Distributed perimeter patrolling and tracking for camera networks

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Abstract— **In this work, we propose a distributed control strategy for perimeter patrolling and target tracking in a multi-camera videosurveillance system with communication, resources and speed constraints. These cameras are required to monitor a perimeter and share common portions of this perimeter to allow redundant coverage. We propose an algorithm that is able to find the global patrolling strategy only through local asynchronous communication and coordination of neighboring cameras even in presence of physical limits of each camera visibility area. The algorithm converges to an optimal solution, and its distribuited implementation is obtainet through an electric circuit analogy. The proposed system also includes a Kalman-based filter for each camera to track moving targets within its areas of competence, and a distributed coordination scheme for target hand-off between different cameras that guarantees target locking at all times. Numerical simulations are provided to test the proposed algorithms.**

I. Introduction

A. Motivations

Patrolling means to keep on traveling around an assigned area to visit every point more and more [1], therefore a good patrolling strategy consists in minimizing the time elapsed between two visits of the same point. Patrolling is a fundamental task in many applications: military defense systems, mobile robots, chemical and nuclear factories and, obviously, surveillance systems in which camera networks are used to monitor perimeters, indoor and outdoor areas. Another important task within this class of application is tracking. In fact when an event occurs, the system has to detect it and then track it. In the context of camera networks, this means that high-resolution and good-dynamic cameras are required, in fact a common trend is the adoption of systems of multiple pan-tilt-zoom (PTZ) cameras which can potentially monitor large areas with high resolution. At present, most surveillance systems are centralized and human-controlled, however the need for scalable systems calls for distributing intelligence on the cameras and for automated patrolling and tracking without human intervention. This is not an easy task since the objective is still to guarantee good global behavior of the whole system, while using only local information and coordination among the different cameras with different resources and constraints.

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B. Previous Work

This work addresses two different but complementary problems: multi-agent patrolling and tracking. Multi-agent patrolling finds applications not only in surveillance problems. A theoretical analysis of this problem is carried out in [1], where the author studies conditions that guarantee the existence of optimal strategies by means of graphs analysis. In [2] the authors introduce a multi-agent patrolling algorithm based on a physical analogy: gas-filled balloons. The agents partition the area and each becomes responsible of its subarea. A very interesting article is [3], in which a multi-agent cooperative solution is realized. The solution is robust and can fit itself to perimeter changes. The main feature of this article is that communication between the agents takes place only when they come in contact, therefore as a result they need to move in phase opposition, i.e their either move towards or away from each other. Another physical analogy is provided in [4] where the authors propose a series of interconnected spring-mass systems. In [5] it is proposed the use of potential functions to coordinate a robot fleet. In [6], [7] the authors provide a detailed algorithm to dynamically cover a region using a team of mobile agents that avoid flocking and collisions. The main feature is that coordination control laws are local since each agent uses information brought only by its neighbors.

As for multi-camera tracking in surveillance networks, most of the literature is concerned with computer-vision problems, such as dedicated algorithms to track faces or specific targets such as cars. This is outside the scope of this work since we assume that each camera is provided with a low-level visual servoing to maintain the target in the filed of view. We addressed the reader interested in review of the field of intelligent surveillance systems to the survey [8]. Nonetheless some relevant works are available. For example in [9] a distributed algorithm for PTZ camera networks is provided. Another notable work is given [10] which proposes a Kalman-based autonomous surveillance system, even if based on a centralized processor. A further application of Kalman filter is developed in [11] and applied successfully to Pan-Tilt-Zoom cameras. The same article also addresses problem of designing a control system for the cameras.

C. Contribution

We address the problem of optimal patrolling and target tracking with multiple cameras by casting it as a real-time control problem. In particular in this work we do not consider the (static) problem of optimal camera placement for patrolling perimeter, sometimes referred to as the "art gallery problem", see [12], [13] for example. Differently, we assume that cameras positions on the perimeter is given as an input of the problem. Thus, our problem is constrained, because our agents (cameras) cannot move freely on the whole patrolling perimeter. Nevertheless, we assume some overlap in the physical range of cameras to justify the search for an optimal coordinated patrolling algorithm. In fact, if there were no overlaps, no coordination would be required. We give an innovative contribution in terms of guaranteed global optimality despite the use of local controllers and despite the cameras constraints, such as their speeds and physical ranges. These two aspects are rarely considered in literature. In particular, we propose a patrolling scheme that is distributed, suitable for asynchronous cammunication, robust to camera failures and parameters variations, and optimal in terms of the global objective, i.e. the worst-case time-to-detection of a random event along the perimeter.

The other main contribution of the paper is the tracking strategy where we adopt a Kalman-based approach. Although the use of a Kalman-filter for tracking is standard for single cameras, we also included a communication and coordination schemes among cameras to avoid that the target gets lost when it goes outside the physical mobility limits of the currently tracking camera, thus guaranteeing continuous target locking. Moreover, when one or more cameras are involved in tracking, the other cameras adaptively optimize their patrolling coverage in order to include also the regions that are not patrolled by the tracking cameras.

II. Problem formulation

Here and in the remainder of the paper we consider a one-dimensional case, which refers to the realistic scenario of perimeter surveillance, while allowing for an easier formulation of the problem and a neater solution design. Also, for the sake of simplicity, we assume: **(a)** 1-d.o.f. cameras, meaning that the field of view (f.o.v.) of each camera is allowed to change due to pan movements only, and **(b)** fixed coverage range, meaning that during pan movements the camera coverage range is not altered by the view perspective.

In this context, the following notation is adopted:

- $L = [0,\, L_{tot}] \subset \mathbb{R}^+$ is the rectified total length of the perimeter to be monitored;
- N is the cardinality of camera set (also referred to as agent set): $\{\mathcal{A}_1, \dots, \mathcal{A}_N\}$;
- $D_i = [D_{i,inf},\, D_{i,sup}] \subset L$ is the total coverage range of i-th camera $\mathcal{A}_i$, due to the scenario topology, the agent configuration and physical constraints;
- $v_i \in [-V_{i,max},\, +V_{i,max}]$ is the (bounded) speed of i-th camera during pan movements;
- $z_i(t) : \mathbb{R}^+ \to D_i$, $z_i(t) \in \mathscr{C}(\mathbb{R}^+)$, is the continuous function mapping the center of the area covered by the i-th camera as a function of the time variable t;
- $A_i = [a_{i-1},\, a_i]$ is the steady state coverage of the i-th camera: a feasible solution is such if $A_i \subseteq D_i\ \forall i$ and, if no overlapping zones are present, $\sum_{i=1}^{N} |A_i| = L_{tot}$, being $|A_i|$ the length of segment A_i.

In order to provide a procedure to solve the area coverage problem, it is a key issue to define an appropriate cost function J and state an optimality criterium. We propose a functional J whose rationale is as follows: at each time instant t and position $x \in L$, J is equal to 0 if location x is currently seen (by any camera), else it takes a positive real value as increasing as the time is passing since the last visit of x. This choice is strictly related to the goal of finding an optimal coverage criterion trading off between the patroling activity and the additional task of tracking detected events.

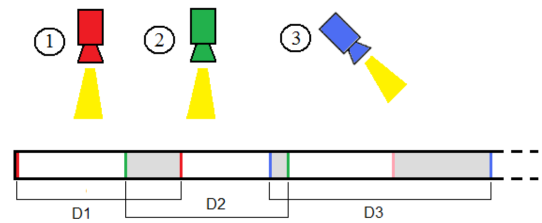

Fig. 1. Perimeter under surveillance. The physical coverage $\{D_i\}$ of three cameras is shown, with some overlapping sections.

More formally, given:

- the set of camera f.o.v. centers $z(t) = [\,z_1(t)\ \dots\ z_N(t)\,]'$;
- the function $\bar{t}(x) : L \to \mathbb{R}^+$, as the elapsed time from the most recent visit of x by a camera (elapsed time from the last time t s.t. $\exists i \in \{1, \dots, N\} | z_i(t) = x$);
- the function $g(\bar{t}(x)) : \mathbb{R}^+ \to \mathbb{R}^+$, as a strictly increasing function with $g(0) = 0$;

the following cost function is assumed:

$$J(x, t, z(t)) : L \times \mathbb{R}^+ \times \mathscr{C}^N(\mathbb{R}^+) \to \mathbb{R}^+, \tag{1}$$

that is equal to:

$$J(x, t, z(t)) = g(\bar{t}(x)). \tag{2}$$

The initial conditions ($t = 0$) are:

- $z_i(0) \sim \mathcal{U}(D_i)$, uniformly distributed random variables in the interval D_i;
- $\bar{t}(x) = 0$, $\forall x \in L$,

so that it holds:

$$J(x, 0, z(0)) = 0, \quad \forall x \in L. \tag{3}$$

Remark 1: We assume implicitly a point f.o.v. for the cameras: in doing so, the pan motion associated to each camera $\mathcal{A}_i$ is $z_i(t)$ and position x is visible if and only if $\exists\, \bar{i}\, |\, z_{\bar{i}}(t) = x$. This assumption nevertheless does not affect the general validity of the approach: if visual cones $\{\text{f.o.v.}(i)\}$ are known (and in general different for each camera), the point object case can be generalized considering a modified length L_{eff} of the trajectory to be monitored w.r.t. L_{tot}, as

$$L_{eff} = L_{tot} - \sum_{i=1}^{N} \text{f.o.v.}(i). \tag{4}$$

The aim is now to design a control law that ensures the minimization of index $J(\cdot, \cdot, \cdot)$ according to some norm. The constrained cameras' dynamics is:

$$\dot{z}_i(t) = v_i(t), \quad \forall\, i \tag{5}$$

$$s.t. \quad \begin{cases} v_i(t) \in [-V_{i,max},\, +V_{i,max}] \\ z_i(t) \in D_i \end{cases}, \tag{6}$$

and the speed set $\{v_i\}$ appears as a natural control input for the system:

$$V(t) = [\, v_1(t) \,\dots\, v_N(t)\,]', \quad t \in [0 + \infty). \tag{7}$$

Therefore, the following minimization problem is posed:

$$\bar{V}(\cdot) = \arg\min_{V(\cdot)} \sup_t \max_x J(x, t, z(\cdot)) \tag{8}$$

constrained to the system dynamics (5)-(6).

Remark 2: The proposed minimization problem (8) is closely related to the ∞-norm minimization of function $J(\cdot,\cdot,\cdot)$ and ensures that by a maximum time T_{max} every point of the perimeter is visited (which is a requirement for the specific surveillance application). Moreover, assuming the event is described by a Poisson Process Counter, and taking $g(\cdot)$ as

$$g(\bar{t}(x)) = 1 - e^{-\lambda \bar{t}(x)}, \tag{9}$$

$J(\cdot,\cdot,\cdot)$ is the probability that at least one event occurs in position x, from last visit in x of a camera. Since an effective surveillance system is required also to detect unexpected events, the definition of such a control law corresponds to minimizing the probability of missing an event.

III. Optimal solution

Due to the many constraints, finding an optimal solution to the camera coordination problem is not straightforward: in this section we compute the optimal solution, supporting the study on the following propositions.

A. Optimal trajectory without coverage bounds

Lemma 1: In the particular case of $N = 1$ camera $\mathcal{A}$ monitoring a perimeter L_{tot}, the problem (8) takes the optimal solution by commanding camera $\mathcal{A}$ to move with periodic motion with period $\bar{T}$ at the maximum speed $\pm V_{max}$ where

$$\bar{T} = \frac{2L_{tot}}{V_{max}}. \tag{10}$$

In this case, the minimum value of functional J is assumed at the perimeter extreme points of the perimeter and its value is

$$\bar{J} = 1 - e^{-\lambda \bar{T}}, \tag{11}$$

assuming g as in (9).

Proof: Considering agent $\mathcal{A}$ moving at speed $v(t)$, it stands:

$$2L_{tot} = \int_0^T |v(t)|dt = T|v_{mean}|. \tag{12}$$

Due to the patrolling policy, T is the elapsed time between two consecutive visits of the extreme perimeter points, which is also the maximum elapsed time between two consecutive visits of any point $x \in L_{tot}$: for any other point to be visited twice camera position $z(t)$ has to move less than $2L_{tot}$. The minimum value of T, called $\bar{T}$, is attained when $|v_{mean}| = V_{max}$, which yields the following relation on g

$$1 - e^{-\lambda \bar{T}} \leq g(\cdot) < 1. \tag{13}$$

Hence, it follows that to minimize $J\ (g(\cdot))$ the optimal choice of input signal $V = [v(t)]$ is $v(t) = \pm V_{max}$.

■

This simple case suggests the following considerations: the optimal coverage of the perimeter, also in the general case, will be achieved by having each camera $\mathcal{A}_i$ to follow a periodical motion at its maximum speed in its coverage area A_i. In doing so, the problem of minimizing the index $J(\cdot,\cdot,\cdot)$ is reduced to the optimal choice of A_i for each camera.

Proposition 1: The optimal coverage of the whole perimeter as the minimization of index $J(\cdot,\cdot,\cdot)$ with criterion (8) and without the coverage constraint (6), is attained assuming that every camera is moving at its maximum speed $|V_{i,max}|$ with a periodical motion of period $\bar{T}$ in non-overlapping coverage areas A_i. The area length $|A_i|$ and optimal period $\bar{T}$ are obtained as

$$|A_i| = V_{i,max} T_o. \tag{14}$$

and

$$\bar{T} = 2T_o = \frac{2L_{tot}}{\sum_{i=1}^{N} V_{i,max}} \tag{15}$$

The perimeter is divided into N separate segments, each one monitored by a camera moving at its maximum speed in T_o, that is half of the optimal period $\bar{T}$.

Proof: From Lemma 1, the length of the A_i interval is

$$|A_i| = V_{i,max} T_o \geq \int_0^{T_o} v_i(t)dt, \tag{16}$$

where T_o is half of the optimal period $\bar{T}$. The stated conditions imply

$$L_{tot} = \sum_{i=1}^{N} |A_i| = \sum_{i=1}^{N} V_{i,max} T_o\ , \tag{17}$$

whence Eq. (15) follows. Modifications to the proposed optimal solution can affect the length of sections $\{A_i\}$, either keeping them or not non-overlapping. It is clear that the overlapping case brings in non-optimality to the solution; nonetheless modification to the $\{|A_i|\}$ cannot be unilateral:

$$if\ \exists\ i\ s.t.\ |A'_i| < |A_i| \Rightarrow \exists\ j\ s.t.\ |A'_j| > |A_j|\ , \tag{18}$$

and since the agents are already moving at their maximum speed

$$T'_o = T'_j > T_o. \tag{19}$$

It follows that an higher value of the functional J would be attained. ■

B. Optimal trajectory with coverage bounds

In general, the optimal solution of problem (8) without any constraint is not equivalent to the constrained optimal solution; this happens if the unconstrained solution is feasible, that is if and only if

$$A_i \subseteq D_i. \tag{20}$$

To cope with the case where the unconstrained solution violate the feasibility condition, we adopt a greedy approach: starting from the optimal trajectory as computed with no constraint, this is adjusted w.r.t. to the introduction of constraints, according to a best available choice strategy.

For simplicity, we show how to modify the optimal trajectory in the case of feasibility violation by the first camera $\mathcal{A}_1$: the case extends similarly to the other agents.

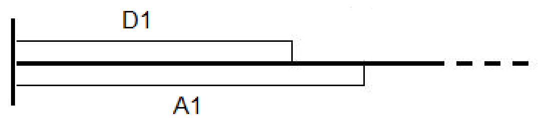

Fig. 2. Feasibility constraint violation for the camera $\mathcal{A}_1$: $A_1 \nsubseteq D_1$.

We introduce in this context the optimal patrolling period with constraints, $T_{o,c}$. In general it holds: $T_{o,c} \geq T_o$, the equality standing only when the feasibility constraint (20) is not violated.

Proposition 2: If the unconstrained solution yields ($A_1 \nsubseteq D_1$), the optimal coverage of the trajectory is attained by assigning to $\mathcal{A}_1$ the maximum feasible length complying to its constraints, and recomputing the optimal solution for the remaining $N-1$ cameras to cover $L \setminus D_1$:

1) $A_1 = D_1$
2) $|A_i| = V_{i,max} T_{o,c} \quad i \neq 1$

with

$$T_{o,c} = \frac{L_{tot} - |D_1|}{\sum_{i=2}^{N} V_{i,max}}, \quad T_{o,c} > T_o. \tag{21}$$

Proof: If $A_1 = D_1$, then the remaining $N-1$ agents arrange to cover $L \setminus D_1$ according to the optimal solution of Prop. 1: the N agent optimal time $T_{o,c}$ will be that of the $N-1$ agent solution, since $\mathcal{A}_1$ will patrol a smaller area than that of the unconstrained solution basically posing a non effective constraint on the problem. Eq. (21) follows.

Suppose, for contradiction, that a $A_1 \subset D_1$ is assigned. The remaining $N-1$ cameras will necessarily cover length $L_{tot} - |A_1| > L_{tot} - |D_1|$, yielding:

$$\bar{T} = \frac{L_{tot} - |A_1|}{\sum_{i=2}^{N} V_{i,max}} > T_{o,c}, \tag{22}$$

with $T_{o,c}$ calculated as in (21). This solution would imply higher values of the g function, hence higher values for the optimization functional J. ∎

IV. Distributed Solution

We shall now find a method to reach the optimal steady-state configuration for patrolling extremes using only local interaction between neighboring cameras. The goal is to let each camera $\mathcal{A}_i$ calculate its A_i patrolling section. This is achieved via analogy with an electric network. First, we present the circuit model with continuous dynamics and then we analyze numeric convergence in realistic conditions such as discrete time control and asynchronous communication. The idea is to relate voltages at circuit nodes to optimal patrolling sections for the surveillance system, and see resistor values as proportional to maximum patrolling speed of cameras. The equivalence of the two systems is described in details below.

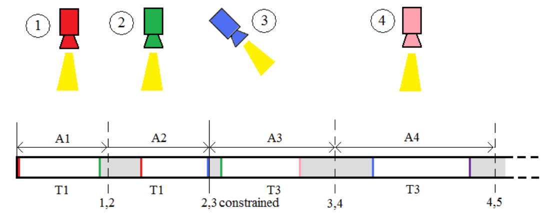

Fig. 3. An example of constrained optimal patrolling coverage, respecting all feasibility constraints.

A. Continuous time version, unconstrained problem

Consider a series of N resistors, and suppose to apply known voltages $\{u_0, u_N\}$ at its ends as shown in Fig. 4.

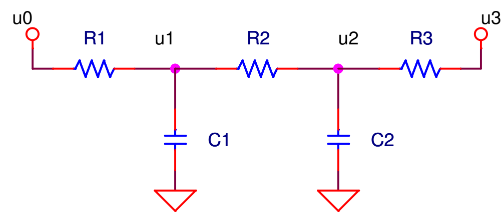

Fig. 4. Electric circuit analogy ($N = 3$). Voltages $\{u_i\}$ relate to the positions of the patrolling sections $\{A_i\}$.

The resistors have local interaction at the circuit nodes, meaning that currents $\{I_i\}$ must sum to zero to obey Kirchhoff's law. If the circuit is composed only by a series of resistors, the voltages at nodes will follow the law of a simple resistive voltage divider. We shall add some capacitors connected to the nodes to model dynamic evolution of the voltages $\{u_1(t), \ldots, u_{N-1}(t)\}$ from general initial conditions to the equilibrium configuration. Since the circuit is passive, there is only one equilibrium point for this circuit, and it is globally asymptotically stable for any initial configuration $\{u_1(0), \ldots, u_{N-1}(0)\}$. We can prove this by means of an appropriate Lyapunov function, that will also be useful for constrained and discrete time versions of this problem. Let us define the vector $U = [u_0 \ u_1 \ \ldots \ u_N]'$ and consider the following function:

$$W(U(t)) = \sum_{i=1}^{N} \frac{1}{2R_i} (u_i(t) - u_{i-1}(t))^2, \tag{23}$$

which represents the power dissipated on resistors. It is a a nonnegative quantity and its time derivative is

$$\dot{W}(U(t)) = \sum_{i=1}^{N} \frac{1}{R_i}(u_i(t) - u_{i-1}(t))(\dot{u}_i(t) - \dot{u}_{i-1}(t))$$
$$= \sum_{i=1}^{N-1} \dot{u}_i(t) \left(\frac{u_i(t) - u_{i-1}(t)}{R_i} + \frac{u_i(t) - u_{i+1}(t)}{R_{i+1}} \right), \quad (24)$$

where we used the fact that $\dot{u}_0(t) = \dot{u}_N(t) = 0$. Moreover, the law for charging capacitors requires that

$$\dot{u}_i(t) = -\frac{1}{C_i} \left(\frac{u_i(t) - u_{i-1}(t)}{R_i} + \frac{u_i(t) - u_{i+1}(t)}{R_{i+1}} \right), \quad (25)$$

so $\dot{W}(U(t))$ is the opposite of a sum of square terms, thus negative semi-definite, and it is equal to zero only at the equilibrium point $\dot{W}(U(t)) = 0$ where all square terms are equal to zero. Since we have the constraints $u_0(t) \equiv 0, u_N(t) \equiv 0$, it is easy to verify that $\dot{W}(U) = 0$ has a unique equilibrium U_{eq} where

$$u_{i,eq} = \frac{u_{i-1,eq}/R_i + u_{i+1,eq}/R_{i+1}}{1/R_i + 1/R_{i+1}}, \quad i = 1, 2, \dots, N-1 \quad (26)$$

that is the equilibrium point of the resistive voltage divider.

One can also describe the circuit in state-space with the following continuous-time free-evolution matrix for $\dot{U}(t) = FU(t),\ U = [u_0\ u_1\ \dots\ u_N]'$:

$$F = \begin{bmatrix} 0 & 0 & 0\dots & 0 \\ \frac{1}{C_1R_1} & -\frac{R_1+R_2}{C_1R_1R_2} & \frac{1}{C_1R_2}\dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots\frac{1}{C_{N-1}R_{N-1}} & -\frac{R_{N-1}+R_N}{C_{N-1}R_{N-1}R_N} & \frac{1}{C_{N-1}R_N} \\ 0 & 0 & 0\dots & 0 \end{bmatrix}$$

and verify that it has a unique equilibrium point that satisfy the constraints and it is asymptotically stable.

B. Continuous time version, constrained problem

We now modify the electric network to model the physical limits of the cameras. More precisely, we add some diodes to the previous circuit to impose saturation limits for capacitors voltages, as shown in Fig. 5.

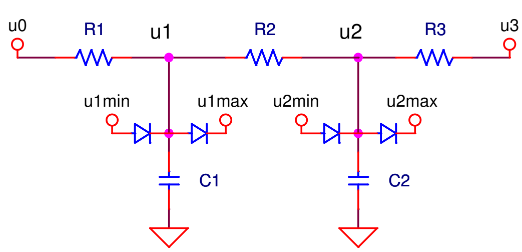

Fig. 5. Electric circuit with saturation constraints.

The complete circuit is still a passive network with constant voltage sources applied to the inputs $\{u_0, u_{1,min}, u_{1,max}, \dots, u_{N-1,min}, u_{N-1,max}, u_N\}$. Energy storing components (capacitors) are connected by dissipative components (resistors), so global asymptotical convergence to the unique equilibrium point is still assured. Let $\dot{u}_i(t)$ be the unconstrained expression for node voltage variation given by (25), and $\dot{\tilde{u}}_i(t)$ be its constrained version:

$$\dot{\tilde{u}}_i(t) = 0 \quad \text{if} \quad u_i(t) = u_{i,max} \wedge \dot{u}_i(t) > 0$$
$$\text{or if} \quad u_i(t) = u_{i,min} \wedge \dot{u}_i(t) < 0.$$

Henceforth, we redefine $\dot{u}_i(t) := \dot{\tilde{u}}_i(t)$ to simplify the notation. Global convergence to a unique minimum can be demonstrated using the same Lyapunov function $W(U(t))$ introduced in the unconstrained case. In fact, whenever a saturation point is reached for the j-th capacitor, the voltage at the saturated node becomes constant, that is $\dot{u}_j(t) \equiv 0$ for $t \geq \bar{t}$ for some $\bar{t}$. As a consequence, the Lyapunov function $\dot{W}(U(t))$ split into the sum of two independent parts:

$$\dot{W}(U(t)) = \sum_{i=1}^{j-1} [\dots] + \sum_{i=j+1}^{N-1} [\dots], \quad t \geq \bar{t} \quad (27)$$

where $[\dots]$ identify the same argument of the sum in (24). This sum can still be expressed as the opposite of the sum of square terms, with the same substitution adopted before, and thus is still a definite negative expression. When more than one capacitor reaches the saturation limit, the sum expressing $\dot{W}(U(t))$ can be split again in more separate terms. If all capacitors reach their saturation limits, this means that the equilibrium point of the circuit is imposed by saturation constraints.

In terms of the state-space representation of the unconstrained electric circuit, the constrained circuit can be obtained by simply setting the row of F to a null vector whenever a saturation point is reached.

C. Discrete time version, unconstrained problem

We now focus on the structure of the dynamics matrix F to obtain a discrete-time version F_d of the same system. F_d is such that $U((m+1)\Delta) = F_d U(m\Delta)$, where $U(m\Delta) = [u_0(m\Delta) \dots u_N(m\Delta)]$ are the state space variables of the system discretized with ZOH-Euler integration and integration step $\Delta > 0$ ($m \in \mathbb{Z}$):

$$F_d = I + \Delta F$$

The matrix F_d has two unitary eigenvalues, and the others are stable with an adequate choice for capacitors C_i and sufficiently small integration time Δ. F_d is also row-stochastic. This characteristic is very interesting in the perspective of treating this problem as a consensus problem. In our scenario, consensus is reached in a configuration that is a convex combination of the steady eigenvectors, that in our system are associated to the voltage values u_0 and u_N. It is important to remark that the only equilibrium points of this discrete-time systems is exactly the same of the continuous-time system.

D. Discrete time version, constrained problem and camera analogy

Before adding more elements to the scenario, let us see how the idea of this physical system can be used to reach optimal patrolling section consensus for our system of cameras. Suppose to identify $u_i = a_i,\ i = 0...N$ and $R_i = V_{i,max}$. This implies that $u_N - u_0 = L_{tot}$. If saturation limits are not strict, the equilibrium point for the electrical system is such that the optimal $A_i = [a_{i-1}, a_i]$ and

$$I_{tot} = \frac{u_N - u_0}{\sum_{j=1}^{N} R_j},\ I_i = \frac{u_i - u_{i-1}}{R_i} = I_{tot}\ \forall i$$

$$u_i - u_{i-1} = I_{tot} R_i = \frac{L_{tot}}{\sum_{j=1}^{N} V_{j,max}} V_{i,max} = a_i - a_{i-1}.$$

If saturation limits are reached, the electric circuit splits at saturation points. In our multi-camera system, that is equivalent to calculating optimal constrained patrolling time. If patrolling sections reach physical limits, the optimal patrolling time is the same for cameras between two saturated patrolling extremes, and the system converges to the optimal constrained solution. Cameras that share the same patrolling period are the equivalent of resistors subject to the same current. This analogy shows that if cameras calculate the equilibrium points for the proposed electrical system, they obtain their optimal patrolling range. This can be performed with communication only between adjacent agents: the i-th camera is ideally represented by the i-th resistor, which knows the voltage between its ends, and can calculate the current flowing through it. Resistors (cameras) then communicate the current flowing through them, and if they adequately compute the current difference at nodes and apply it to the system composed by capacitor with saturation diodes (ideally modeling the optimal patrolling extreme), the equilibrium point is reached with only local interactions. Moreover, the convergence speed is influenced by the choice of the C_i values, acting as consensus weights.

As compared to the electric circuit analogy, the camera networks present a peculiar different since it is not clear *which* of the two neighboring cameras must calculate position of the common patrolling boundary. We solve this problem by adopting a redundant strategy, that however fits well a real-world scenario: each camera calculates positions of *both* patrolling extremes, so each limit is modeled twice. It is then important that the two versions of the shared patrolling extremes are consistent in neighboring cameras. This is achieved by periodically comparing the values of the extremes, which are eventually reset to a mean value if consensus has been lost. However, this scenario should occur only at the system initialization: if communication is bidirectional (as we assume), cameras can apply simultaneously the same control law to the shared patrolling extremes, and consensus on the patrolling extremes is never lost.

As for stability, one can say that the constrained discrete model will still converge, based on the fact that we have a discretization of a stable continuous system and by choosing Δ such that $\frac{\Delta}{C_i}$ is "small enough" $\forall i$, stability would still be preserved. However, a Lyapunov function can be obtained also for the discrete-time system, on the same line of the previous derivations, and convergence can be readily verified.

E. Asynchronous problem

In a real scenario, communication can often be asynchronous. We assume that when adjacent cameras communicate, they do it in a bidirectional fashion. In this way, if cameras i and $i+1$ separated by patrolling extreme a_i communicate at time instants in $\mathcal{T}_i = \{t_{i,1}, t_{i,2}, \dots t_{i,M_i}\}$ where $t_{i,j} < t_{i,j+1}$, at any j-th instant of communication they can calculate how to update the position of their common optimal patrolling extreme a_i adopting the same control law to determinate its successive position a_i^+. The law is defined as a linear combination:

$$a_i^+(t_{i,j}) = k_{i-1} a_{i-1}(t_{i,j}) + k_i a_i(t_{i,j}) + k_{i+1} a_{i+1}(t_{i,j}), \quad (28)$$

where the coefficients are computed so as to balance between the different $V_{i,max}$ of the two adjacent agents, similarly to the voltage divider weights:

$$k_{i-1} = \frac{\Delta}{C_i R_i} \quad k_i = 1 - \frac{\Delta(R_i + R_{i+1})}{C_i R_i R_{i+1}} \quad k_{i+1} = \frac{\Delta}{C_i R_{i+1}}$$

As previously stated, $a_i(\cdot)$ have here the same role of the $u_i(\cdot)$ variables, but the control law is only applied at time instants in $\mathcal{T}_i$ and not at every discrete time instant $m\Delta$ as it is for the discrete-time version model of the initial electric circuit.

With these positions, the proof of convergence is more difficult. However simulations show that the following function of $\{a_0(t), \dots, a_N(t)\}$

$$\bar{W}(a_0(t), \dots, a_N(t)) = \sum_{i=1}^{N} \frac{1}{2R_i} (a_i(t) - a_{i-1}(t))^2$$

decreases in time even with the asynchronous update law expressed by (28).

Remarkably, the performed simulations confirm equilibrium stability and global convergence even with asynchronous communication between cameras and with random time delay between two consecutive communications.

V. Tracking

In this section we address the problem of tracking a mobile target once it is detected. A common approach is to model the unknown motion of the target by a second-order linear system driven by white noise. A state-space representation of this model is given by the following equations:

$$\begin{aligned} x(t+1) &= \begin{bmatrix} 1 & 1 \\ 0 & \alpha \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ \beta & 0 \end{bmatrix} \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & \gamma \end{bmatrix} \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} \end{aligned} \quad (29)$$

where $\alpha \approx 1$ to model the inertia of the target, β, γ regulate the intensity of process and measurement noise, respectively, and $n_1(t), n_2(t)$ are uncorrelated white noise

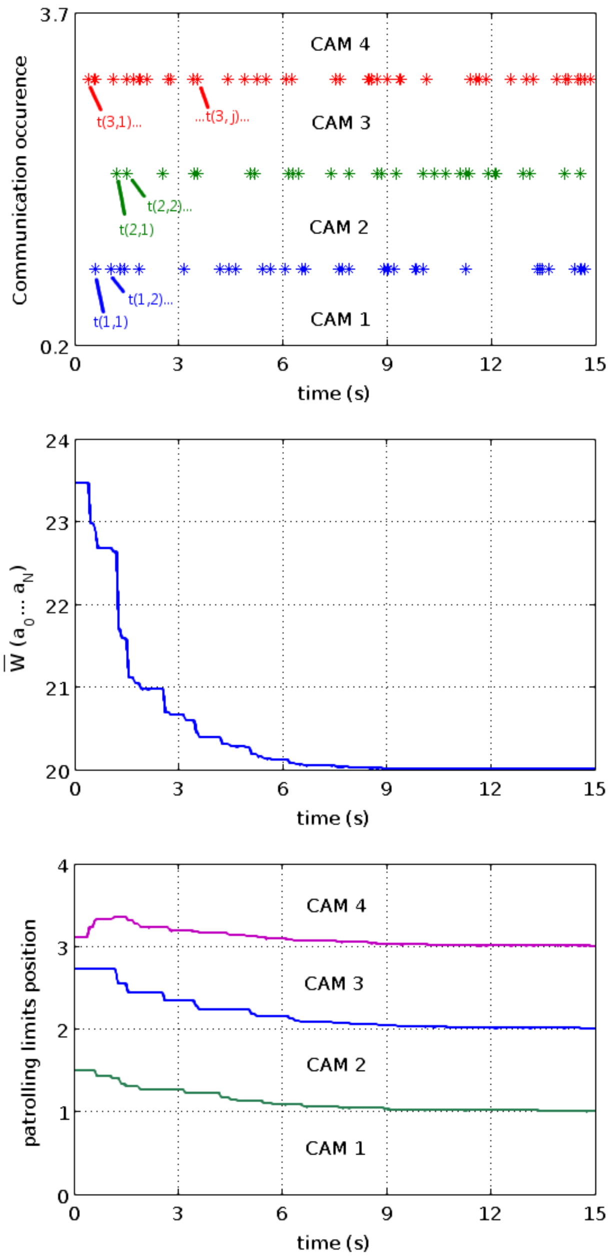

Fig. 6. Asynchronous communication pattern between cameras (*top*), and corresponding global Lyapunov-like function (*center*) and patrolling boundaries (*bottom*) for 4 cameras.

with unit variance. Parameters α, β, γ are set in order to match the statistical information about target motion. Based on this model, each camera processes measurements with a Kalman filter which provides an estimate of the current target position and speed. The equations for the Kalman filter are standard and are therefore omitted. Although tracking based on a Kalman filter is rather standard, in this setting there is an interesting trade-off between patrolling and tracking: if a camera is tracking, it cannot patrol its area any more. However, tracking needs to be given higher priority over patrolling since a target should never be lost, even when it passes from a camera's f.o.v. to that of a neighboring one. Nonetheless, patrolling of the regions of competence of the tracking cameras should be inherited by neighboring cameras as much as possible to detect future events. The strategy we propose to cope with coordinated tracking and adaptive patrolling is the following:

1) the tracking camera produces a higher current on the equivalent electric model to its boundaries in order to attract towards itself the coverage boundaries of its neighboring cameras within their physical limits, thus promoting patrolling of regions left un-monitored by the tracking cameras;
2) each camera tracks a target till it reaches its physical boundaries, i.e. tracking has always higher priority over patrolling;
3) each tracking camera calculates the estimated time-of-arrival for the target to reach the physical boundaries, and passes this information to its neighboring cameras;
4) the neighboring camera, according to estimated time-of-arrival received by the tracking camera, stops patrolling and starts moving towards the boundary to catch the arriving target only when the expected time-of-arrival of a target becomes smaller than the time required for the camera to reach the boundary.

This strategy guarantees that the target is never lost, as the cameras choose in the best way if they have to move towards the boundaries or otherwise keep on patrolling. Moreover, the update of the optimal patrolling boundaries takes place on-line and distributively according to the theory developed in the previous sections, thus providing very good performance.

VI. Simulations

The system described above has been implemented and tested in very general conditions. Communication has been modeled as asynchronous and real-world parameters for cameras have been taken in account. Some of these parameters include maximum patrolling speed, limited field of view, and the overlapping patrolling range between adjacent cameras.

In the top plot of Fig. 6, we show an example of asynchronous communication pattern between cameras, and the associated Lyapunov function $\overline{W}(a_0(t), ..., a_N(t))$ that is monotonically decreasing with time. As expected, the optimal patrolling range division is asymptotically reached.

Next, camera motion between patrolling extremes is shown in top plot of Fig. 7. Every camera is represented by two parallel lines (representing the field of view) that move up and down the patrolling range. The physical range of every camera is delimited by two dashed lines of the same color. Optimal patrolling range limits are instead represented by thick black lines. As expected, patrolling extremes converge to a configuration in which all cameras have the same patrolling time. The fact that cameras in this simulation have different maximum patrolling speed is correctly taken into account. Note that all saw-tooth camera trajectories have the same period, which is a necessary condition for optimality.

Then, in middle plot of Fig. 7 we show an example of camera lineup where physical coverage limits of camera $\mathcal{A}_2$ induce a constraint in the optimal patrolling of the other

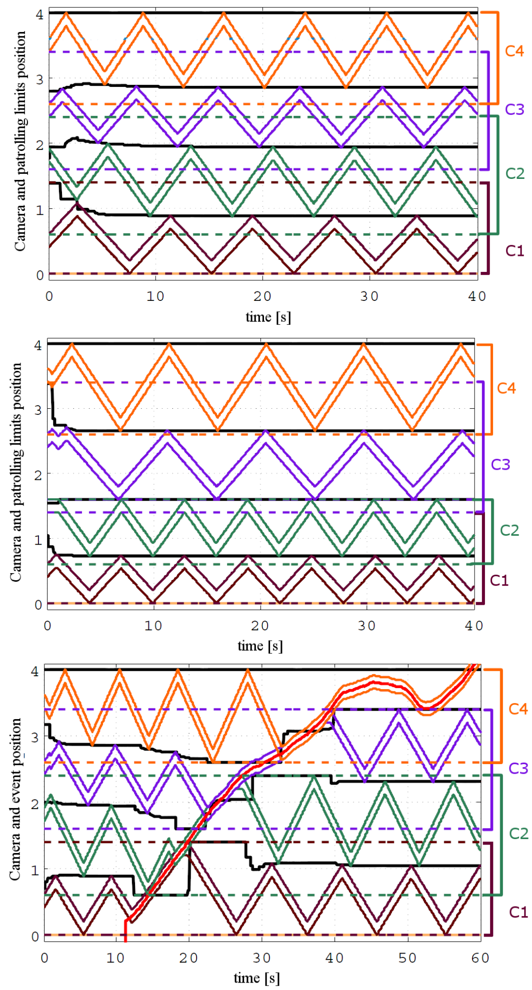

Fig. 7. Camera motion between unconstrained (*top*) and constrained (*center*) optimal patrolling limits. Concurrent camera motion and event tracking (*bottom*).

cameras. The optimal configuration is such that camera $\mathcal{A}_1$ and camera $\mathcal{A}_2$ have the same patrolling period, i.e. $T_{\mathcal{A}_1} = T_{\mathcal{A}_2}$, and so do camera $\mathcal{A}_3$ and camera $\mathcal{A}_4$, $T_{\mathcal{A}_3} = T_{\mathcal{A}_4}$, but these are different , i.e. $T_{\mathcal{A}_2} \neq T_{\mathcal{A}_3}$. Basically, when a camera reaches one of its physical coverage limits, it splits the perimeter patrolling problem into two separated and independent patrolling problems.

Finally, we demonstrate the ability of the system to correctly locate and track a moving target in the bottom plot of Fig. 7. The use of a Kalman filter and a good time-of-arrival estimate ensures that the target is never lost. In addition to that, the great advantage of an on-line calculation of the optimal patrolling limits is evident: patrolling limits correctly redistribute when the target passes from one camera to another one. When one camera is engaged in tracking, the others extend their patrolling ranges in the overlapping zone. In this way patrolling coverage is maximized when one agent is "distracted" from the patrolling task. This can also be useful in case one of the cameras stops working, and a self-adjustment of the system is needed in order to mitigate the detrimental effects of the failure.

VII. Conclusions

In this work we address the problem of optimally patrolling a one-dimensional perimeter using only local controllers by means of an analogy with a passive electric network. This analogy provides an intuitive proof of optimality even in presence of mobility constraints. Its implementation proves to be very robust also in a discrete time setting where communication among cameras can be asynchronous or subject to packet loss. We also propose a Kalman-based adaptive target tracking algorithm that provides continuous target locking and optimal perimeter patrolling for the cameras not involved in tracking. The proposed solution naturally extends to real problems such as the coverage of closed perimeters, of production line, or of roads with crossings. Future work involves the formalization of the proposed algorithms and their proofs of convergence to the optimal solution in the asynchronous discrete time implementation, as the numerical simulations seem to support.

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