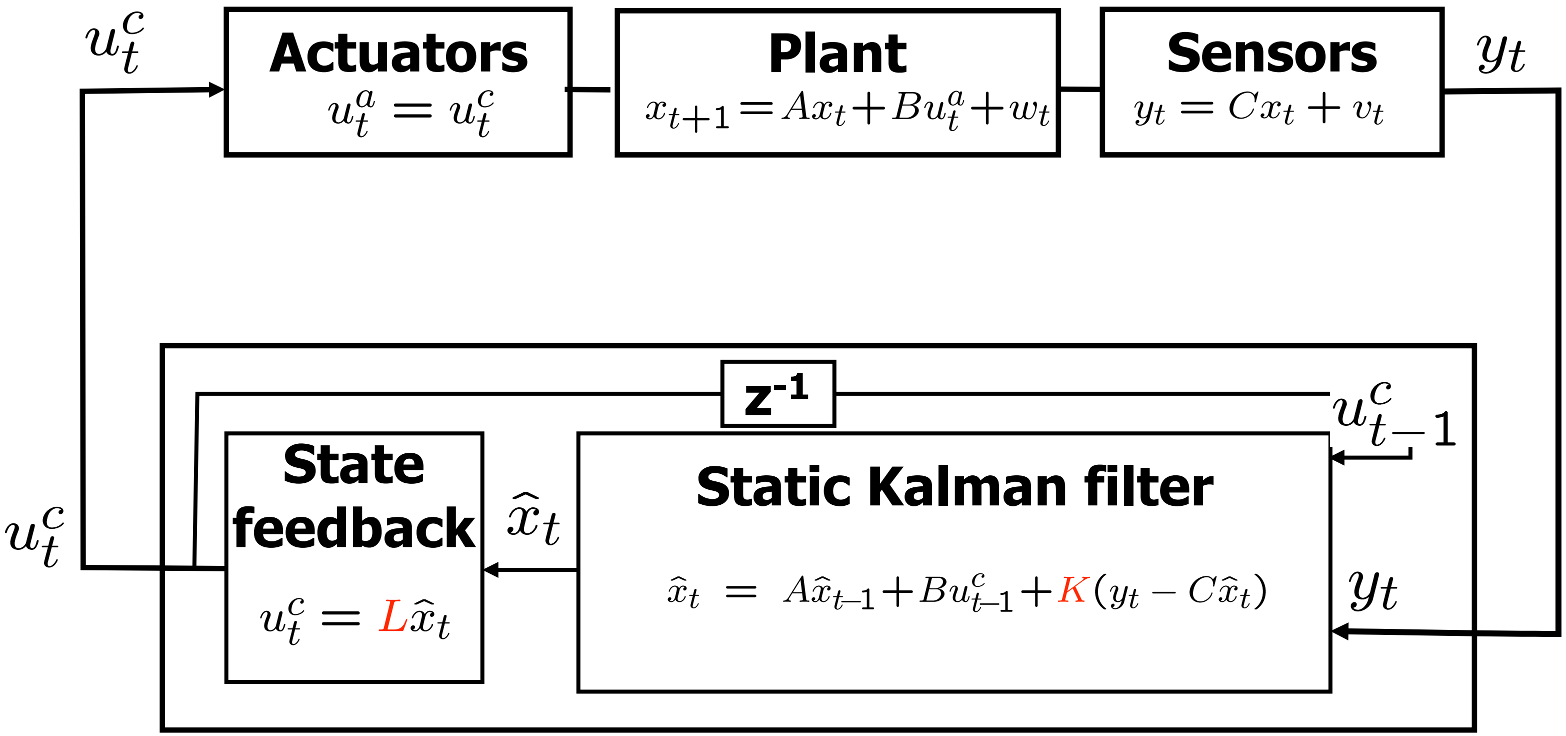
$$\lambda^2 \qquad t > 50$$

Multiple sensors

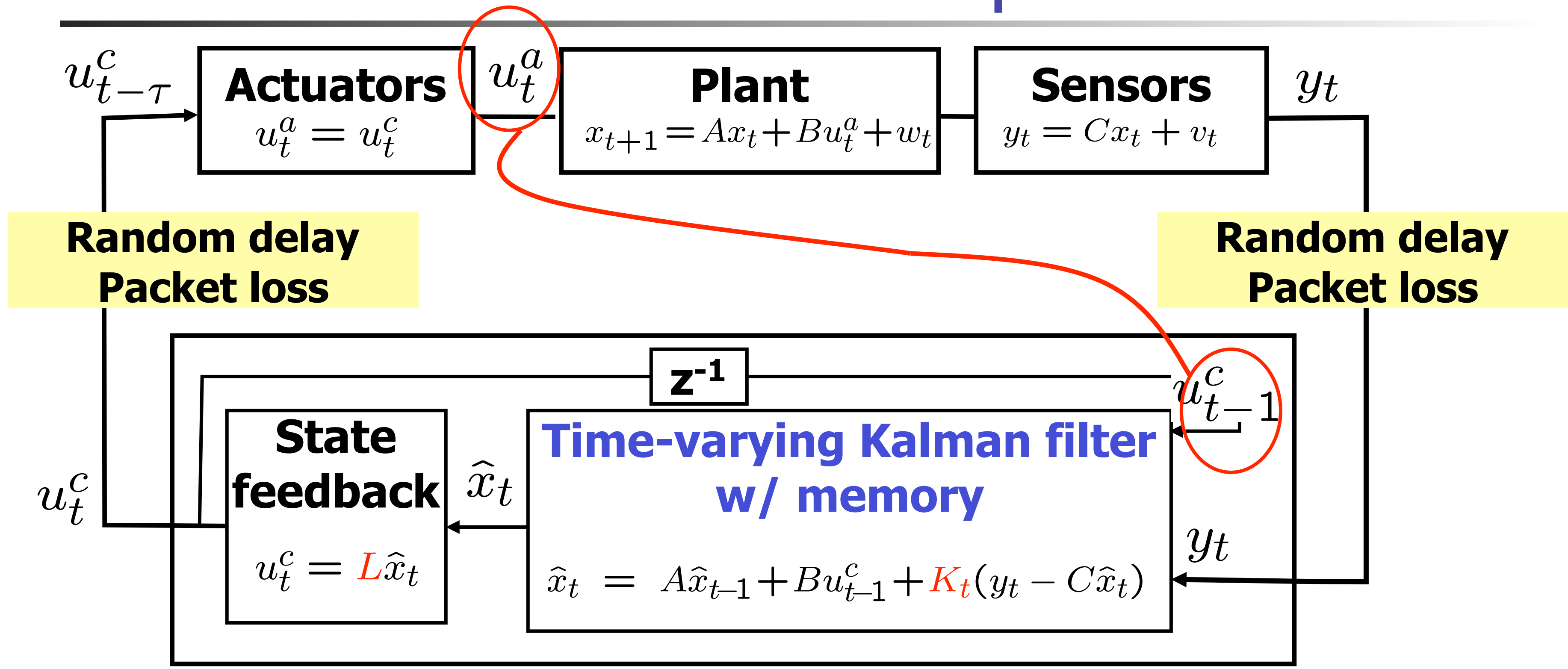




Back to the control problem



Back to the control problem



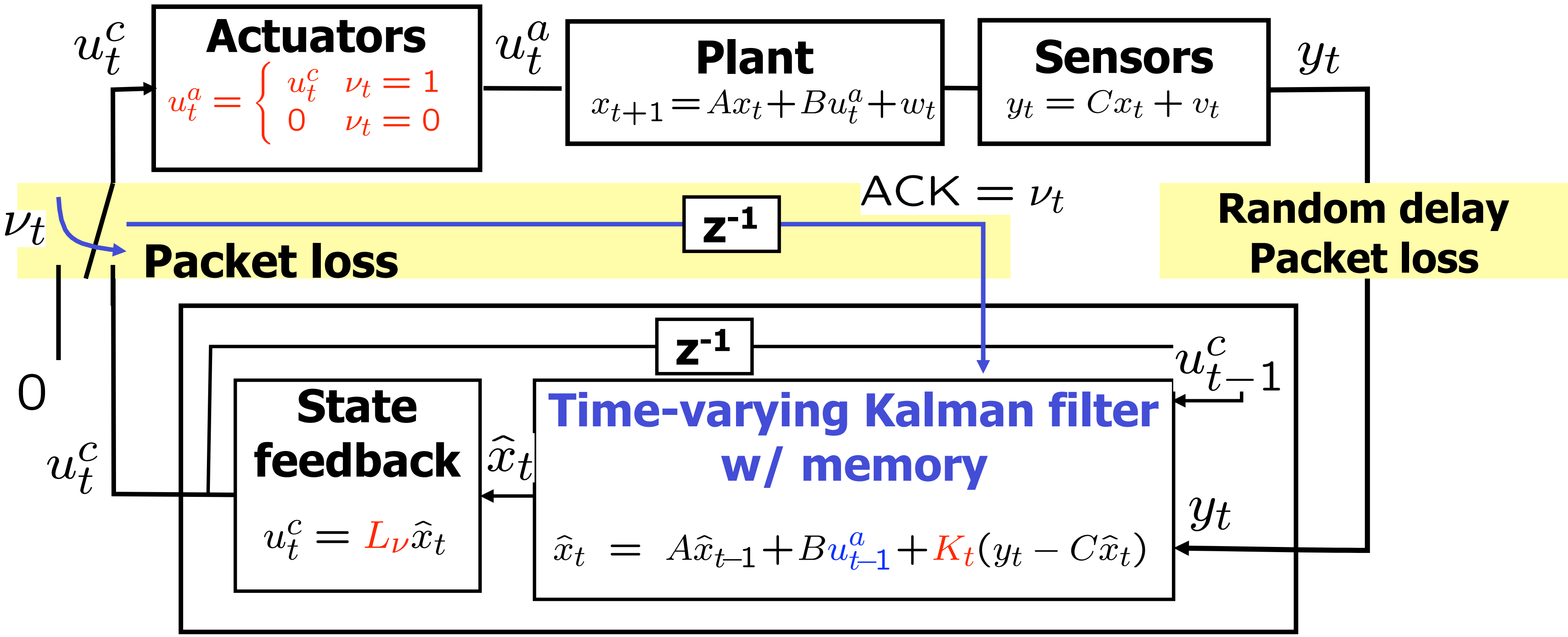
$$\hat{x}_t = E[x_t | y_t, y_{t-1}, .., y_0, u^a_{t-1}, .., u^a_1]$$

if $u^c_{t-1} \neq u^a_{t-1} \implies e_t = x_t - \hat{x}_t = f(y_t, .., y_0, u^c_t, .., u^c_0, u^a_t, .., u^a_0)$

$$P_{t|t-1} = AP_{t-1|t-1}A^T + Q + B(u^a_{t-1} - u^c_{t-1})(u^a_{t-1} - u^c_{t-1})^T B^T$$

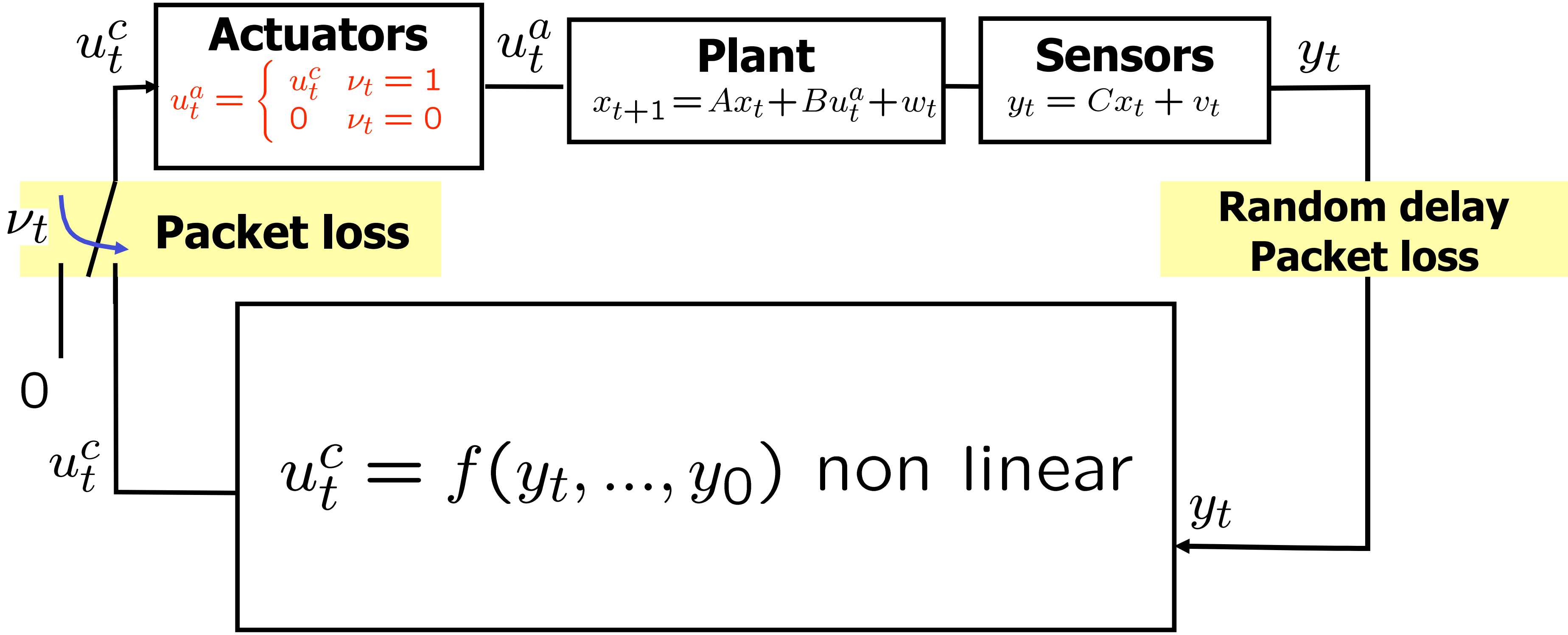
Estimation error coupled with control action → no separation principle

LQG over TCP-like (ACK-based) protocols



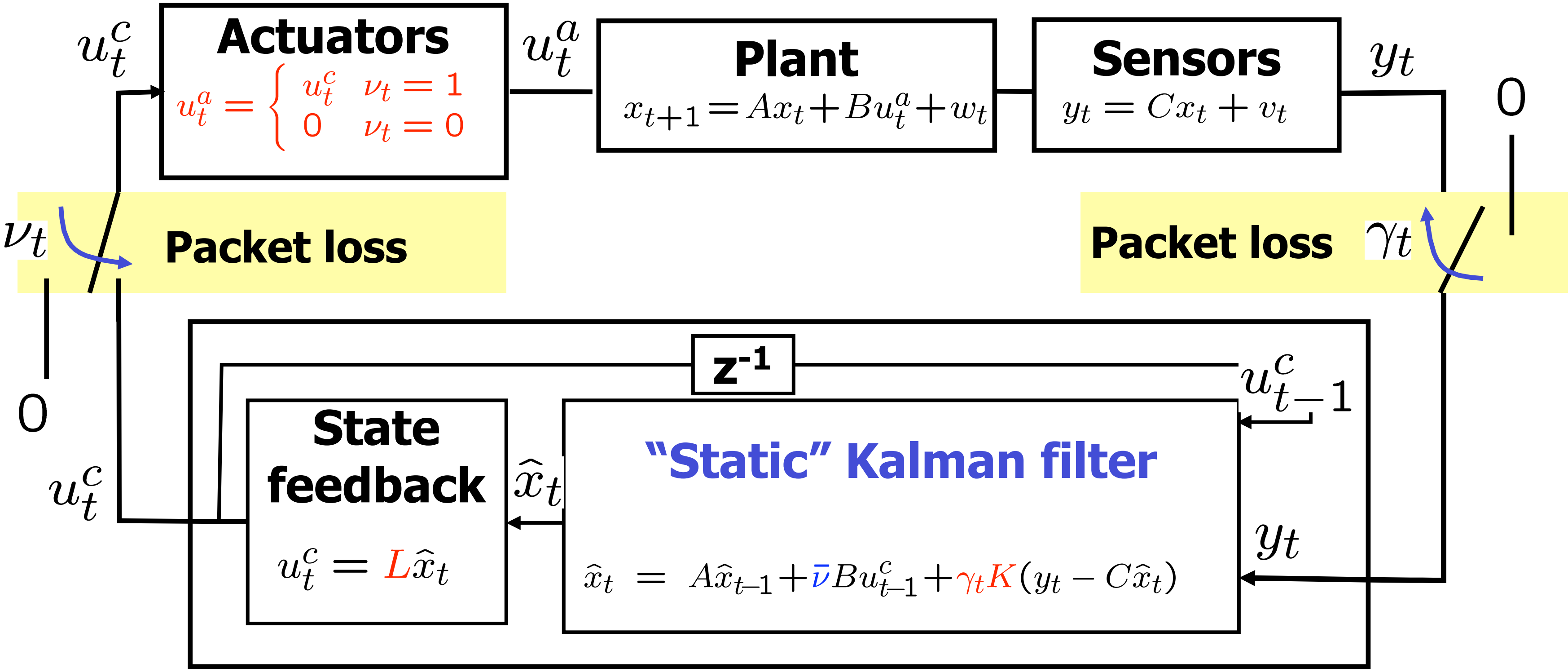
- Separation principle hold (I know exactly u^a_{t-1})
- ν_t Bernoulli rand. var and independent of observation arrival process
- Static state feedback, L_ν solution of dual MARE

LQG over UDP-like (no-ACK) protocols



- LQG problem still well defined: $\min_{u_t^c,\ldots,u_1^c} E[\sum_{h=1}^{t} x_t^T W x_t + (u_t^a)^T U u_t^a]$
- No separation principle hold (u^a_{t-1} NOT known exactly)
- … but still have some statistical information about u^a_{t-1}

LQG over UDP-like (no-ACK) protocols



- Bernoulli arrival process $P[\nu_t = 1] = \bar{\nu}, P[\gamma_t = 1] = \bar{\gamma}$
- $\bar{\nu}u_{t-1}^c = E[u_{t-1}^a]$
- Sub-optimal controller forced to be state estimator+state feedback
- Optimal choice of K,L is unique solution of 4 coupled Riccati-like equations

"Compensability and Optimal Compensation of systems with white parameters", De Koning, TAC'92

LQG as optimization problem

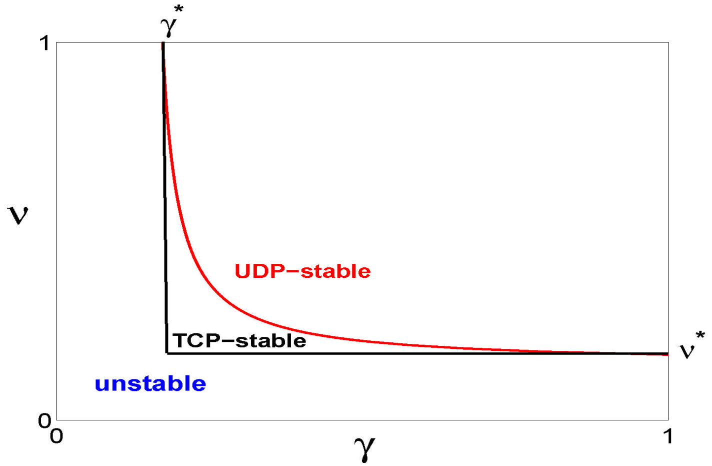
$$P \triangleq \mathbb{E}\left[\begin{bmatrix} x \\ \hat{x} \end{bmatrix} \begin{bmatrix} x^T & \hat{x}^T \end{bmatrix}\right] = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$$

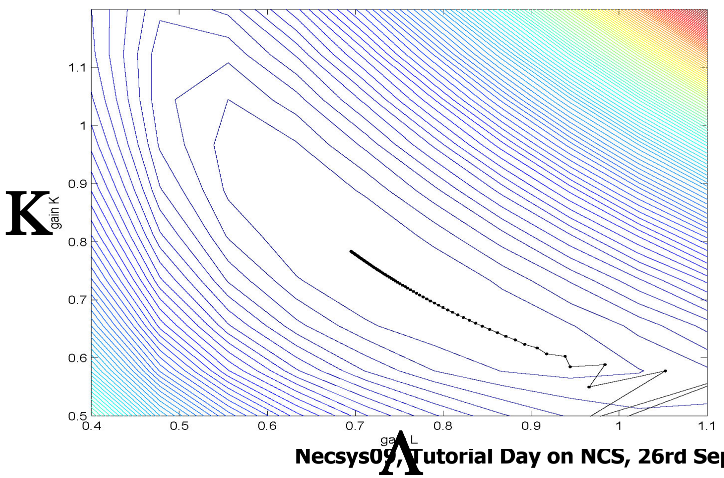
$$\text{Min}_{K,L} \quad \text{Trace}\left(\begin{bmatrix} W & 0 \\ 0 & \bar{\nu} L^T U L \end{bmatrix} P\right)$$

$$s.t. \quad P = \mathbb{E}\left[\begin{bmatrix} A & -\nu_k BL \\ \gamma_k KC & A - \bar{\nu} BL - \gamma_k KC \end{bmatrix} P \begin{bmatrix} A & -\nu_k BL \\ \gamma_k KC & A - \bar{\nu} BL - \gamma_k KC \end{bmatrix}^T\right] + \begin{bmatrix} Q & 0 \\ 0 & \bar{\gamma} KRK^T \end{bmatrix}$$

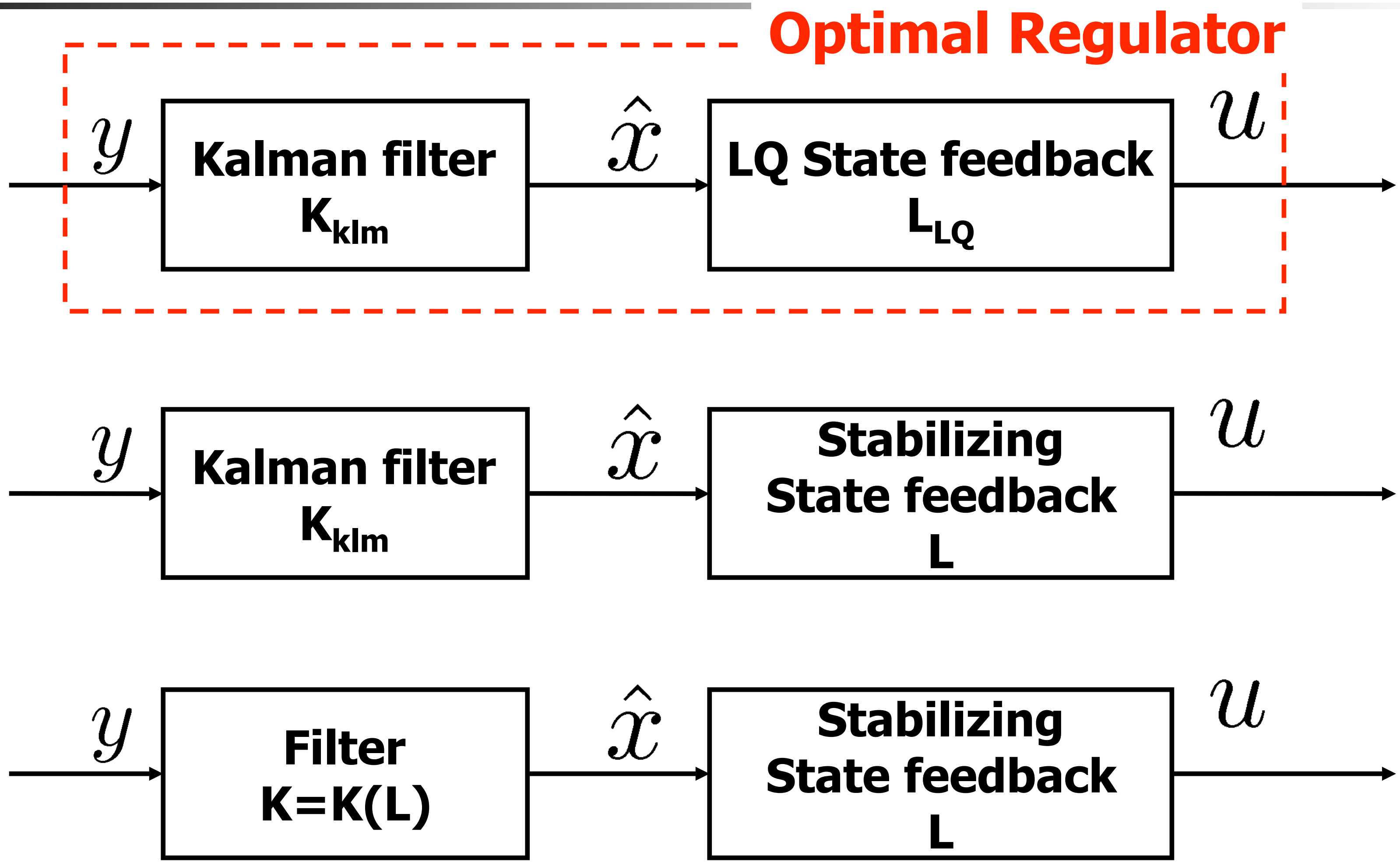
$$P \geq 0$$

- Non convex problem even for ν=γ=1, i.e. classic LQG
- Classic and TCP-based LQG become convex when exploiting optimality conditions like uncorralation between estimate and error estimate $\mathbb{E}[x(x-\hat{x})^T] = 0$
- For UDP-like problem non convex but unique solution using Homotopy and Degree Theory (DeKoning,Athans,Bernstain) (maybe using Sum-of-Squares?)
- Stability on ν and γ is coupled



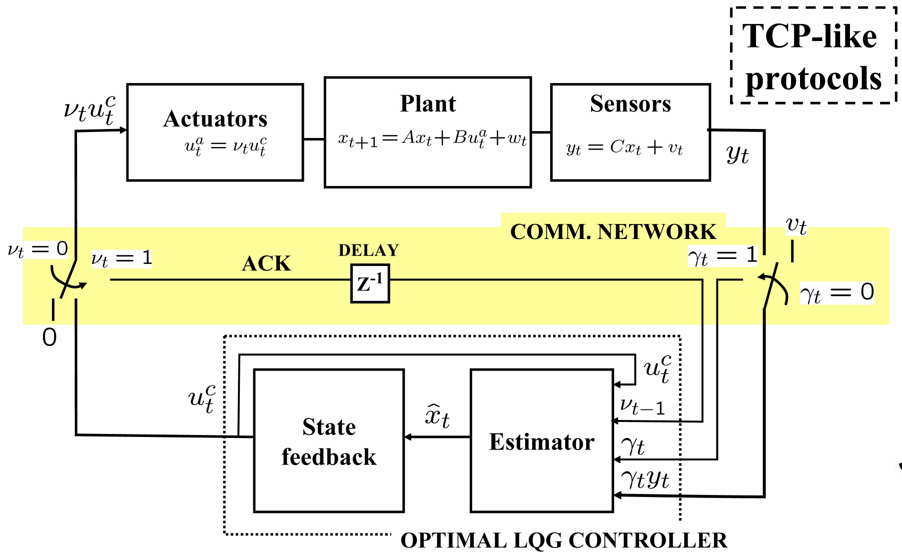


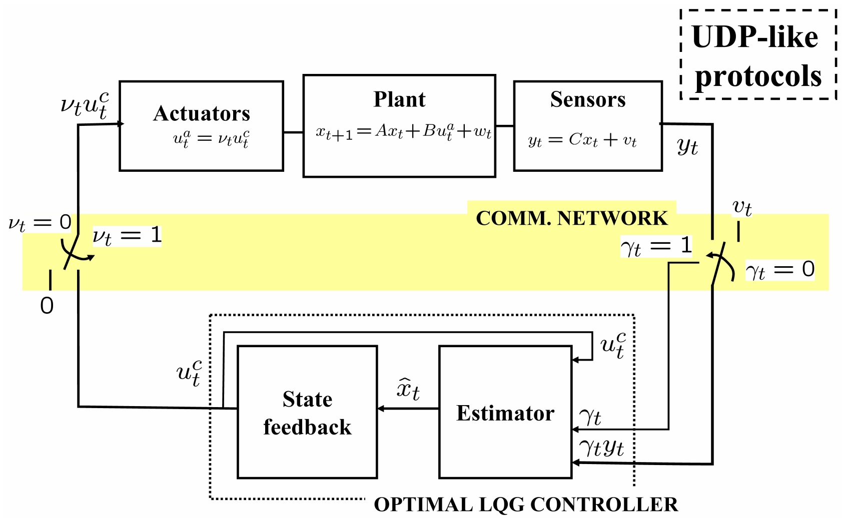
Paradox: Kalman filter is not always optimal !

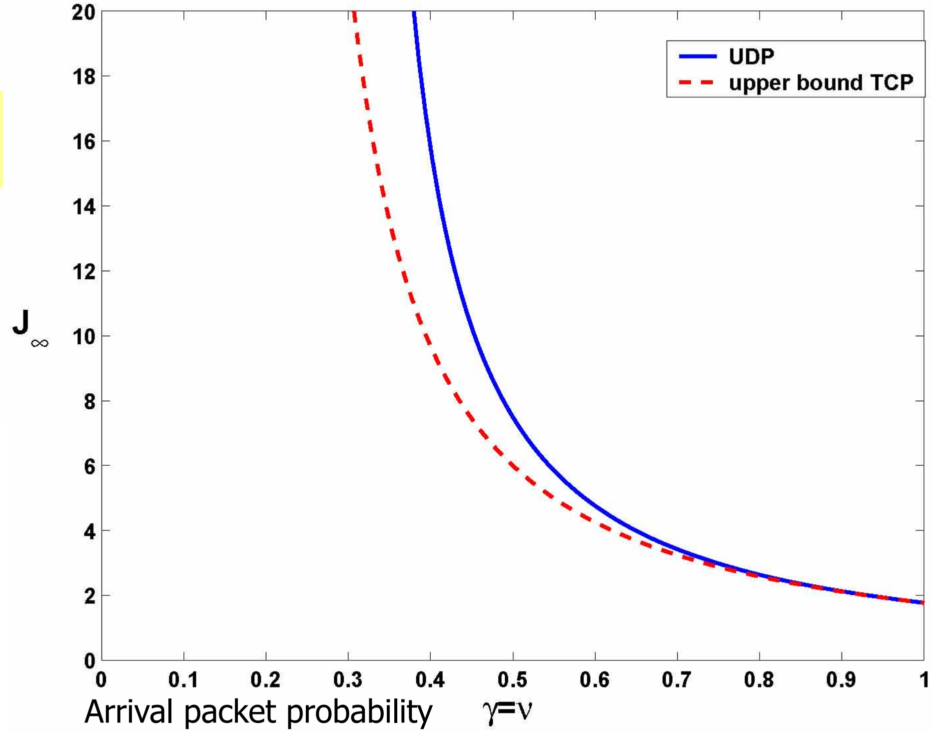


- Kalman filter always gives smallest estimate error **regardless** of controller L
- If controller $L \neq L_{LQ}$, then performance improves if my estimate is "bad" !

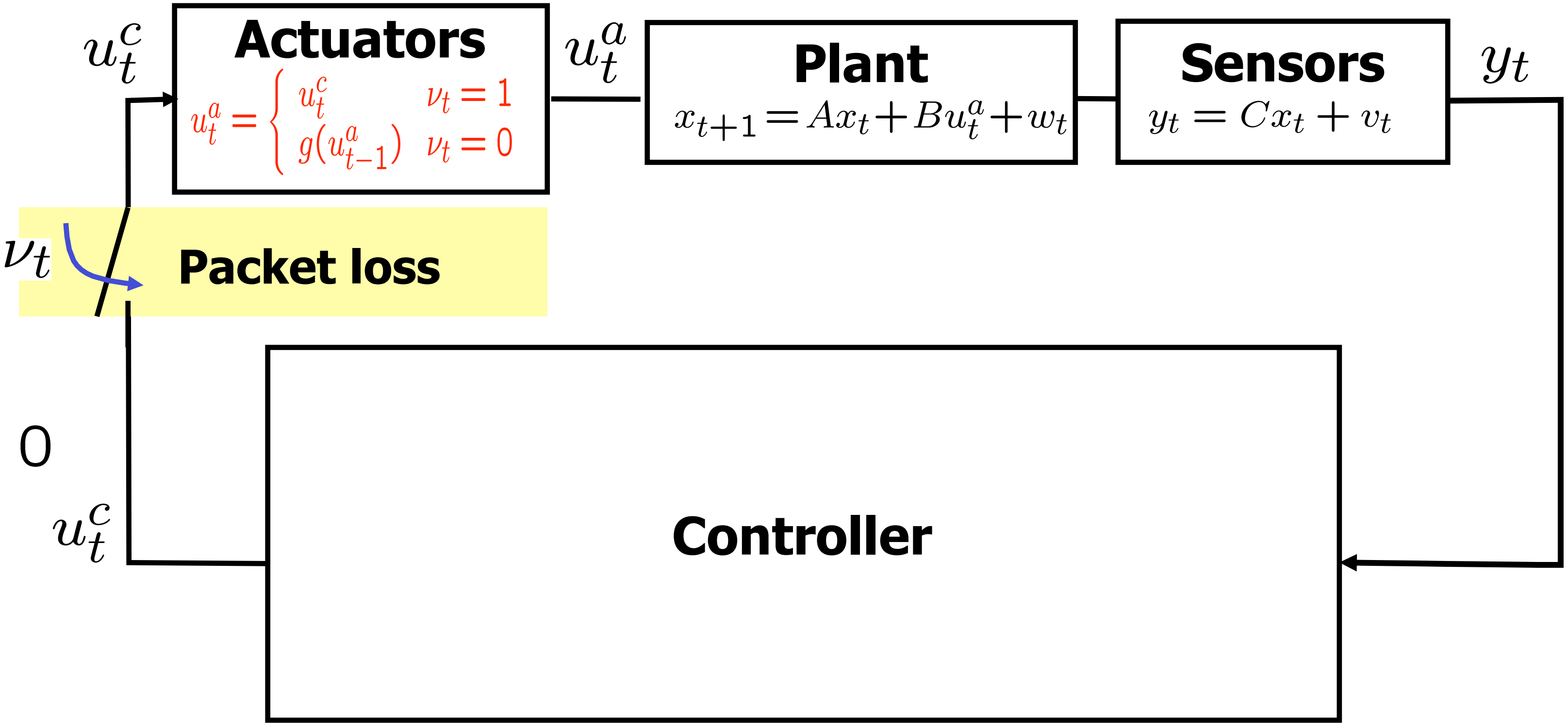
Numerical example: TCP vs UDP







To hold or to zero control input?



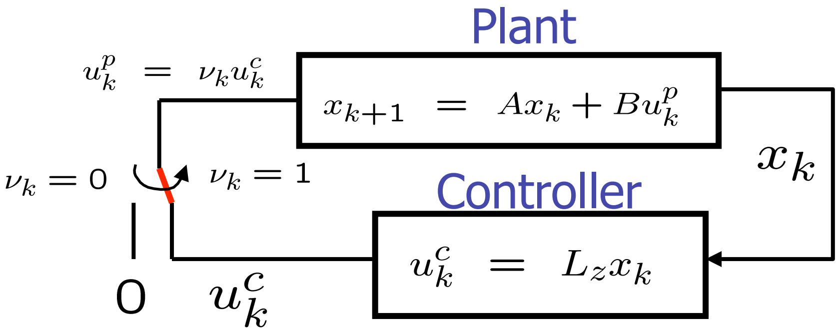
Most common strategy:

$g(u_{t-1}^a) = 0$ zero-input strategy **(mathematically appealing)**

$g(u_{t-1}^a) = u_{t-1}^a$ hold-input strategy **(most natural)**

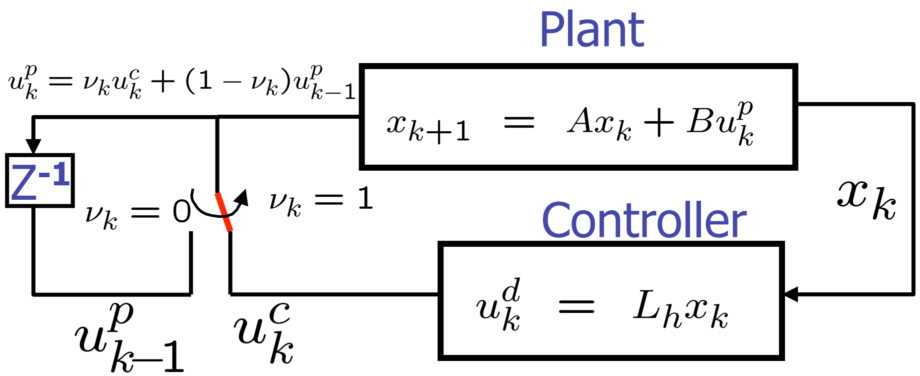
To hold or to zero control input: no noise (jump linear systems)

Zero-input Strategy



$$J_z^* = \min_{L_z} E[\sum_{t=1}^{\infty} x_t^T W x_t + (u_t^a)^T U u_t^a]$$

Hold-input Strategy



$$J_h^* = \min_{L_h} E[\sum_{t=1}^{\infty} x_t^T W x_t + (u_t^a)^T U u_t^a]$$

Using cost-to-go function (dynamic programming)

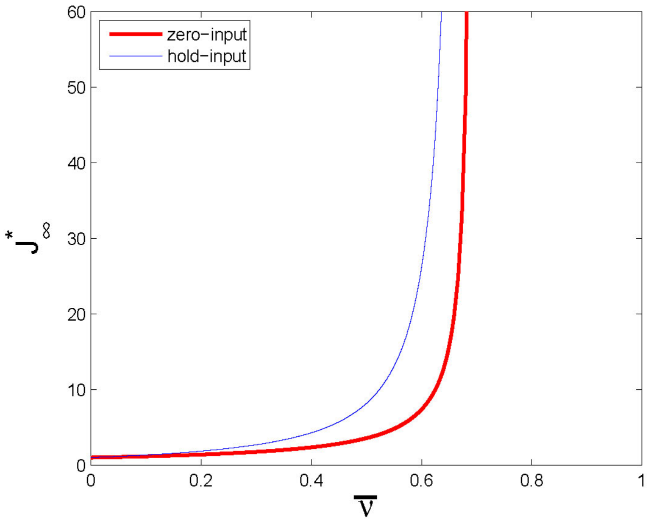
$$J_z^* = E[x_0^T S_z x_0] \qquad J_h^* = E[x_0^T S_h x_0]$$

$$S_z = \Phi_z(S_z) \longleftarrow \textbf{Riccati-like equation} \longrightarrow S_h = \Phi_h(S_h)$$

$$L_z^* = f_z(S_z) \qquad L_h^* = f_h(S_h)$$

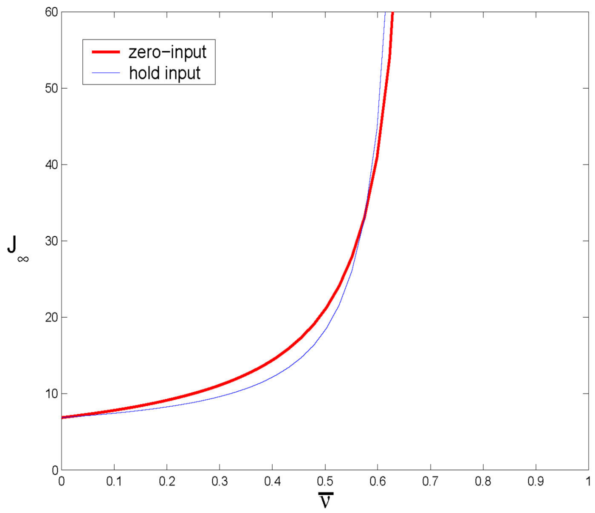
Example: unstable scalar system

A=1.2, U=0
(fastest convergence)



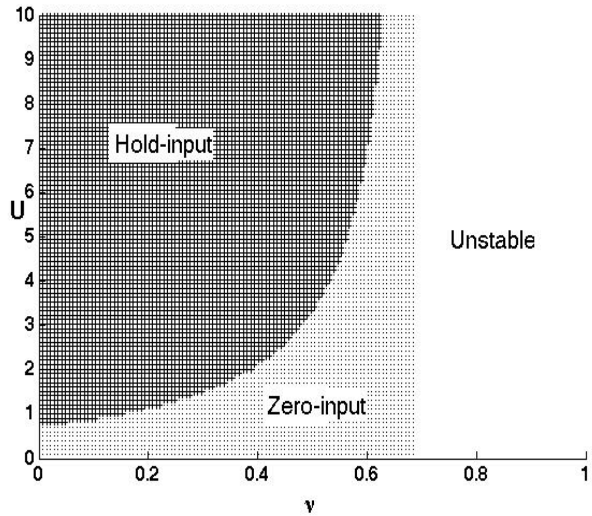
Loss probability

A=1.2, U=10
(small input)



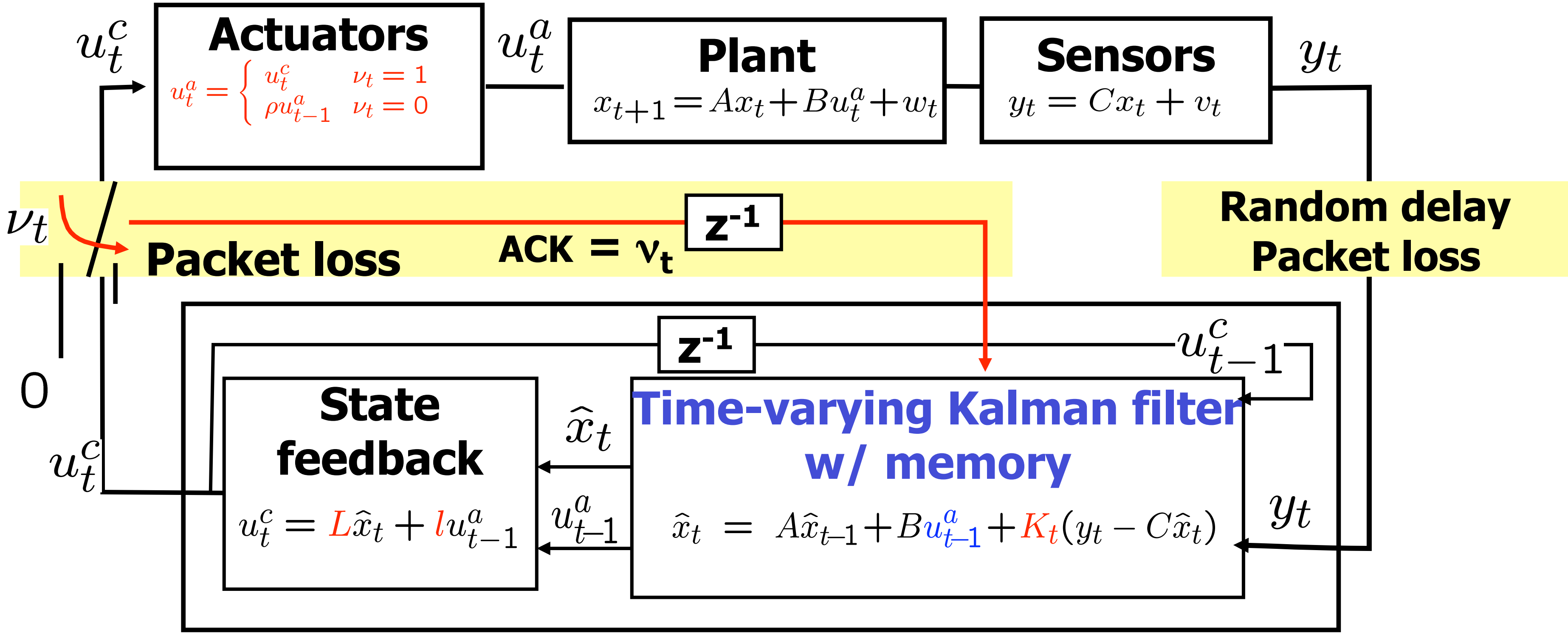
Loss probability

Optimal strategy
regions



Loss probability

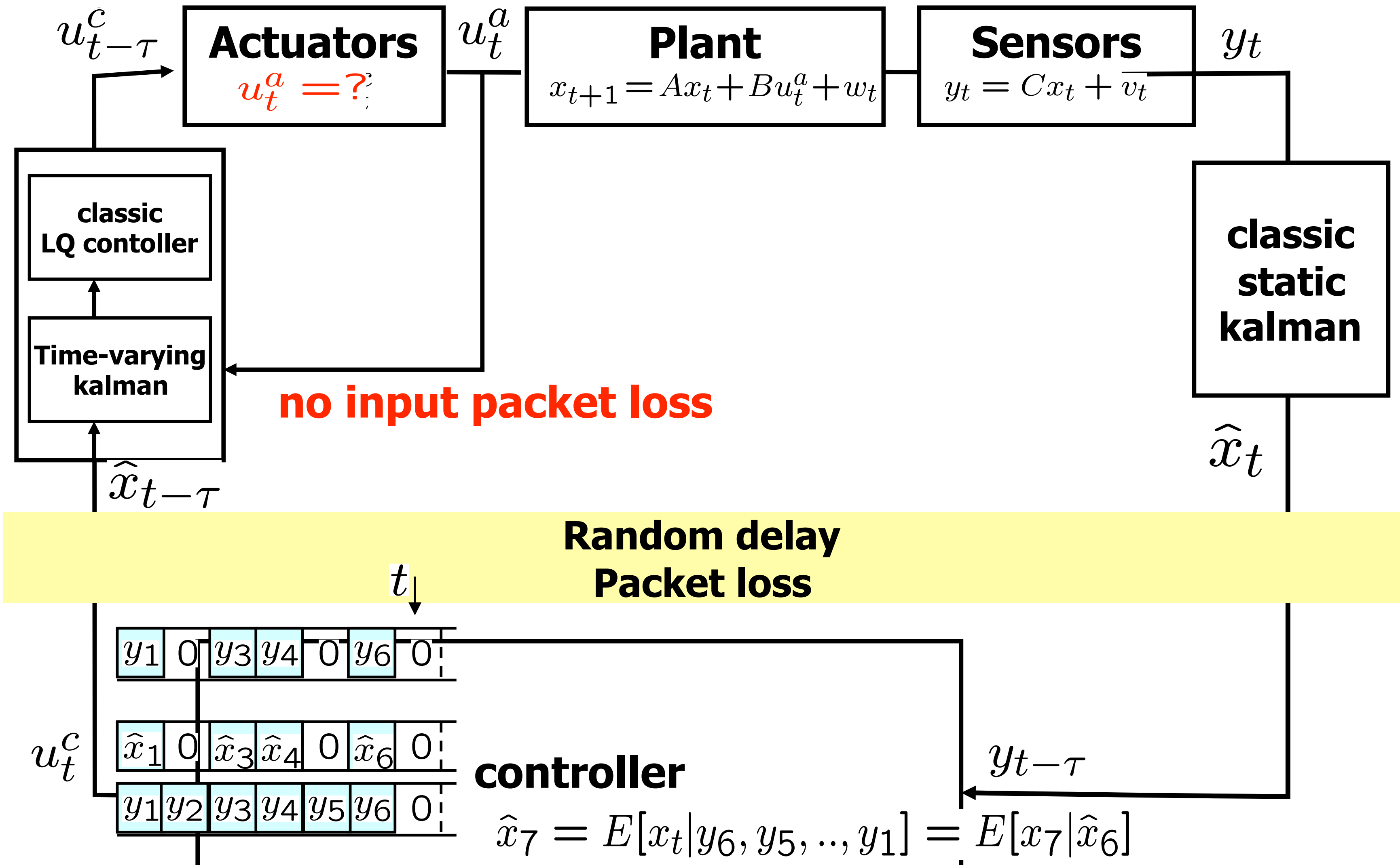
LQG over TCP-like protocols revised



Conjecture:

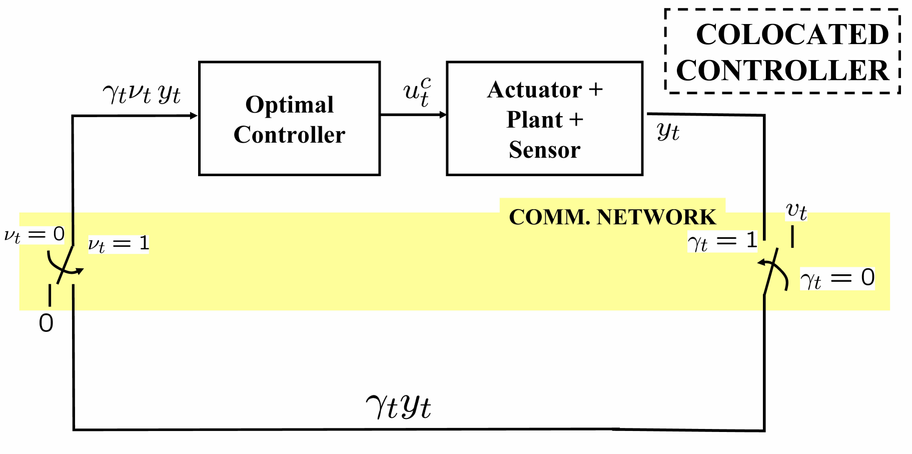
- Separation principle hold
- Optimal function $g(u_{t-1}^a) = \rho u_{t-1}$
- Design parameter L, l, ρ obtained via LQ-like optimal state feedback

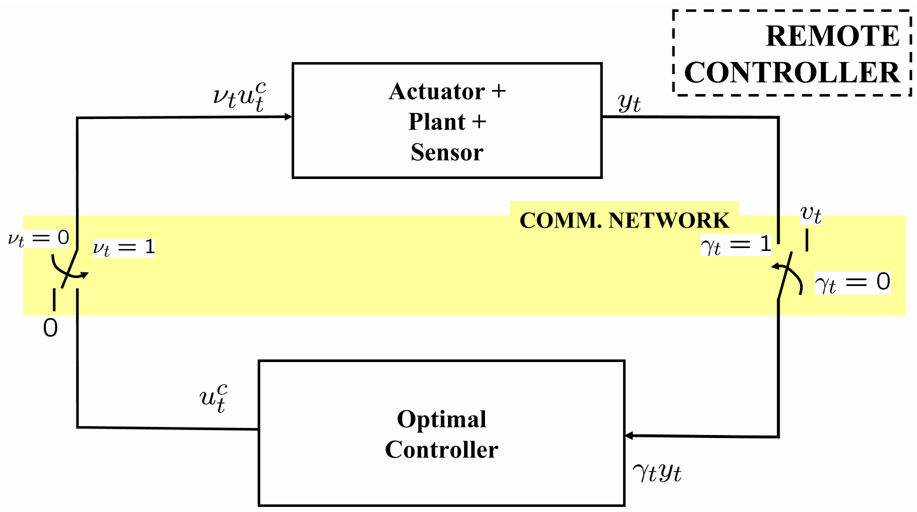
Smart sensors & smart actuators

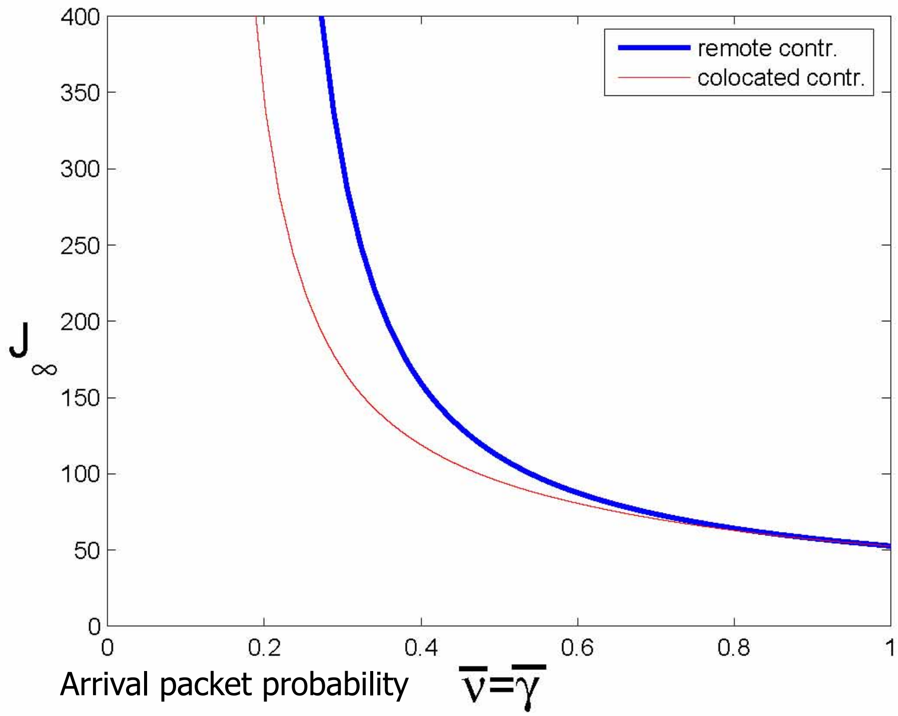


"Optimal LQG control across a packet-dropping link", Gupta, Spanos, Murray, Submitted to Sys.Cont.Lett. 05
"Estimation under controlled and uncontrolled communications in networked control systems", Xu, Hespanha, CDC 05

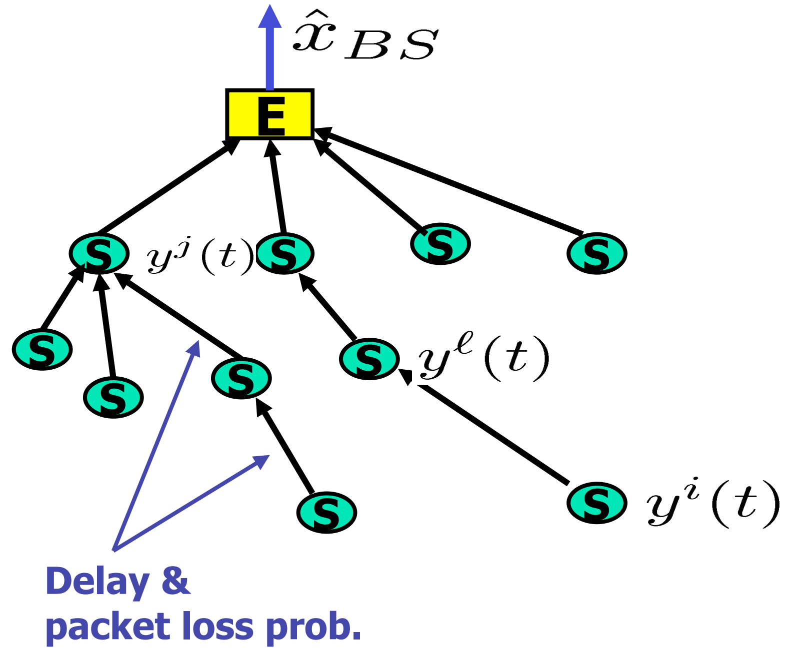
Numerical example: remote vs co-located controller

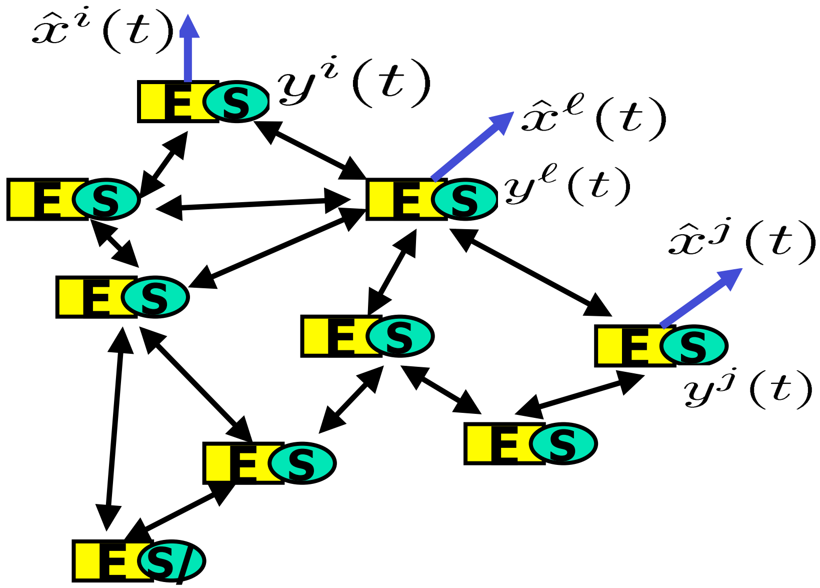






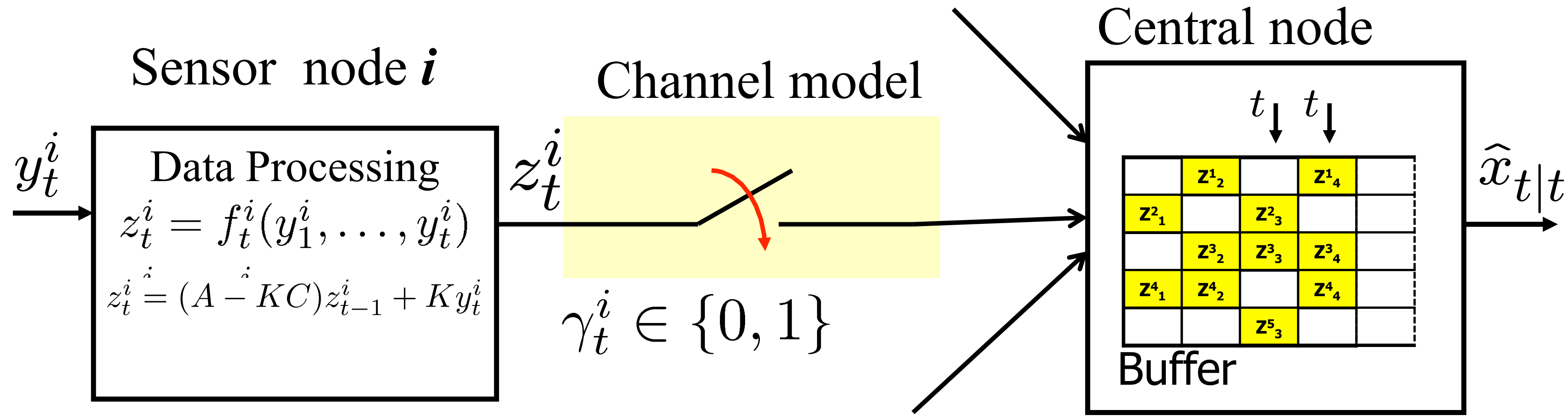
Distributed estimation: previous work





- Distributed estimation is old problem (see Levy, Willsky 80's, Bar-Shalom 90's)
- Consensus-based estimation (Olfati-Saber et al. 07, Carli et al. 08)
- Many results on optimal estimation under perfect communication
- Distributed estimation with packet loss still open problem

Modeling

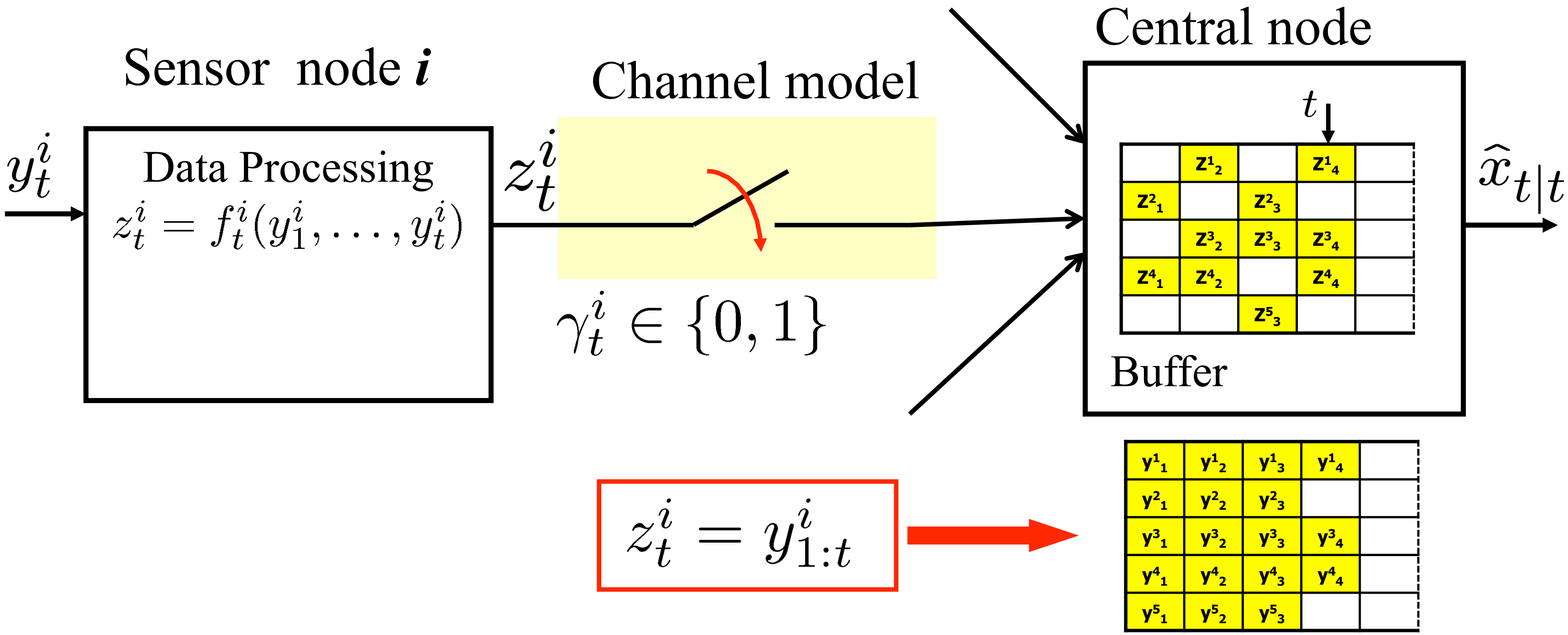


$$
\begin{aligned}
x_{t+1} &= Ax_t + w_t \\
y_t^i &= C^i x_t + v_t^i \quad i = 1, \ldots, M \\
E[w_t] &= E[v_t^i] = 0, \;\; E[w_t w_t^T] = Q, \;\; E[v_t^i (v_t^j)^T] = R_{ij} \\
P[\gamma_t^i] &= \bar{\gamma}
\end{aligned}
$$

Objective:

$$\hat{x}_{t|t}^{BS} = E[x_t \,|\, \text{information } z_{1:t}^i \text{ available at base station}]$$

Optimal strategy: Infinite Bandwidth Filter



$$\hat{x}_{t|t}^{IBF}, \quad P_{t|t}^{IBF} = \mathrm{Var}(\hat{x}_{t|t}^{IBF} - x_t \,|\, \text{sequence})$$

$$P_{t|t}^{IBF} \leq P_{t|t}, \quad \forall \gamma_t^i, \forall f_t^i$$

Necsys 2009

A negative result

Theorem Let us consider the state estimate $\hat{x}_{t|t}$ and $\hat{x}_{t|t}^{IBF}$ defined as above. Then there do not exist (possibly nonlinear) functions $z_t^i = f_t^i(y_{1:t}^i) \in \mathbb{R}^\ell$ with bounded size $\ell < \infty$ such that $P_{t|t}^{IBF} = P_{t|t}$ for any possible packet loss sequence, i.e.

$$\nexists f_t^i() \,|\, P_{t|t} = P_{t|t}^{IBF}, \ \forall \gamma_t^i$$

A negative result

Theorem Let us consider the state estimate $\hat{x}_{t|t}$ and $\hat{x}^{IBF}_{t|t}$ defined as above. Then there do not exist (possibly nonlinear) functions $z^i_t = f^i_t(y^i_{1:t}) \in \mathbb{R}^\ell$ with bounded size $\ell < \infty$ such that $P^{IBF}_{t|t} = P_{t|t}$ for any possible packet loss sequence, i.e.

$$\nexists f^i_t() \,|\, P_{t|t} = P^{IBF}_{t|t}, \ \forall \gamma^i_t$$

Sketch of proof:

$$\begin{aligned} x_{t+1} &= x_t + w_t \\ y^1_t &= x_t + v^1_t \\ y^2_t &= x_t + v^2_t \end{aligned}$$

$\ell \in \mathbb{R}$ and $f^i_t()$ linear, $\mathbb{E}[x_0] = 0$

$\sigma_x = \sigma_w = \sigma_{v_1} = \sigma_{v_2}$

$$\begin{bmatrix} \alpha_1^{1,a} \\ \alpha_2^{1,a} \end{bmatrix} \neq \beta \begin{bmatrix} \alpha_1^{1,b} \\ \alpha_2^{1,b} \end{bmatrix}$$

Scenario a

	z_2^1

$$z^1_2 = f^1_2(y^1_1, y^1_2) = \bar{\alpha}^1_1 y^1_1 + \bar{\alpha}^1_2 y^1_2$$

y_1^1	y_2^1

$$\hat{x}^{IBF,a} = \alpha_1^{1,a} y^1_1 + \alpha_2^{1,a} y^1_2$$

Scenario b

	z_2^1
	z_2^1

$$z^1_2 = \bar{\alpha}^1_1 y^1_1 + \bar{\alpha}^1_2 y^1_2$$

y_1^1	y_2^1
y_1^2	y_2^2

$$\hat{x}^{IBF,b} = \alpha_1^{1,b} y^1_1 + \alpha_2^{1,b} y^1_2 + \alpha_1^{2,b} y^2_1 + \alpha_2^{2,b} y^2_2$$

Necsys 2009

Suboptimal strategies

- Measurement fusion:
  - $z_t^i = y_t^i$ at sensor
  - $\hat{x}_{t|t}^{MF} = E[x_t \,|\, \text{all } z_t^i \text{ arrived}]$: base station
- Optimal Kalman Filter Fusion
  - $z_t^i = \hat{x}_t^i = (A - C^i K^{i,loc})\hat{x}_{t-1}^i + K^{i,loc} y_t^i$
  - $\hat{x}_{t|t}^{OKFF} = E[x_t \,|\, \text{latest } z_t^i \text{ arrived } \forall i] = \sum_i \Psi_t^i z_{t-\tau_t^i}^i$
- Optimal Partial Estimate Fusion
  - $z_t^i = \hat{x}_t^i = (A - \sum_i C^i K^{i,cent})\hat{x}_{t-1}^i + K^{i,cent} y_t^i$
  - $\hat{x}_{t|t}^{OPEF} = E[x_t \,|\, \text{latest } z_t^i \text{ arrived } \forall i] = \sum_i \Phi_t^i z_{t-\tau_t^i}^i$
- Open Loop Partial Estimate Fusion
  - $z_t^i = \hat{x}_t^i = (A - C^i K^{i,cent})\hat{x}_{t-1}^i + K^{i,cent} y_t^i$
  - $\hat{x}_{t|t}^{OLPEF} = \sum_i A^{\tau_t^i} z_{t-\tau_t^i}^i$

Single sensor & packet loss

$\downarrow t = 6$

y_1			y_4	y_5	

$$\hat{x}_{6|6}^{MF} = E[x_6 | y_1, y_4, y_5]$$

y_1	y_2	y_3	y_4	y_5	

$$\hat{x}_{6|6}^{IBF} = E[x_6 | y_{1:5}] = E[x_6 | \hat{x}_{5|5}] = A\hat{x}_{5|5}$$

$\hat{x}_{1|1}$			$\hat{x}_{4|4}$	$\hat{x}_{5|5}$	

$$\hat{x}_{6|6}^{OKFF} = E[x_6 | \hat{x}_{5|5}] = A\hat{x}_{5|5}$$

$$P_{t|t}^{IBF} = P_{t|t}^{OKFF} = P_{t|t}^{OLPEF} = P_{t|t}^{OPEF} < P_{t|t}^{MF}$$

Multi sensor & no packet loss

$\downarrow t = 3$

$y^1{}_1$	$y^1{}_2$	$y^1{}_3$
$y^2{}_1$	$y^2{}_2$	$y^2{}_3$
$y^3{}_1$	$y^3{}_3$	$y^3{}_3$

$$\hat{x}^{MF}_{t|t} = E[x_t | y^i_{1:t} \, \forall i] = \hat{x}^{IBF}_{t|t} = \hat{x}^{cent}_{t|t}$$

$\downarrow t = 3$

$\hat{x}^{1,loc}_{3\|3}$
$\hat{x}^{2,loc}_{3\|3}$
$\hat{x}^{3,loc}_{3\|3}$

$$\hat{x}^{i,loc}_t = (A - K^{i,loc} C^i)\hat{x}^{i,loc}_{t-1} + K^{i,loc} y^i_t$$

$$\hat{x}^{OKFF}_{t|t} = E[x_t | \hat{x}^{i,loc}_{t|t} \, \forall i] \neq \hat{x}^{IBF}_{t|t} = \hat{x}^{cent}_{t|t}$$

A.S. Willsky, D. Castanon, B. Levy, and G. Verghese,“ Combining and updating of local estimates and regional maps along sets of one-dimensional track ” IEEE Trans. on Aut. Cont.,1982

Centralized Kalman Filter

$$\begin{aligned} x_{t+1} &= A x_t + w_t \\ y_t^i &= C^i x_t + v_t^i \end{aligned} \qquad C = \begin{bmatrix} C^1 \\ C^2 \\ \vdots \\ C^M \end{bmatrix}, \; y_t = \begin{bmatrix} y_t^1 \\ y_t^2 \\ \vdots \\ y_t^M \end{bmatrix}, \; v_t = \begin{bmatrix} v_t^1 \\ v_t^2 \\ \vdots \\ v_t^M \end{bmatrix}, \; E[v_t v_t^T] = R$$

$$K^{cent} = [K^{1,cent} \;\; K^{2,cent} \;\; \ldots \;\; K^{M,cent}]$$

$$\begin{aligned} \hat{x}_t^{cent} &= (A - K^{cent} C)\hat{x}_{t-1}^{cent} + K^{cent} y_t \\ &= \underbrace{\left(A - \textstyle\sum_i K^{i,cent} C^i\right)}\hat{x}_{t-1}^{cent} + \textstyle\sum_i K^{i,cent} y_t^i \end{aligned}$$

$$\hat{x}_t^{i,cent} = F \hat{x}_{t-1}^{i,cent} + K^{i,cent} y_t^i, \qquad \text{local filter}$$

$$\boxed{\hat{x}_{t|t}^{cent} = \textstyle\sum_i \hat{x}_{t|t}^{i,cent}}$$

J. Wolfe and J. Speyers,"A low-power filtering scheme for distributed sensor networks," CDC'03

Multi sensor & no packet loss

$\downarrow t = 3$

$y^1{}_1$	$y^1{}_2$	$y^1{}_3$
$y^2{}_1$	$y^2{}_2$	$y^2{}_3$
$y^3{}_1$	$y^3{}_3$	$y^3{}_3$

$$\hat{x}^{MF}_{t|t} = E[x_t | y^i_{1:t}\, \forall i] = \hat{x}^{IBF}_{t|t} = \hat{x}^{cent}_{t|t}$$

$\downarrow t = 3$

$\hat{x}^{1,cent}_{3\|3}$
$\hat{x}^{2,cent}_{3\|3}$
$\hat{x}^{3,cent}_{3\|3}$

$$\hat{x}^{OKFF}_{t|t} = E[x_t | \hat{x}^{i,loc}_{t|t}\, \forall i] \neq \hat{x}^{IBF}_{t|t} = \hat{x}^{cent}_{t|t}$$

$$\hat{x}^{i,loc}_t = (A - K^{i,loc}C^i)\hat{x}^{i,loc}_{t-1} + K^{i,loc}y^i_t$$

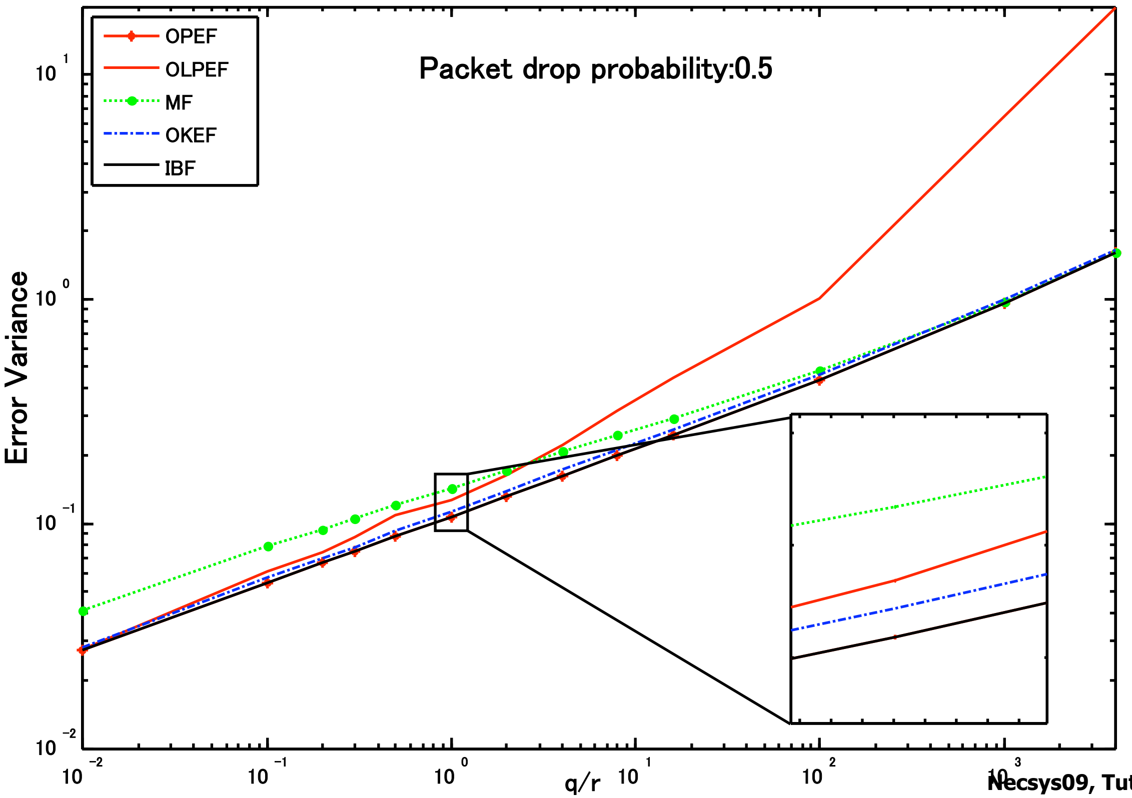
$$\hat{x}^{i,cent}_t = (A - \sum_i C^i K^{i,cent})\hat{x}^{i,cent}_{t-1} + K^{i,cent}y^i_t$$

$$P^{IBF}_{t|t} = P^{MF}_{t|t} = P^{OLPEF}_{t|t} = P^{OPEF}_{t|t} < P^{OKFF}_{t|t}$$

Multi sensor & packet loss

$$Q = 0 \implies P_{t|t}^{IBF} = P_{t|t}^{OLPEF} = P_{t|t}^{OPEF} < P_{t|t}^{OKFF}, P_{t|t}^{MF}$$

6 sensors, double integrator dynamics, uncorrelated noise



Strategy summary

	Estimation error	**Sensor complex.**	**Base station complex**
Measurement fusion	Almost optimal for R/Q small, Acceptable for R/Q large	none	Medium (inversion of n-dimensional matrix)
Optimal Kalman filter Fusion	Almost optimal always	Medium (local Kalman filter)	High (inversion of many matrices)
Optimal Partial Estimate Fusion	Optimal for Q/R small, almost optimal elsewhere	Medium (local Kalman-like filter)	High (inversion of many matrices)
Open loop partial estimate fusion	Optimal for Q/R small, very poor for R/Q small	Medium (local Kalman-like filter)	None

Strategy summary (con'd)

- Distributed estimation is old problem (Willsky, Bar-Shalom)
- Packet loss makes distributed estimation hard: optimal sensor preprocessing depends on future loss sequence
- No optimal strategy for all scenarios
- Some results based on simulations only: no theoretical proofs

- A.S. Willsky, D. Castanon, B. Levy, and G. Verghese," Combining and updating of local estimates and regional maps along sets of one-dimensional tracks," IEEE Trans. on Aut. Cont.,1982
- J. Wolfe and J. Speyers,"A low-power filtering scheme for distributed sensor networks," CDC'03
- Alessandro Agnoli, Alessandro Chiuso, Pierdomenico D'Errico, Andrea Pegoraro,L. Schenato "Sensor fusion and estimation strategies for data traffic reduction in rooted wireless sensor networks", ISCCSP08,
-A. Chiuso, L. Schenato, "Information fusion strategies from distributed filters in packet-drop networks," CDC'08
-A. Chiuso, L. Schenato, "Performance bounds for information fusion strategies in packet-drop networks," to appear in ECC'09

Takeaway points

- Input packet loss more dangerous than measurement packet loss
- TCP-like protocols help controller design as compared to UDP-like (but harder for communication designer)
- If you can, place controller near actuator
- If you can, send estimate rather than raw measurement
- Zero-input control seems to give smaller closed loop state error ($||x_t||$) than hold-input (but higher input)
- Trade-off in terms of performance, buffer length, computational resources (matrix inversion) when random delay
- Can help comparing different communication protocols from a real-time application performance
- Packet loss makes problem extremely hard
- No good-for-all-scenarios strategy when packet loss

References

- General and survey papers on NCS

  - M.S. Branicky W. Zhang and S.M. Phillips. Stability of networked control systems. IEEE Control Systems Magazine, 21(1):84–99, February 2001
  - R. Murray, K.J. Astrom, S.P. Boyd, R.W. Brockett, and G. Stein. Control in an information rich world. IEEE Control Systems Magazine, 23(2):20–33, April 2003.
  - J.P. Hespanha, P. Naghshtabrizi, and Y. Xu. A survey of recent results in networked control systems. Proceedings of the IEEE, 95(1):138–162, January 2007
  - ----, Technology of networked control systems. Proceedings of the IEEE, Special issue, 95(1):5–312, January 2007
  - James R. Moyne, Dawn M. Tilbury, "The Emergence of Industrial Control Networks for Manufacturing Control, Diagnostics, and Safety Data," Proceedings of IEEE, January 2007, 95(1), pp. 29-47

- Related workshops and slides
  - WIDE'09 Ph.D. School: http://ist-wide.dii.unisi.it/school09/school_program.htm
  - Frontiers in Distributed Communication, Sensing and Control in http://www.eng.yale.edu/dcsc/schedule.html

References

- **Rate Limited Control**

  - Ishii et al. (2008), , Fu & Xie (2005), Ishii & Francis (2002),
  - N. Elia. Remote stabilization over fading channels. *Systems & Control Letters*, 54:238–249, 2005.
  - N. Elia and S. K. Mitter. Stabilization of linear systems with limited information, *IEEE Trans. Autom. Control*, 46:1384–1400, 2001.
  - A. S. Matveev and A. V. Savkin. The problem of LQG optimal control via a limited capacity communicationchannel. *Systems & Control Letters*, 53:51–64, 2004
  - G. N. Nair and R. J. Evans. Stabilizability of stochastic linear systems with finite feedback date rates. *SIAM J. Contr. Optim.*, 43:413–436, 2004.
  - S. Tatikonda and S. K. Mitter. Control under communication constraints. *IEEE Trans. Autom. Control*, 49:1056–1068, 2004.
  - S. Tatikonda, A. Sahai, and S. K. Mitter. Stochastic linear control over a communication channel. *IEEE Trans. Autom. Control*, 49:1549–1561, 2004.
  - W. S. Wong and R.W. Brockett. Systems with finite communication bandwidth constraints II: Stabilization with limited information feedback. *IEEE Trans. Autom. Control*, 44:1049–1053, 1999.
  - S. Yuksel and T. Basar. Minimum rate coding for LTI systems over noiseless channels. *IEEE Trans. Autom. Control*, 51:1878–1887, 2006
  - H. Ishii and B. A. Francis. *Limited Data Rate in Control Systems with Networks*, volume 275 of *Lect. Notes Contr. Info. Sci.* Springer, Berlin, 2002.
  - H. Ishii, C. Ohyama, and K. Tsumura. Performance analysis of control systems under limited data rates. *Trans. SICE*, 44:396–404, 2008.
  - M. Fu and L. Xie. The sector bound approach to quantized feedback control. *IEEE Trans. Autom. Control*, 50:1698–1711, 2005.
  - G. Nair, F. Fagnani, S. Zampieri, J.R. Evans, Feedback control under data constraints: an overview, Proceedings of the IEEE, pp.108-137, 2007.

References

Signal-to-noise Limited Control

- Elia (2004), Martins & Dahleh (2008), Braslavsky at al 2006), Okano et al. (2009)
- N. C. Martins and M. A. Dahleh. “Feedback control in the presence of noisy channels: “Bode-like” fundamental limitations of performance”. *IEEE Trans. Autom. Control*, 52:1604–1615, 2008.
- E.I. Silva. “A Unified Framework for the Analysis and Design of Networked Control Systems”. PhD Thesis, Department of Electrical Engineering and Computer Science, The University of Newcastle, Australia, February 2009
- J. H. Braslavsky, R. H. Middleton and J. S. Freudenberg, “Feedback Stabilization over Signal-to-Noise Ratio Constrained Channels”, to appear, IEEE Transactions on Automatic Control, October, 2007
- N. Elia, "When bode meets shannon: control-oriented feedback communication schemes", IEEE Trans. Automatic Control, 49(9), pages 1477-1488, 2004
- K. Okano, S. Hara, and H. Ishii. “Characterization of a complementary sensitivity property in feedback control: An information theoretic approach” *Automatica*, 45:504–509, 2009.

References

- **Estimation and control: unknown delay & packet loss**

  - D. Nešic and A.R. Teel. "Input-output stability properties of networked control systems". IEEE Trans. Autom. Control, 49(10):1650–1667, 2004.
  - G.C. Walsh, O. Belidman, and L.G. Bushnell. "Stability analysis of networked control systems". IEEE Trans. Control Syst. Techn., 10:438–446, 2002
  - P. Naghshtabrizi and J.P. Hespanha. "Stability of network control systems with variable sampling and delays". In Proc. Annual Allerton Conf. Communication, Control, and Computing, 2006.
  - Heemels, A.R. Teel, N. van de Wouw and D. Nešic, "Networked control systems with communication constraints: Tradeoffs between transmission intervals and delays", European Control Conference 2009.
  - M. Cloosterman, N. van de Wouw, W.P.M.H. Heemels, and H. Nijmeijer. "Robust stability of networked control systems with time-varying network-induced delays". In Proc. Conf. on Decision and Control, pages 4980–4985, San Diego, USA, 2006.
  - L. Hetel, J. Daafouz, and C. Iung. "Stabilization of arbitrary switched linear systems with unknown time-varying delays". IEEE Trans. Autom. Control, 51(10):1668–1674, 2006.
  - J. Skaf and S. Boyd, "Analysis and Synthesis of State-Feedback Controllers With Timing Jitter", IEEE Transactions on Automatic Control, Vol. 54, No. 3, p. 652-657, 2009
  - M. Garcia-Rivera, A. Barreiro, "Analysis of networked control systems with drops and variable delays", Automatica 43 (2007) 2054–2059.

References

- …… (cont'd)

  - Johan Nilsson. Real-Time Control Systems with Delays. PhD thesis, Department of Automatic Control, Lund Institute of Technology, 1998.
  - L.A. Montestruque and P.J. Antsaklis. Stability of model-based networked control systems with time-varying transmission times. IEEE Transactions on Automatic Control, 49(9):1562–1572, 2004
  - Anton Cervin, Dan Henriksson, Bo Lincoln, Johan Eker, and Karl-Erik Arzen. How does control timing affect performance? IEEE Control Systems Magazine, 23(3):16–30, June 2003.
  - D. Giorgiev and D.M. Tilbury. Packet-based control. In Proc. of the 2004 Amer. Contr. Conf., pages 329–336, June 2004
  - A. V. Savkin and I. R. Petersen, "Robust filtering with missing data and a deterministic description of noise and uncertainty," *Int. J. Syst. Sci.,* vol. 28, no. 4, pp. 373–390, 1997.

Necsys 2009

References

- Estimation and control with measurable random delay and packet loss

  - Johan Nilsson. “Real-Time Control Systems with Delays”. PhD thesis, Department of Automatic Control, Lund Institute of Technology, 1998.
  - V. Gupta, D. Spanos, B. Hassibi, and R. M. Murray, “Optimal LQG control across a packet-dropping link,” *Systems and Control Letters, 2005.*
  - S. Smith and P. Seiler, “Estimation with lossy measuraments: jump estimators for jump systems,” *IEEE Transaction on Automatic Control,* vol. 48, no. 12, pp. 1453–1464, 2003.
  - N. Elia and J. Eisembeis, “Limitation of linear control over packet drop networks,” in *Proceedings of IEEE Conference on Decision and Control,* vol. 5, Bahamas, December 2004, pp. 5152–5157.
  - O. C. Imer, S. Yuksel, and T. Basar, “Optimal control of dynamical systems over unreliable communication links,” *To appear in Automatica,* July 2006.
  - B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. Jordan, S. Sastry, “Kalman filtering with intermittent observations,” *IEEE Transactions on Automatic Control, vol. 49, no. 9, pp. 1453–1464,* September 2004.
  - T. Katayama, “On the matrix Riccati equation for linear systems with a random gain,” *IEEE Transactions on Automatic Control, vol. 21, no. 2,* pp. 770–771, October 1976.

References

- ... cont'd

  - W. Koning, "Infinite horizon optimal control for linear discrete time systems with stochastic parameters," *Automatica, vol. 18, no. 4, pp.* 443–453, 1982.
  - B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, and S. Sastry "*Optimal Linear LQG Control Over Lossy Networks Without Packet Acknowledgment*", Asian Journal of Control 2009
  - Luca Schenato *"To zero or to hold control inputs with lossy links?"* IEEE Transactions on Automatic Control, 2009
  - Luca Schenato "*Optimal estimation in networked control systems subject to random delay and packet drop*" , IEEE Transactions on Automatic Control, 2009
  - V. Gupta, S. Adlakha, B. Sinopoli, and A. Goldsmith, "Towards receding horizon networked control," *The Forty-Fourth Annual Allerton Conference on Communication, Control and Computing.* Monticello, IL, September 2006.
  - M. Epstein, L. Shi, S. D. Cairano, and R. M. Murray, "Control over a network: Using actuation buffers and reducing transmission frequency," in *2007 European Control Conference (ECC'07), Kos, Greece, July 2007.*
  - N. Nahi, "Optimal recursive estimation with uncertain observation," *IEEE Transaction on Information Theory, vol. 15, no. 4, pp.* 457–462, 1969.
  - Q. Ling and M. Lemmon, "Optimal dropout compensation in networked control systems," in *IEEE conference on decision and control,* Maui, HI, December 2003.

References

- … cont'd

  - S. Dey, A.S. Leong and J.S. Evans, ``Kalman Filtering with Faded Measurements,'' *Automatica*, vol. 45, no. 10, pp. 2223-2233, Oct. 2009
  - M. Huang and S. Dey, `` Stability of Kalman Filtering with Markovian Packet Losses '', *Automatica,* vol. 43, no. 4, pp. 598-607, March 2007
  - Michael Epstein, Ling Shi, Abhishek Tiwari and Richard M. Murray "Probabilistic Performance of State Estimation Across a Lossy Network", Automatica, 2008

References

- Distributed Estimation with packet loss

  - A.S. Willsky, D. Castanon, B. Levy, and G. Verghese,“ Combining and updating of local estimates and regional maps along sets of one-dimensional tracks,” IEEE Trans. on Aut. Cont.,1982
  - J. Wolfe and J. Speyers,“A low-power filtering scheme for distributed sensor networks,” CDC’03
  - S. Yuksel and T. Basar. Optimal signaling policies for decentralized multicontroller stabilizability over communication channels. *IEEE Trans. Autom. Control*, 52:1969–1974, 2007.
  - Alessandro Agnoli, Alessandro Chiuso, Pierdomenico D’Errico, Andrea Pegoraro,L. Schenato “Sensor fusion and estimation strategies for data traffic reduction in rooted wireless sensor networks”, ISCCSP08,
  - Chiuso, L. Schenato, “Information fusion strategies from distributed filters in packet-drop networks,” CDC’08
  - Chiuso, L. Schenato, “Performance bounds for information fusion strategies in packet-drop networks,” ECC’09
  - B.C. Levy, D.A. Castanon, G.C. Verghese, and A.S. Willsky. “A scattering framework for decentralized estimation problems” Automatica, 19(4):373–384, 1983
  - L. Shi, K. H. Johansson, and R.M. Murray. “Estimation over wireless sensor networks: Tradeoff between communication, computation and estimation qualities”. In Proc. of IFAC World Congress, volume 17, Seoul, Korea, 2008
  - Y. Bar-Shalom, X.R. Li, and T. Kirubarajan. “Estimation with Applications to Tracking and Navigation”. John Wiley & Sons, Inc., 2001.
  - Gupta, N. C. Martins, and J. S. Baras. “Stabilization over erasure channels using multiple sensors”. IEEE Transactions on Automatic Control, 57(7):1463–1476, July 2009