To zero or to hold control inputs in lossy networked control systems?

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***Abstract*— This paper studies the LQ-like performance of networked control systems where control packets are subject to losses. In particular we explored the two simplest compensation strategies commonly found in the literature: the zero-input strategy where the input to the plant is set to zero if a packet is dropped, and the hold-input strategies where the previous control input is used if packet is lost. We derived numerical solutions for computing the optimal static gain for both strategies and we compared their performance on some numerical examples. Interestingly, none of the two can be claimed superior to the other, even for simple scalar systems, as there are scenarios where one performs better then the other and vice-versa.**

I. Introduction

Today's technological advances in wireless communications and in the fabrication of inexpensive embedded electronic devices, are creating a new paradigm where a large number of systems are interconnected, thus providing an unprecedented opportunity for totally new applications. This is particularly true for real-time control systems where access to information from many sensors and distributed actuators can potentially lead to better performances. These systems are commonly referred as networked control systems. However, these advantages come at the price of unreliable or at least not-ideal communication links which lead to packet drops, random delay, quantization errors, thus leading to degradation from the ideal performance. Recently, a great effort has been given to understand and analyze these systems with respect to the interaction of communications and control, which has been recently surveyed in the nice paper [1].

In particular, one of the most common problem in networked control systems, especially in wireless sensor networks, is packet loss, i.e. packets can be lost due to communication noise, interference, or congestion. If the controller is not co-located with the sensor and the actuator and it is placed in a remote location, then both sensor measurement packets and control packets can be lost. This would be the case, for example, in a pursuit-evasion-game scenario where locations of evaders are obtained using a wireless sensor networks, then processed in some centralized controller, and then optimal control inputs are dispatched to the mobile pursuers [2]. A large number of works in the literature have analyzed estimation performance in lossy systems [3][4][5][6][7] [8] [9], where the performance of the closed loop system is not considered. However, there are also several papers which considered the close loop performance where control packet can be dropped [10][11][12][13][14]. In general, two different strategies are considered. In the first one, that we refer as zero-input, the control input to the plant is sent to zero when the control packet from the controller to the actuator is lost [12][13][14], while in the second, that we refer as hold-input, the pervious control input is used when a packet is lost [10][11]. If smart actuators are available, i.e. if actuators are provided with computational resources, the controller or some effective compensation strategy can be placed on the actuator as suggested in [15] and [14], however this is not always possible and therefore only very simple strategies as the two mentioned above can be adopted.

To the author's knowledge there is no studies available in the literature which directly compare these two strategies, except for a simple empirical example in [14]. In particular, it seems that the zero-input strategy is mainly used for mathematical convenience as it gives simpler equations than the hold-input strategy, rather than based on performance considerations. Indeed, intuitively one is lead to consider more effective the use the previous control input rather than zero using continuity arguments, at least during the transient. The zero-input strategy, however, it is not so awkward, as the optimal control input at steady state is zero for a stable closed loop system. Motivated by these observations, the goal of this paper is to qualitatively quantify the performance of these two strategies by adopting an LQ-like approach on discrete time linear system where the control input packet is dropped according to a Bernoulli stochastic process as described in details the Section II. In particular, we derive equations to compute the optimal static control gains for both strategies. While the equations for optimal control under the zero-input strategy in Section III have been previously derived [14], the equations for the hold-input strategy presented in Section IV are novel, to the author's knowledge. The equations are then used to compared the performance of the two strategies for an unstable scalar system in Section V. Interestingly, even in this simple scenario, none of the two strategies is always superior to the other, but the performance depends on the packet loss probability and the systems parameters.

II. Problem formulation

Consider the following linear stochastic system:

$$x_{k+1} = Ax_k + Bu_k^a \tag{1}$$

where u_k^a is the control input to the actuator. We assume that the full state x_k is available to a remote controller which adopt a simple linear feedback:

$$u_k^c = Lx_k$$

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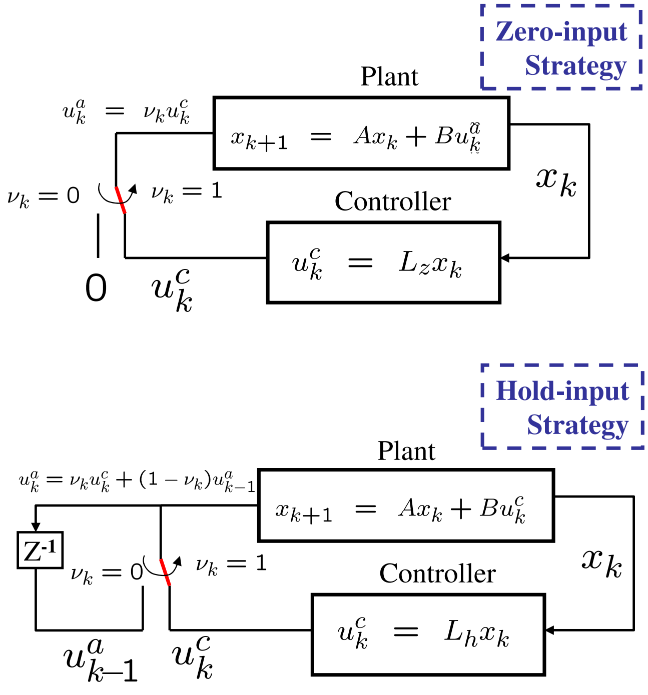

Fig. 1. Compensation approaches for actuators with no computational resources when a control packet is lost: zero-input approach $u_k^a = 0$ (*top*) and hold-input approach $u_k^a = u_{k-1}^a$ (*bottom*).

The link between the controller and the actuator is lossy, and stochastic variable ν_k models the packet loss between the controller and the actuator. We consider two control strategies. In the zero-input strategy, if the packet is correctly delivered then $u_k^a = u_k^c$, otherwise if it is lost then the actuator does nothing, i.e. $u_k^a = 0$, which gives the following closed loop system:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k^a \\ u_k^a &= \nu_k u_k^c \\ u_k^c &= L_z x_k \end{aligned} \tag{2}$$

In the hold-input strategy instead, when the packet is lost we use the previous control value stored in actuator, i.e. $u_k^a = u_{k-1}^a$, which leads to the following closed loop dynamics:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k^a \\ u_k^a &= \nu_k u_k^c + (1-\nu_k)u_{k-1}^a \\ u_k^c &= L_h x_k \end{aligned} \tag{3}$$

These two control packet loss compensation strategies are graphically illustrated in Figure 1.

We compare the performance in terms of the infinite horizon expected total cost

$$J_\infty = \mathbb{E}[\sum_{k=0}^{\infty} x_k^T W x_k + u_k^{aT} U u_k^a] \tag{4}$$

III. LQ OPTIMAL CONTROL: ZERO-INPUT STRATEGY

The optimal control equations are obtained using the standard dynamic programming approach, i.e. we compute the cost-to-go function iteratively. First note that Equations (2) can be written as

$$\begin{aligned} x_{k+1} &= (A + \nu_k BL)x_k & (5)\\ u_k^a &= \nu_k L x_k & (6) \end{aligned}$$

Let us define the cost-to-go function C_k as follows

$$C_k^N(x_k) = \mathbb{E}[\sum_{h=k}^{N} x_k^T W_k x_k + u_k^{aT} U_k u_k^a | x_k] \tag{7}$$

where $W_k = W$ and $U_k = U$ except for the terminal cost $U_N = 0$. We claim that the cost-to-go function can be written as

$$C_k^N(x_k) = \mathbb{E}[x_k^T S_k x_k | x_k] \tag{8}$$

This is clearly true for $k = N$ with $S_N = W$. Then by induction, we show this is true for all k. Suppose it is true for $k+1$, then we have:

$$\begin{aligned} C_k^N(x_k) &= \mathbb{E}[\textstyle\sum_{h=k}^{N} x_k^T W x_k + u_k^{aT} U u_k^a | x_k] \\ &= \mathbb{E}[x_k^T W x_k + u_k^{aT} U u_k^a + C_{k+1}^N | x_k] \\ &= \mathbb{E}[x_k^T W x_k + \nu_k x_k^T L^T U L x_k + \\ &\quad + x_k^T (A+\nu_k BL)^T S_{k+1} (A + \nu_k BL) x_k | x_k] \\ &= \mathbb{E}\big[x_k^T \big(W + (1-\bar\nu) L^T U L + \bar\nu A^T S_{k+1} A + \\ &\quad + (1-\bar\nu)(A+BL)^T S_{k+1}(A+BL)\big) x_k | x_k\big] \end{aligned} \tag{9}$$

where we used the fact that ν_k is independent of x_k. Therefore the claim above is true and the matrix S_k is given by:

$$\begin{aligned} S_k &= W + \bar\nu A^T S_{k+1} A + (1-\bar\nu)\big(L^T U L + (A+BL)^T S_{k+1}(A+BL)\big) \\ &= \mathcal{F}(S_{k+1}, L) \end{aligned} \tag{10}$$

where the operator $\mathcal{F}(S, L)$ is affine in S for fixed L, and quadratic in L for fixed S. The infinite horizon cost can be obtained from the cost-to-go function as follows:

$$J_\infty(L) = \lim_{N\to\infty} C_0^N(x_0) = x_0^T S_\infty x_0$$

where S_∞ is the solution of the Lyapunov-like equation $S_\infty = \mathcal{F}(S_\infty, L)$, if such solution exists. The optimal gain L^* is defined as the minimizer of the infinite horizon cost $L^* = \mathrm{argmin}_L x_0^T S_\infty x_0$. It was shown in [14] that the optimal gain is independent of the initial condition x_0 and can be obtained by solving a Riccati-like equation. We summarize those results in the following theorem:

Theorem 3.1 ([14]): Consider the system defined by Equations (2) and the infinite horizon cost defined in Equation (4). Assume that the pairs (A, B) and $(A^T, W^{\frac{1}{2}})$ are stabilizable. Then the optimal infinite horizon cost $J_\infty^* = \min_L J_\infty(L)$ is given by $J_\infty^* = x_0 S_\infty^* x_0$ where S_∞^* is the unique strictly positive solution of the Riccati-like equation:

$$\begin{aligned} S_\infty^* &= A^T S_\infty^* A + W - (1-\bar\nu) A^T S_\infty^* B (B^T S_\infty^* B + U)^{-1} B^T S_\infty^* A \\ &= \Phi(S_\infty^*) \end{aligned} \tag{11}$$

and the optimal gain is given by

$$L^* = -(B^T S_\infty^* B + U)^{-1} B^T S_\infty^* \tag{12}$$

The Riccati-like equation $S_\infty^* = \Phi(S_\infty^*)$ has a positive definite solution if and only if $\bar\nu < \nu_c$, where ν_c is a critical packet loss probability, which depends on the pair

(A,B). The critical loss probability ν_c satisfies the following bounds:

$$\begin{array}{ll} \nu_m & \leq \nu_c \leq \nu_M \\ \nu_m & = \frac{1}{\max|\lambda_i^u|^2}, \qquad \nu_M = \frac{1}{\prod_i |\lambda_i^u|^2} \end{array} \tag{13}$$

where λ_i^u are the unstable eigenvalues of the matrix A.

IV. LQ OPTIMAL CONTROL: HOLD-INPUT STRATEGY

We now derive the equations to compute the infinite horizon cost for the hold-input strategy. We proceed similarly to the previous section by computing the cost-to-go function. We fist defined the augmented state $z_k = [x_k \ u_{k-1}^a]^T$. Then the system defined by Equations (3) can be written as:

$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ u_k^a \end{bmatrix} &= \begin{bmatrix} A+\nu_k BL & (1-\nu_k)B \\ \nu_k L & (1-\nu_k)I \end{bmatrix} \begin{bmatrix} x_k \\ u_{k-1}^a \end{bmatrix} && (14) \\ &= F(\nu_k) z_k && (15) \end{aligned}$$

where I is the identity matrix. We define the cost-to-go function as in the previous section:

$$C_k^N(z_k) = \mathbb{E}[\sum_{h=k}^{N} x_k^T W_k x_k + u_k^{aT} U_k u_k^a | z_k] \tag{16}$$

where $W_k = W$ and $U_k = U$ except for the terminal cost $U_N = 0$. We claim that the cost-to-go function can be written as

$$C_k^N(z_k) = \mathbb{E}[z_k^T V_k z_k | z_k] \tag{17}$$

This is clearly true for $k = N$ with $V_N = \begin{bmatrix} W & 0 \\ 0 & 0 \end{bmatrix}$. Then by induction, we show this is true for all k. Suppose it is true for $k+1$, then we have:

$$\begin{aligned} C_k^N(z_k) &= \mathbb{E}[\textstyle\sum_{h=k}^N x_k^T W x_k + u_k^{aT} U u_k^a | z_k] \\ &= \mathbb{E}[x_k^T W x_k + u_k^{aT} U u_k^a + C_{k+1}^N | x_k] \\ &= \mathbb{E}[z_k^T \begin{bmatrix} W + \nu_k L^T U L & \nu_k(1-\nu_k) L^T U \\ \nu_k(1-\nu_k) U L & (1-\nu_k)^2 U \end{bmatrix} z_k + \\ &\quad + z_k^T F^T(\nu_k) V_{k+1} F(\nu_k) z_k | z_k] \\ &= \mathbb{E}[z_k^T \begin{bmatrix} W + (1-\bar{\nu}) L^T U L & 0 \\ 0 & \bar{\nu} U \end{bmatrix} z_k + \\ &\quad + \bar{\nu} z_k^T F^T(0) V_{k+1} F(0) z_k + \\ &\quad + (1-\bar{\nu}) z_k^T F^T(1) V_{k+1} F(1) z_k | z_k] \end{aligned} \tag{18}$$

where we used the fact that ν_k is independent of x_k. Therefore the claim above is true and the matrix V_k is given by:

$$\begin{aligned} V_k &= \begin{bmatrix} W + (1-\bar{\nu}) L^T U L & 0 \\ 0 & \bar{\nu} U \end{bmatrix} + \\ &+ \bar{\nu} \begin{bmatrix} A^T & 0 \\ B^T & I \end{bmatrix} V_{k+1} \begin{bmatrix} A & B \\ 0 & I \end{bmatrix} + \\ &+ (1-\bar{\nu}) \begin{bmatrix} (A+BL)^T & L^T \\ 0 & 0 \end{bmatrix} V_{k+1} \begin{bmatrix} A+BL & 0 \\ L & 0 \end{bmatrix} \\ &= \mathcal{G}(V_{k+1}, L) \end{aligned} \tag{19}$$

where the operator $\mathcal{G}(V,L)$ is affine in V for fixed L, and quadratic in L for fixed V. The infinite horizon cost can be obtained from the cost-to-go function as follows:

$$J_\infty(L) = \lim_{N\to\infty} C_0^N(x_0) = z_0^T V_\infty z_0$$

where V_∞ is the solution of the Lyapunov-like equation $V_\infty = \mathcal{G}(V_\infty, L)$, if such solution exists.

Let us partition the matrix V_∞ as follows

$$V_\infty = \begin{bmatrix} V_1 & V_{12} \\ V_{12}^T & V_2 \end{bmatrix}$$

then the Lyapunov-like equation $V_\infty = \mathcal{F}(V_\infty, L)$ can be expanded as:

$$\begin{aligned} V_{12} &= \nu(A^T V_1 B + A^T V_{12}) && (20) \\ V_2 &= \nu(U + B^T V_1 B + V_{12}^T B + B V_{12} + V_2) && (21) \\ V_1 &= W + \nu A^T V_1 A + (1-\nu)\Big(L^T U L + (A+BL)^T V_1 (A+BL) + \\ &\quad + L^T V_{12}^T (A+BL) + (A+BL)^T V_{12} L + L^T V_2 L\Big) && (22) \end{aligned}$$

After some simple algebraic manipulations, the previous equations can be rewritten in terms of V_1 as follows:

$$\begin{aligned} V_{12} &= \nu(I-\nu A)^{-T} A^T V_1 B && (23) \\ V_2 &= \frac{\nu}{1-\nu}\Big(U + B^T V_1 B + \nu B^T\big(V_1 A (I-\nu A)^{-1} + (I-\nu A)^{-T} A^T V_1\big) B\Big) && (24) \\ V_1 &= W + A^T V_1 A + L^T U L + L^T B^T V_1 B L + (1-\nu) A^T V_1 B L + \\ &\quad + (1-\nu) L^T B^T V_1 A + \nu(1-\nu) L^T B^T V_1 A (I-\nu A)^{-1} A + \\ &\quad + \nu(1-\nu) A' (I-\nu A)^{-T} A' V_1 B L + \\ &\quad + \nu L^T B^T V_1 A (I-\nu A)^{-1} B L + \nu L^T B^T (I-\nu A)^{-T} A^T V_1 B L && (25) \end{aligned}$$

Note that only the last equation depends on the control gain L. In particular it is quadratic in L, in fact it can be written as follows:

$$\begin{aligned} V_1 &= P_1 + P_{12}^T L + L^T P_{12} + L^T P_2 L && (26) \\ &= \mathcal{L}(L, V_1) \end{aligned}$$

where:

$$\begin{aligned} P_1(V_1) &= W + A^T V_1 A \\ P_{12}(V_1) &= (1-\nu) B^T V_1 \Big(I + \nu A (I-\nu A)^{-1}\Big) A \\ P_2(V_1) &= U + B^T\big(V_1 + \nu (I-\nu A)^{-T} A^T V_1 + \nu V_1 A (I-\nu A)^{-1}\big) B \end{aligned}$$

Note that the matrices P_1, P_{12}, P_2 are linear function of the matrix V_1, and that the operator $\mathcal{L}(L, V_1)$ is linear in V_1 for fixed L. Moreover, this operator and can be written as follows:

$$\begin{aligned} \mathcal{L}(L,V_1) &= P_1 - {P_{12}}^T P_2^{-1} P_{12} + (L + P_2^{-1} P_{12})^T P_2 (L + P_2^{-1} P_{12}) \\ &= \Psi(V_1) + (L - L_V)^T P_2 (L - L_V) && (27) \\ \Psi(V_1) &= P_1 - {P_{12}}^T P_2^{-1} P_{12} && (28) \\ L_V &= -P_2^{-1} P_{12} && (29) \end{aligned}$$

If $P_2 > 0$, then we have:

$$\Psi(V_1) \leq \mathcal{L}(L, V_1), \quad \forall L$$

where the operator $\Psi_\nu(V_1)$ is nonlinear in V_1. The condition $P_2 > 0$ is a necessary condition for stability, otherwise we could choose a gain L that would give a nonpositive definite V_1 which is clearly unfeasible. The previous inequality can be used to find the optimal gain L that minimizes the matrix V_1. In fact it is possible to show that if $S = \mathcal{L}(L, S)$ and $T = \Psi(T)$, then $S \geq T$.

Theorem 4.1: Consider the system defined by Equations (3) and the infinite horizon cost defined in Equation (4). Assume that the pairs (A,B) and $(A^T, W^{\frac{1}{2}})$ are stabilizable. Also assume that $u_{-1}^a = 0$. Then the optimal infinite horizon cost $J_\infty^* = \min_L J_\infty(L)$ is given by $J_\infty^* = x_0 T_\infty^* x_0$ where

T_∞^* is the unique strictly positive solution of the Riccati-like equation:

$$T_\infty^* \; = \Psi(T_\infty^*) \tag{30}$$

where $\Psi(T)$ is defined in Equation (28) and the optimal gain is given by

$$L^* = L_{T_\infty^*} \tag{31}$$

where L_V is defined in Equation (29). The Riccati-like equation $T_\infty^* = \Psi(T_\infty^*)$ has a positive definite solution if and only if $\bar{\nu} < \nu_c$, where ν_c is a critical packet loss probability, which depends on the pair (A, B). The T_∞^* can be obtained as the limit of the sequence $T_{k+1} = \Psi(T_k)$, for $T_0 \geq 0$, i.e. $\lim_{k\to\infty} T_k = T_\infty^*$.

Note that the hypothesis $u_{-1}^a = 0$ is a natural choice which for a fair comparison between the zero-input strategy and the hold-input strategy. Note that is this case, the optimal choice for L is independent of the initial condition x_0. If we consider $u_{-1}^a \neq 0$, it is still possible to derive a similar optimization based on a different Riccati-like equation which might depend on the initial conditions.

So far we have shown how to compute the optimal gain for both the zero-input strategy and for hold-input strategy. However, we have not yet shown wether one strategy is better than the other. In the next section, we will compare the performance of the two strategies for scalar unstable systems.

V. Hold-input vs zero-input: the scalar case

Without loss of generality, we assume that $B = 1$, $A = a$, $W = w$, and $x_0 = 1$. Also we consider at first $U = 0$, which corresponds to a scenario where we look for the fastest converging controller. In fact, for $\nu = 0, U = 0$ we obtain the usual dead-beat controller. If we substitute these values into Equations (11) and (12) for the zero-input strategy we get:

$$\begin{aligned} s_\infty^* &= w + a^2 s_\infty^* - (1-\bar{\nu})a^2 = w + \bar{\nu}a^2 s_\infty^* \\ &= \frac{w}{1-\bar{\nu}a^2} && (32) \\ l_z^* &= -a && (33) \end{aligned}$$

Note that if the open loop system is unstable, i.e. $|a| > 1$, then the optimal solution exists, i.e. $s_\infty^* \geq 0$, if and only if $\bar{\nu}a^2 < 1$.

Similarly, if we substitute these values into Equations (30) and (31) for the hold-input strategy we get:

$$\begin{aligned} t_\infty^* &= w + a^2 t_\infty^* - (1-\bar{\nu})^2 \left(1+\frac{\bar{\nu}a}{1-\bar{\nu}a}\right)^2 \left(1+\frac{2\bar{\nu}a}{1-\bar{\nu}a}\right)^{-1} a^2 t_\infty^* \\ &= w + a^2 t_\infty^* - \frac{(1-\bar{\nu})^2}{1-\bar{\nu}^2a^2} a^2 t_\infty^* \\ &= \frac{w}{1-\left(1-\frac{(1-\bar{\nu})^2}{1-\bar{\nu}^2a^2}\right)a^2} && (34) \\ l_h^* &= -\frac{(1-\bar{\nu})a}{1+\bar{\nu}a} && (35) \end{aligned}$$

If the open loop system is unstable then the optimal solution exists if and only if the denominator $\frac{w}{1-\left(1-\frac{(1-\bar{\nu})^2}{1-\bar{\nu}^2a^2}\right)a^2} = \frac{w(1-\bar{\nu}^2a^2)}{(1-\bar{\nu}a^2)^2}$ is positive, which leads to the constraint $\bar{\nu}|a| < 1$. The constraint $\bar{\nu}|a| < 1$ is less restrictive that $\bar{\nu}a^2 < 1$, therefore it seems that the hold-input strategy can stabilize the system for larger packet loss probability than the zero-input strategy. However, we need not to forget that a necessary and sufficient stability condition for the hold-input strategy is that $V_\infty \geq 0$, which is equivalent to the conditions

$$V_1 \geq 0 \text{ and } V_1 - V_{12}V_2^{-1}V_{12}^T \geq 0, \quad V_1 = t_\infty^*$$

where V_1, V_{12}, V_2 are defined in Equations (23)-(25). The first inequality is obviously satisfied, while the second, after some simple algebraic manipulation is given by:

$$V_1 - V_{12}V_2^{-1}V_{12}^T = t_\infty^* \frac{1-\bar{\nu}a^2}{1-\bar{\nu}^2a^2}$$

which is positive if and only if $\bar{\nu}a^2 < 1$, thus recovering the same stability condition of the zero-input strategy.

We now show that the zero-input strategy gives a better performance than the hold-input strategy. This is equivalent to show that:

$$\begin{aligned} & 0 \leq s_\infty^* \leq t_\infty^* \iff \bar{\nu} \leq 1 - \tfrac{(1-\bar{\nu})^2}{1-\bar{\nu}^2a^2} \\ & \iff \tfrac{(1-\bar{\nu})(1-\bar{\nu}^2a^2)-(1-\bar{\nu})^2}{1-\bar{\nu}^2a^2} \geq 0 \\ & \iff \tfrac{\bar{\nu}(1-\bar{\nu})(1-\bar{\nu}a^2)}{1-\bar{\nu}^2a^2} \geq 0 \iff \bar{\nu} \leq \tfrac{1}{a^2} \end{aligned} \tag{36}$$

which is always true since the feasibility condition is $\bar{\nu}a^2 < 1$.

Figure 2 shows a graphical representation of Equations (32) and (34), where $A = 1.2$, $B = W = 1$, and $U = 0$. In Figure 3 it is shown a typical realization for an unstable system, $A = 1.2$, with packet loss probability $\bar{\nu} = 0.5$, using optimal gain $l_z^* = -a = -1.2$ for the zero-input strategy and $l_h^* = -(1-\bar{\nu})a/(1+\bar{\nu}a) = -0.375$ for the hold-input strategy. Note that the first control packet is lost and the state x starts to diverge, however as soon as a packet arrives the zero-hold strategy drives the system to zero, while the hold-input requires a longer time.

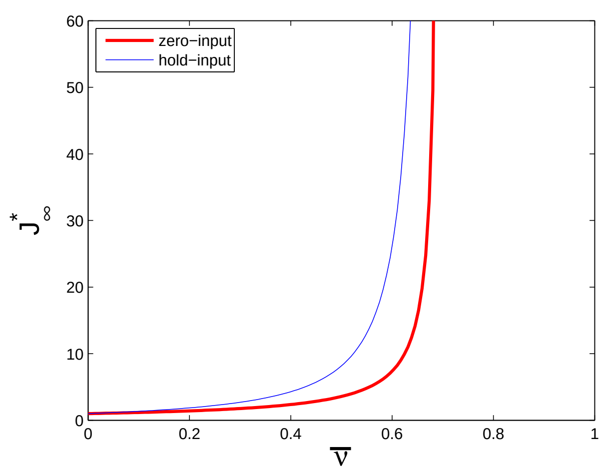

Fig. 2. Minimum cost J_∞ for $A = 1.2$, $B = W = x_0 = 1$, $U = 0$ under zero-input and hold-input control architectures. The critical loss probability for this systems is $\nu_c = 1/1.2^2 = 0.69$.

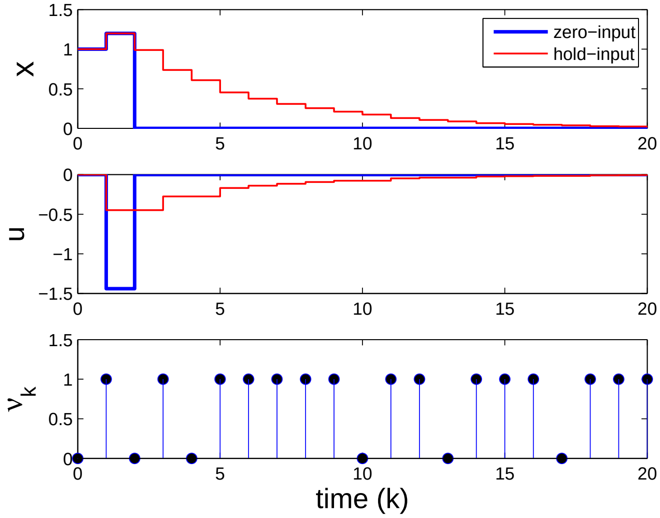

Fig. 3. A specific realization for $A = 1.2$, $x_0 = 1$, $\bar{\nu} = 0.5$ under under optimal zero-input control, $l_z^* = -a = -1.2$ and optimal hold-input control, $l_h^* = -(1-\bar{\nu})a/(1+\bar{\nu}a) = -0.375$.

To validate the analytical equations derived in this paper, we computed the empirical total cost J_∞^{emp} by averaging 10000 run starting with the initial condition $x_0 = 1$ and $u_{-1} = 0$, for $A = 1.2$, $B = W = x_0 = 1$, $U = 0$ and $\bar{\nu} = 0.5$ for different values of the feedback gain l. The analytical optimal gains l_z^* and l_h^*, and the corresponding minimum cost $J_{\infty,z}^* = s_\infty^*$ and $J_{\infty,h}^* = t_\infty^*$ given by Equations (32)-(35) are computed and shown in Figure 4, which appear to be consistent with the empirical values.

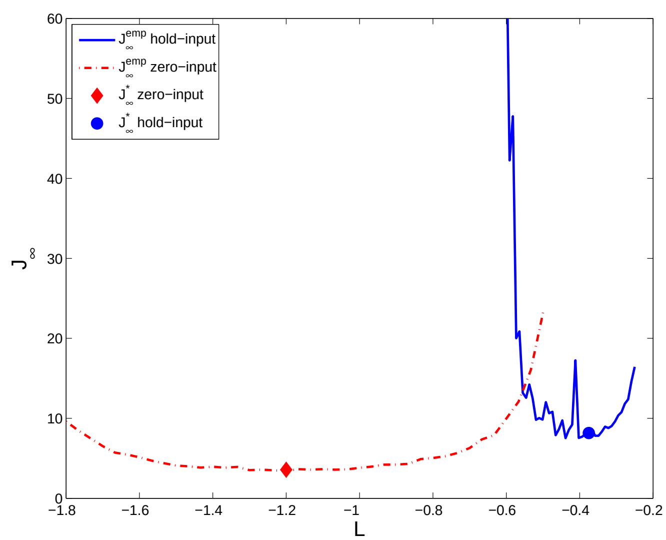

Fig. 4. Empirical total cost J_∞^{emp} for $A = 1.2$, $x_0 = 1$, $\bar{\nu} = 0.5$ and obtained by averaging 10000 Monte Carlo runs under zero-input and hold-input control architectures. The analytical optimal gains l^* and minimum total costs J_∞^* are also shown corresponding to the two strategies are shown.

So far we have considered the case $U = 0$, i.e. the case when the input it is not penalized. Figure 5 shows the minimum cost obtained for the system where $A = 1.2$, $B = W = 1$, and $U = 10$. Very interestingly, there is range of values of the packet loss probability $\bar{\nu}$ for which the hold-input strategy performs better than the zero-input strategy, while there is another range of values for which it is the opposite. This implies that in general it is not possible to state

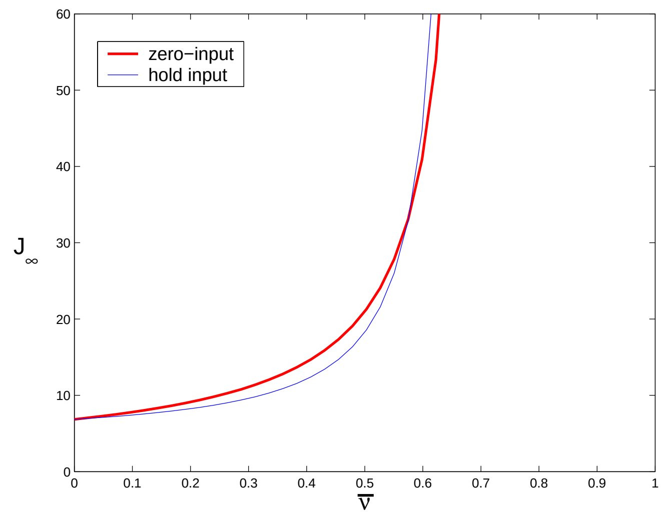

Fig. 5. Minimum cost J_∞ for $A = 1.2$, $B = W = x_0 = 1$, $U = 10$ under zero-input and hold-input control architectures.

wether the hold-input strategy is better than the hold-input strategy or viceversa, even for simple scalar linear systems.

VI. Conclusion

In this paper we studied LQ-like performance of the hold-input and zero-input strategy for control systems for which the control packets are subject to loss. These are the most commonly adopted strategies in the literature. We derive explicit expressions for computing the optimal static controller gain when control packets are lost according to a Bernoulli process. Interestingly, we showed that none of these two control schemes can be claimed to be superior to the other, even in simple scalar systems. However, the tools developed in this paper can be used to evaluate which architecture performs once the packet loss statistics are known.

We want to remark that although the zero-input strategy has been proposed in the literature, the hold-input strategy is much more popular because it is rather natural and easy to implement. The fact that in many situations the zero-hold strategy performs better than the hold-input strategy, encourage further investigation in experimental settings.

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