Impact of a realistic transmission channel on the performance of control systems

F. Parise, L. Dal Col, A. Chiuso, N. Laurenti, L. Schenato, A. Zanella

***Abstract*—In this paper we study the effect of communication nonidealities on the control of unstable stochastic linear systems. The communication protocol links the sensors to the actuators and should be studied by taking into account several limitations such as quantization errors, limited channel capacity, decoding delay and packet loss which, to the best of our knowledge, have not been considered jointly in the control theory literature. We restrict our study to a special class of linear plants and controllers and to a transmission with signal-to-quantization noise ratio limitations and packet loss, and derive their impact on the stability conditions and optimal design parameters for the controller. In particular, we characterize a tradeoff among quantization and packet loss depending on the state coefficient of the system to be controlled. Through this analysis we are also able to recover several results already available in the literature that have treated packet loss and quantization error separately.**

I. Introduction

Traditionally, control theory and communication theory have been developed independently and have reached considerable success in developing fundamental tools for designing information technology systems. The major objective of control theory was to develop tools to stabilize unstable plants and to optimize some performance metrics in closed loop under the assumption that the communication channel between sensors and controller and between the controller and the plant were ideal, i.e. without distortion, packet loss or delay. This assumption actually holds in many control applications where the non idealities of the communication channel have negligible impact compared to the effects of noise and uncertainty in the plants. With the advent of wireless communication, the Internet and the need for high performance control systems, however, the sharp separation between control and communication has been questioned and a growing body of literature has appeared from both the communication and the control communities trying to analyze the interaction between control and communication. This recent branch of research is known as *Networked Control System* (NCS) and considers control systems wherein the control loops are closed through a real-time network, and feedback signals are exchanged in the form of data packets.

Recent results in this area have revealed the existence of a strict connection between the performance of the controlled plant and the Shannon capacity of the feedback channel. However, this is not sufficient to completely characterize the communication channel from a control perspective [1], [2]. For instance, it has been proved that in order to stabilize an unstable plant through a control loop, the signal-to-noise ratio (SNR) of the feedback channel must be larger than some threshold depending on the unstable eigenvalues of the plant [3], [4], [5]. Another line of research has addressed the problem of stabilizing an unstable plant in the presence of a feedback channel that is prone to random packet losses [6], [7], [8], [9], or that is rate-limited [10], [11], [12]. A subsequent step has been made to include multiple channel limitations into the model, such as packet loss and quantization [13], [14], which however results in complex optimization problems.

In this work, we address the problem of performance optimization in a NCS with a realistic feedback channel. More specifically, we consider the Linear-Quadratic-Gaussian (LQG) control problem, which consists in finding the control signal of a linear time-invariant (LTI) plant that minimizes a quadratic cost function of the system state, when both the system state and the output signal are affected by Gaussian noise. While the optimal solution to the LQG problem in LTI systems with *ideal feedback channel* is known to be achieved by a controller formed by a Kalman filter and a linear-quadratic regulator, the solution to the problem in NCS systems with *realistic feedback channels* has only been investigated for specific feedback channel models, while the general solution still remains unknown.

Our feedback channel model takes into account packet loss, code rate limitations, signal quantization and delay, while still being mathematically amenable to analysis. By using this model, we find a stability condition that depends on both the packet loss probability and the signal to quantization noise ratio (SQNR). The LQG architecture proposed in this paper actually generalizes those considered in the previous literature; in fact we recapture several conditions available in the literature for more specific channel models as special cases of our model.

II. System model and problem formulation

In this section, we first cast the LQG problem into the NCS framework. Then, we introduce the model of the feedback transmission channel that completes the structure of the NCS considered in this work. Finally, we formally define the LQG problem in the considered NCS architecture.

A. LQG problem definition

We consider a plant, modeled as a discrete-time, scalar, LTI system, subject to additive white Gaussian measurement and

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All authors are with the Department of Information Engineering, University of Padova, Via Gradenigo 6/b, 35131 Padova, Italy *name.lastname*@dei.unipd.it

process noise. More specifically, the state of the system at step t, denoted as x_t, evolves according to the following linear model:

$$x_{t+1} = ax_t + bu_t + w_t \quad (1)$$
$$y_t = cx_t + v_t \quad (2)$$

where u_t and y_t represent the input and output signals of the plant, respectively, whereas w_t and v_t are two independent discrete-time Gaussian white noise processes with variance σ_w^2 and σ_v^2, respectively. Finally, a, b and c are the state, input and output coefficients, respectively.

We consider the *steady state* variance as performance index[1]

$$J = \limsup_{t\to+\infty} \mathbb{E}[y_t^2]\,. \quad (3)$$

The objective of the LQG problem is then to minimize J by means of a suitable control signal u_t, which only depends causally on the output signal $\{y_m,\ m \le t\}$, and possibly on its previous values $\{u_m,\ m \le t-1\}$.

B. Feedback channel modeling

In the NCS framework, the plant output y_t is not directly accessible to the controller, but must be delivered by means of a suitable transmission scheme. The feedback channel will thus comprise analog to digital conversion of y_t and source coding of the corresponding bitstream into packets, channel coding and transmission over the physical channel. At the receiver, after forward error correction, typically a further frame check is performed to drop packets that have not been successfully corrected (packet erasures). Accepted packets are then decoded to yield the correct digital values.

We model the feedback channel as represented in Fig. 1, where n_t represents the quantization noise. Assuming that a packet is sent at each $t = 0, 1, \ldots$, $\gamma_t \in \{0, 1\}$ is a Bernoulli process that models the erasure events ($\gamma_t = 0$), occurring with probability ϵ at each packet transmission, independently of previous events. Finally, the delay block $z^{-\tau}$ which accounts for the encoding/decoding delay.

The quantization noise n_t accounts for the distortion due to the quantization of the real-valued signal y_t before transmission. If quantization is fine enough, n_t can be effectively modeled as a zero-mean additive random process, with identically distributed uncorrelated samples of power $\sigma_n^2 = \mathbb{E}\big[n_t^2\big]$. The SQNR, $\Lambda = \mathbb{E}\big[y_t^2\big]/\sigma_n^2$, is related to the information rate R_{q} of the quantized signal, and increases with it. Since the maximum information rate R_{q} is upper limited by the channel code rate R_{c}, then the SQNR cannot be increased above a certain threshold Λ^*, which depends on R_{c}.

The channel model considered in this paper is, hence, completely characterized by three parameters, namely ϵ, τ, and Λ^*. These parameters are clearly related, as, for instance, reducing the erasure probability ϵ may require increasing the delay τ or reducing the code rate R_{c}, i.e., decreasing the maximum achievable SQNR Λ^*. Therefore some trade-offs are expected in the context of feedback control , since all three

[1]Strictly speaking the terminology "steady state" should be used only when the limit is finite; this will actually hold under suitable conditions, see Theorem 1.

terms negatively impact the performance of the closed loop system. Unfortunately, the exact form of the relation among these parameters is not available, though some tight bounds have recently been derived in [15].

For the ease of mathematical treatment, in our analysis we will assume that these parameters can be set independently. We can thus sort out the impact of each single parameter on the system performance. Note that, the interdependencies among the channel model parameters will only shrink the design parameter space, without affecting the validity of our analysis. An extension of our approach that keeps into account this aspect is left for future work.

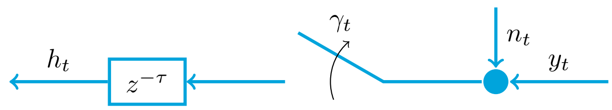

Fig. 1. Equivalent model of the feedback channel, accounting for the presence of quantization noise, packet loss and decoding delays.

C. Problem statement

In this work, we address a special case of the general LQG optimization problem by considering a channel model that includes packet loss, quantization noise and limited SQNR, but assumes no decoding delay, i.e.,

$$\tau = 0, \quad \mathbb{P}[\gamma_t = 0] = \epsilon, \quad \sigma_n^2 = \mathbb{E}\big[y_t^2\big]/\Lambda \ge \mathbb{E}\big[y_t^2\big]/\Lambda^* \quad (4)$$

While delays do not influence the possibility to stabilize a system, they do play a major role when performance (e.g. measured by the variance of certain error signals) is of interest. However, transmitting close to channel capacity with small delays, will make packet drops non-negligible. The erasure channel is modeled by setting

$$h_t = (y_t + n_t)\gamma_t\,, \quad (5)$$

according to which, in case of erasure, no signal is received.

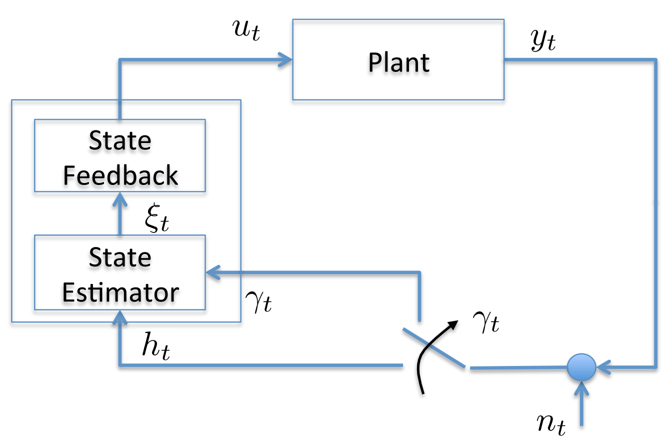

Fig. 2. NCS scheme for scalar output plants, where the plant decoder is given by the cascade of a linear state estimator and a state feedback

We restrict our attention to the classical LQG structure for the plant decoder, which is given by the cascade of a linear state estimator and a state feedback, as represented in Fig. 2. The state estimator is governed by the following law

$$\xi_t = a\xi_{t-1} + bu_{t-1} + \gamma_t k\big(h_t - c(a\xi_{t-1} + bu_{t-1})\big) \quad (6)$$

where k is the predictor gain and the estimator (6) is time-varying since it depends on the sequence γ_t. In fact, if a packet is not received correctly, i.e. $\gamma_t = 0$, then the estimator updates its state using the model only, while when $\gamma_t = 1$ the estimate is adjusted by a correction term, based on the output innovation, similarly to a Kalman filter. The state feedback module, in turn, will simply return a control signal proportional to the predicted state through a gain ℓ, i.e.,

$$u_t = \ell \xi_t \quad (7)$$

This scheme is the same as was proposed in [16] and, although it does not yield the optimal Kalman filter [6], it has the advantage of being computationally simpler and allowing for the explicit computation of the performance J, as will be shown in the next section.

In this framework, the objective is to solve the following optimization problem:

$$\min_{k,\ell} \quad J \quad (8)$$

$$\text{s.t.} \lim_{T\to+\infty} \frac{\sum_{t=0}^{T} \mathbb{E}[||y_t||^2]}{\sum_{t=0}^{T} \mathbb{E}[||n_t||^2]} \le \Lambda^* \quad (9)$$

Constraint (9) sets an upper bound on the SQNR, which cannot exceed the maximum value Λ^* allowed by the channel code rate.

Although in this study we limit our attention to the case of scalar systems, the approach we proposed is amenable to be extended to the multidimensional case, though we leave the investigation of the more general case to future work.

III. Analysis of the scalar case

As a first step, we derive the dynamical equations that govern the state as well as the error evolution for the estimator in equation (6). If we let

$$\hat{x}_t = a\xi_{t-1} + bu_{t-1} \quad (10)$$

be the optimal (constant gain) one-step-ahead predictor of the state x_t, we can rewrite (6) and (7) as

$$\begin{aligned} \hat{x}_{t+1} &= a\hat{x}_t + bu_t + a\gamma_t k\,(h_t - c\,\hat{x}_t) && (11)\\ u_t &= \ell\,(1-\gamma_t kc)\,\hat{x}_t + \gamma_t \ell k h_t && (12) \end{aligned}$$

Substituting the input given by the controller, (12), into the predictor equation (11) we get:

$$\begin{aligned} \hat{x}_{t+1} &= a\hat{x}_t + b\ell\,(1-\gamma_t kc)\,\hat{x}_t + \gamma_t b\ell k h_t + \gamma_t ak\,(h_t - c\hat{x}_t)\\ &= (a + b\ell)\,\hat{x}_t + \gamma_t\,(b\ell k + ak)\,(h_t - c\hat{x}_t)\\ &= (a+b\ell)\,\hat{x}_t + \gamma_t\,(b\ell + a)\,k\,(c\tilde{x}_t + w_t + n_t) \quad (13) \end{aligned}$$

where, in the last row, we introduced the error term $\tilde{x}_t = x_t - \hat{x}_t$. In turn, $\tilde{x}_t$ evolves as follows

$$\begin{aligned} \tilde{x}_{t+1} &= ax_t + bu_t + w_t - a\hat{x}_t - bu_t - \gamma_t ak\,(h_t - c\hat{x}_t)\\ &= a\tilde{x}_t + w_t - \gamma_t ak\,(c\tilde{x}_t + w_t + n_t)\\ &= a(1-\gamma_t kc)\tilde{x}_t + w_t - \gamma_t ak(w_t + n_t) \quad (14) \end{aligned}$$

For the sake of a more compact notation, and without loss of generality, we let $b = c = 1$ in the following[2], so that (13) and (14) can be combined in matrix form together with (2)

$$\begin{aligned} \begin{bmatrix} \hat{x}_{t+1} \\ \tilde{x}_{t+1} \end{bmatrix} &= A_{\gamma_t} \begin{bmatrix} \hat{x}_t \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_t + \begin{bmatrix} \gamma_t(a+\ell)k \\ -\gamma_t ak \end{bmatrix} (w_t + n_t)\\ y_t &= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_t \\ \tilde{x}_t \end{bmatrix} + v_t \quad (15) \end{aligned}$$

where we set

$$A_{\gamma_t} = \begin{bmatrix} (a+\ell) & \gamma_t(a+\ell)k \\ 0 & a(1-\gamma_t k) \end{bmatrix}.$$

We observe that the cost function (3) is equal to the steady state power of the system output y_t that can be expressed as

$$\begin{aligned} J(k,\ell) &= \begin{bmatrix} 1 & 1 \end{bmatrix} P \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \sigma_v^2\\ &= p_{11} + 2p_{12} + p_{22} + \sigma_v^2 \quad (16) \end{aligned}$$

where the matrix P is defined as

$$P = \mathbb{E}\left[\begin{bmatrix} \hat{x} \\ \tilde{x} \end{bmatrix} [\hat{x}\,\tilde{x}]\right] = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}. \quad (17)$$

Using the fact that the $[\hat{x}_t\,\tilde{x}_t]$, v_t, $(w_t + n_t)$ are uncorrelated, it follows that

$$\begin{aligned} P = (1-\epsilon)A_1 P A_1^\top + \epsilon A_0 P A_0^\top + \begin{bmatrix} 0 & 0 \\ 0 & \sigma_w^2 \end{bmatrix} + \quad (18)\\ + (1-\epsilon) \begin{bmatrix} (a+\ell)k \\ -ak \end{bmatrix} (\sigma_v^2 + \sigma_n^2) \begin{bmatrix} (a+\ell)k & -ak \end{bmatrix} \end{aligned}$$

We are now in the position to compute the optimal ℓ and k that minimizes the cost J.

Theorem 1: The minimum of the cost function J as given in Eqn. (16) under constraint (9) is

$$J^* = \min_{k,\ell} J(k,\ell) = J(k^*, \ell^*) = \overline{p}_{22} + \sigma_v^2$$

where $\overline{p}_{22}$ is the unique positive solution of

$$\overline{p}_{22} = a^2 \overline{p}_{22} + \sigma_w^2 - \frac{1-\epsilon}{1 + 1/\Lambda^*} \frac{a^2 \overline{p}_{22}^2}{\overline{p}_{22} + \sigma_v^2} \quad (19)$$

and is achieved with the following gains:

$$\ell^* = -a, \qquad k^* = \frac{1}{1 + 1/\Lambda^*} \frac{\overline{p}_{22}}{\overline{p}_{22} + \sigma_v^2} \quad (20)$$

The positive solution $\overline{p}_{22}$ exists finite if and only if

$$\frac{1-\epsilon}{1 + 1/\Lambda^*} > 1 - \frac{1}{a^2} \quad (21)$$

Proof: See Appendix. ■

Theorem 1 provides some interesting results: first, it gives the minimum cost function J^* that can be reached with the control system depicted in Fig. 2; second, it defines the values of the gains ℓ and k that achieve such a minimum; third, and most importantly, it sets a necessary and sufficient condition on the packet loss rate ϵ and the SQNR Λ^* of the feedback

[2] Note that c can always be set equal to 1 via a suitable change of basis in the state space. In addition b can be fixed equal to one via a rescaling of the control input u, which in our setup does not influence the performance criteria.

channel for the system to be controllable and the cost function J to be reduced to J^*.

This last result is a generalization of similar results available in the literature. In fact, by letting $\Lambda^* \to +\infty$, which is equivalent to consider a channel with infinite code rate, (21) returns $\epsilon < 1/a^2$ that is the same stability condition found in the lossy network literature [6], [9]. Alternatively, if we assume no packet loss in the channel, i.e. $\epsilon = 0$, the stability condition can be rewritten as

$$1 - \frac{1}{a^2} < \frac{1}{1 + 1/\Lambda^*} = 1 - \frac{1}{1 + \Lambda^*}$$

that leads to

$$\Lambda^* > a^2 - 1,$$

which is the same stability condition presented in the context of SNR-limited control system in [3].

Therefore, the bound provided by (21) will be useful to compare different communication protocols. In fact, by using a corse quantizer it is possible to reduce the transmission rate R_q, thus allowing more redundant channel coding schemes and consequently a smaller packet loss probability ϵ. On the other hand a coarser quantizer gives a smaller Λ. In conclusion, Λ and ϵ are coupled and cannot be designed separately.

IV. Conclusions and future work

We have considered an LQG control problem; the model we have proposed accounts for code rate limitations, as well as for packet drops and delays. We have argued in fact that there is a tight connection between the actual rate at which one can transmit information, the decoding delay (due to long block coding) and the packet-drop probability.

We have restricted our attention to a specific control architecture in which the plant outputs are transmitted via a rate limited channel and then processed through the cascade of a state estimator followed by a linear (state) feedback controller; for ease of exposition we did not consider delays, while both limited rate and packet drops have been included in our analysis. We have considered a scalar model and found that the optimal controller has a dead-beat structure and the optimal estimator is a Kalman-like constant gain estimator (which accounts for the packet drop probability). Conditions for stability are derived in terms a modified algebraic Riccati equation and recapture results from the literature as special cases.

Future work will include a detailed analysis of the multivariable case as well as the inclusion of delays in our model.

Appendix

A. Proof of Theorem 1

As the quantization noise variance is related to the signal power by $\sigma_n^2 = J/\Lambda$, by using (16), we can express (18) in terms of the entries in the matrix P as

$$p_{11}=(a+\ell)^2[(1-\epsilon)(p_{11}+2kp_{12}+k^2p_{22}+k^2(\sigma_v^2+ J/\Lambda))+\epsilon p_{11}]$$
$$p_{12}=(a+\ell)[(1-\epsilon)a((1-k)(p_{12}+kp_{22})-k^2(J/\Lambda+\sigma_v^2))+\epsilon p_{12}]$$
$$p_{22}=(1-\epsilon)(a^2(1-k)^2p_{22}+a^2k^2(\sigma_v^2+J/\Lambda))+\epsilon a^2 p_{22}+\sigma_w^2$$

which provide implicit expressions for the steady state solution. We then have to show that the necessary conditions for optimality

$$\frac{\partial J}{\partial k} = 0 \quad \frac{\partial J}{\partial \ell} = 0 \tag{22}$$

hold for $\ell = -a$, and $k = k^*$ that satisfies (20).

Using the implicit expressions for p_{11}, p_{12} and p_{22} above it is easy to check that, setting $\ell = -a$, we have

$$\frac{\partial p_{11}}{\partial \ell} = \frac{\partial p_{11}}{\partial k} = \frac{\partial p_{12}}{\partial k} = 0 \quad \forall k. \tag{23}$$

Note also that for $\ell = -a$, $k = k^*$ is obtained by solving:

$$k^* = \arg\min_k p_{22} = \frac{p_{22}}{(1 + 1/\Lambda)(p_{22} + \sigma_v^2)} \tag{24}$$

Therefore also $\frac{\partial p_{22}}{\partial k} = 0$ must hold for $\ell = -a$ and $k = k^*$; now, using (23) also $\frac{\partial J}{\partial k} = 0$ for $k = k^*$, $\ell = -a$ which proves the first of (22).

From the implicit expression for p_{12} above, we obtain (recall that $p_{12} = p_{11} = 0$ for $a = -\ell$)

$$\frac{\partial p_{12}}{\partial \ell} = (1 - \epsilon)ak^*[(1 - k^*)p_{22} - k^*(J/\Lambda + \sigma_v^2)] \tag{25}$$

and, using the explicit value of k^* in (20), it is easy to see that $\frac{\partial p_{12}}{\partial \ell} = 0$ for $a = -\ell$ and $k = k^*$.

Using the fact that $p_{12} = p_{11} = 0$ for $a = -\ell$, we have that $J = p_{22} + \sigma_v^2$ and, therefore, the Riccati equation for p_{22} takes the form $p_{22} = \beta^2 p_{22} + \delta$ where the specific expressions for β and δ are immaterial; we just need to recall that, since k^* is a stabilizing gain, $\beta^2 < 1$. Therefore

$$\frac{\partial p_{22}}{\partial \ell} = \beta^2 \frac{\partial p_{22}}{\partial \ell}$$

which, using $\beta^2 < 1$, admits the unique solution $\frac{\partial p_{22}}{\partial \ell} = 0$. We have now proved that $\frac{\partial p_{11}}{\partial \ell} = \frac{\partial p_{12}}{\partial \ell} = \frac{\partial p_{22}}{\partial \ell} = 0$ and, therefore, $\frac{\partial J}{\partial \ell} = 0$; this proves the second condition in (22).

Let us now observe that substituting the optimal value k^* in p_{22} we get:

$$p_{22} = a^2 p_{22} + q - \frac{1 - \epsilon}{1 + 1/\Lambda} \frac{a^2 p_{22}^2}{p_{22} + r} \tag{26}$$

which is the same modified algebraic Riccati equation (MARE) that appears in [6] with $\bar{\eta} = \frac{1-\epsilon}{1+1/\Lambda}$.
Notice also that, since the system is scalar, the critical probability for the solvability of the MARE is known:

$$\eta_c = 1 - \frac{1}{a^2}$$

Given that the problem has solution, i.e. p_{22} converges, iff:

$$\bar{\eta} = \frac{1 - \epsilon}{1 + 1/\Lambda} > \eta_c = 1 - \frac{1}{a^2} \tag{27}$$

which is equivalent to

$$\Lambda > \frac{a^2 - 1}{1 - \epsilon a^2} \tag{28}$$

It is now self evident that the problem has a solution for some SQNR $\Lambda \le \Lambda^*$ iff (28) holds with Λ^* replacing Λ, that is iff (21) holds. Moreover, since $\bar{p}_{22}$ is a non increasing function of Λ, the optimal solution is obtained with $\Lambda = \Lambda^*$ in (24) and (26), thus yielding (19) and (20).

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