The Consensus algorithm in distributed multi-agent systems

M.A.g.I.C.
Multi Agent Intelligent Control

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University of Padova

17 November 2011, Seoul

DEPARTMENT OF INFORMATION ENGINEERING
UNIVERSITY OF PADOVA

University of Padova

- Founded 1222: 2nd oldest university
- 60K students out of 200K citizens
- First graduate woman in 1678
- Alumni: Galileo, Copernicus, Riccati, Bernoulli
- Department of Information Engineering (EE&CS) 3K students

Networked Control Systems Group in Padova

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Angelo Cenedese


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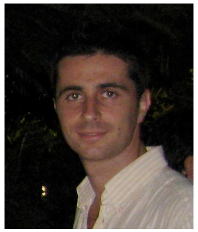
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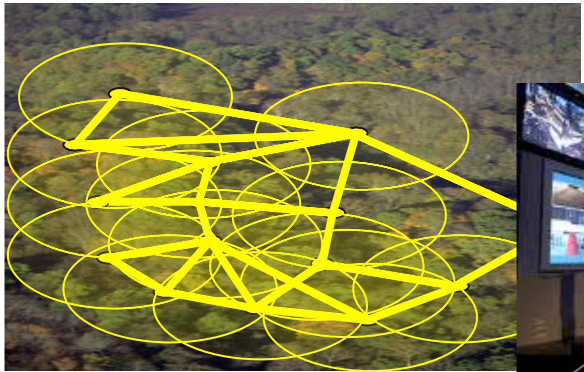
Enrico Lovisari

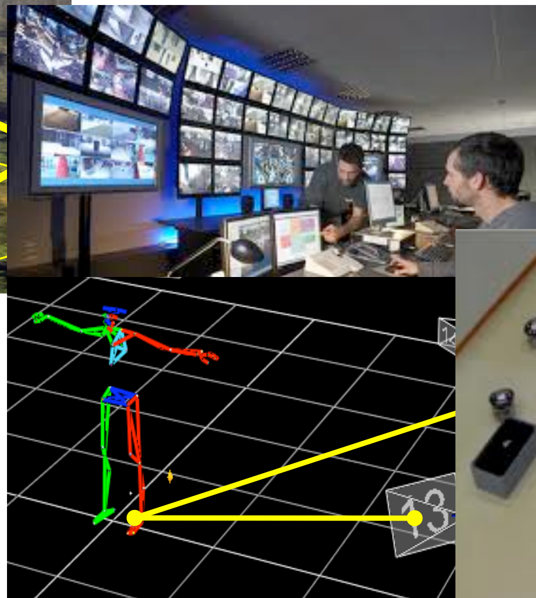

Filippo Zanella

NCS applications at Padova: MAgIC Lab

Wireless Sensor Actuator Networks



Smart Camera Networks



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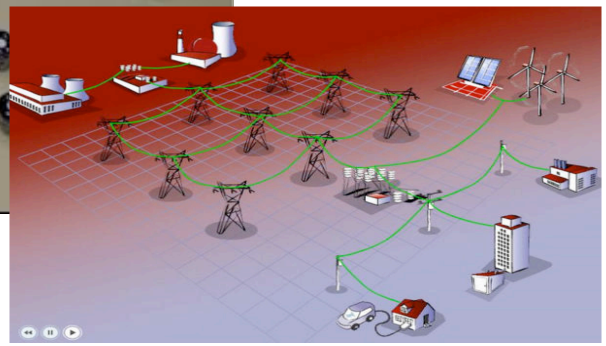
Robotic Networks

Smart Energy Grids



NCSs: physically distributed dynamical systems interconnected by a communication network

Outline

- Motivations and target applications
- Overview of consensus algorithms
- Application of consensus:
  - Sensor calibration in WSN
  - Clock-synchronization in WSN
  - Cooperative map-building in Robotic Networks
  - Perimeter patrolling in camera networks
- Open vistas and conclusions

Outline

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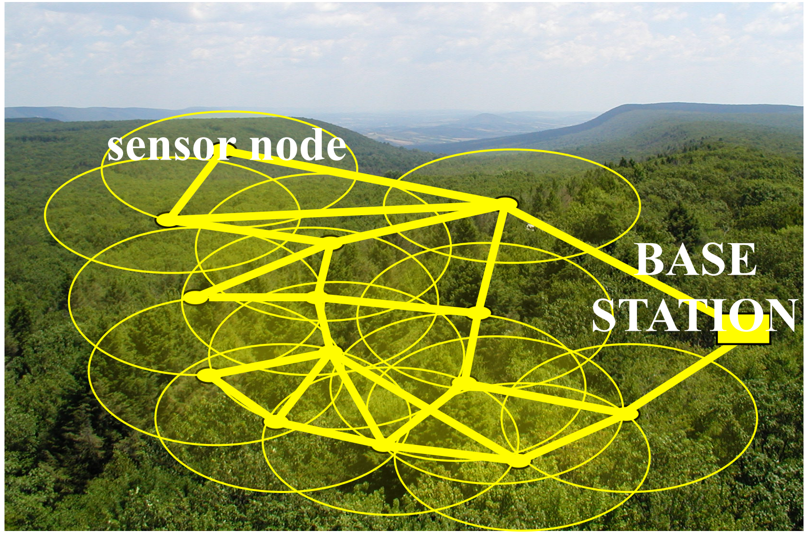
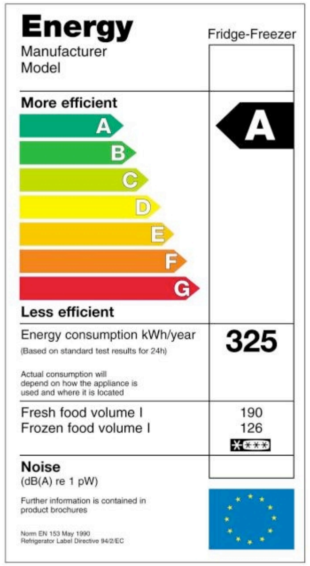
Wireless Sensor Actuator Networks (WSANs)

- Small devices
  - μController, Memory
  - Wireless radio
  - Sensors & Actuators
  - Batteries
- Inexpensive
- Multi-hop communication
- Programmable (micro-PC)
- Self-configuring

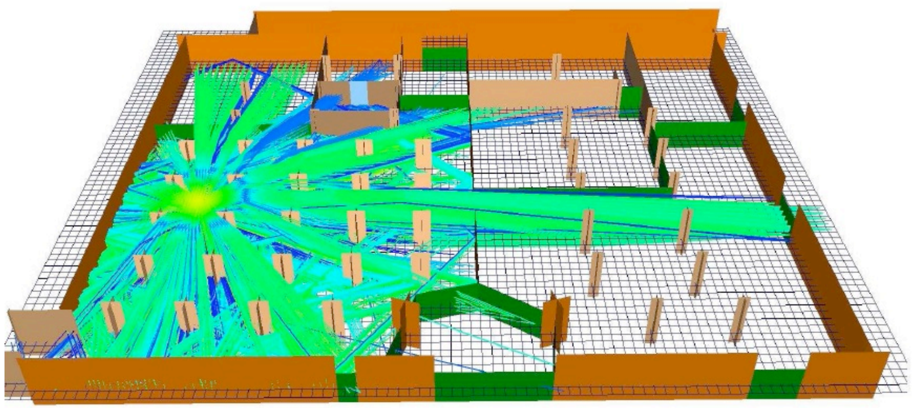


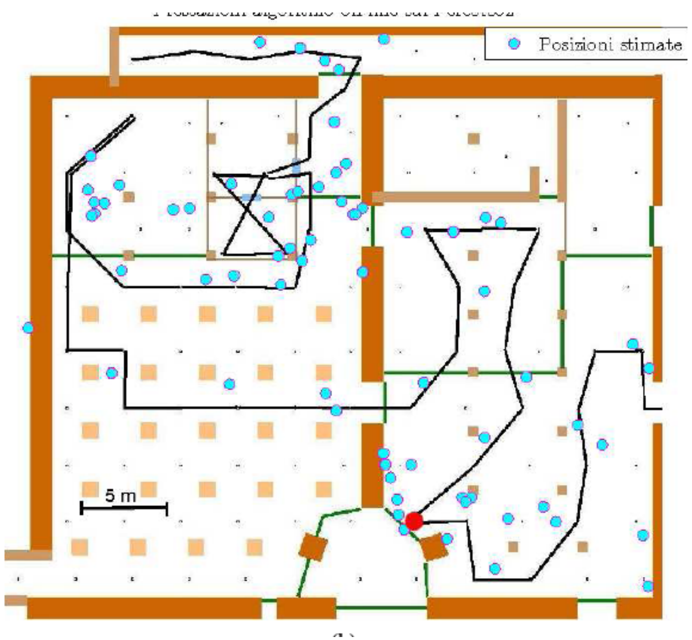
WSAN applications: Smart Buildings & greenhouses



- Building thermodynamics model identification
- Sensor selection for identification
- Optimal sensor placement
- Optimal control
- Energy efficiency certification

WSAN applications:
RF Localization & Tracking



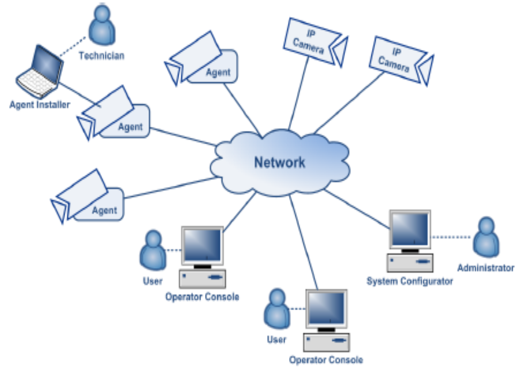


- Indoor radio signal modeling
- Real-time localization
- Distributed tracking
- Coordination

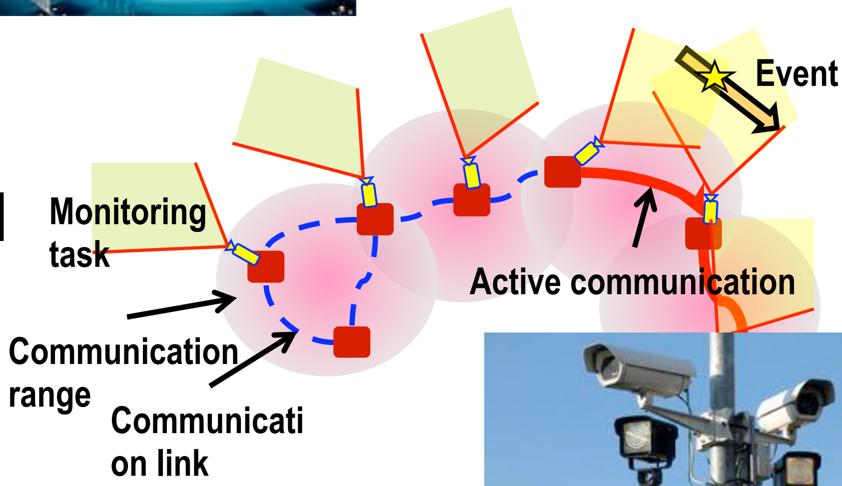
Smart camera networks apps: surveillance systems

today

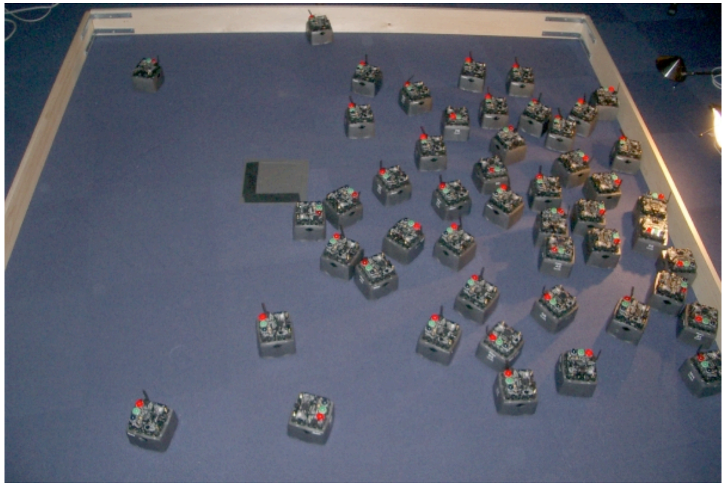
tomorrow

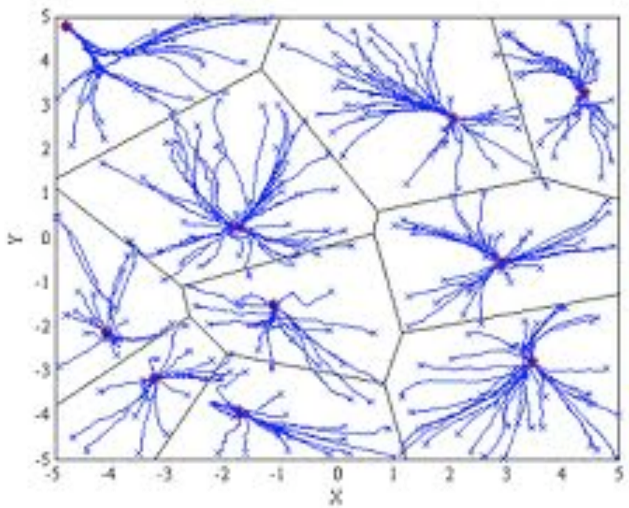


- Distributed camera calibration
- Real-time adaptive patrolling
- Cooperative event detection and tracking
- Distributed fault detection and compensation
- Virtual world navigation

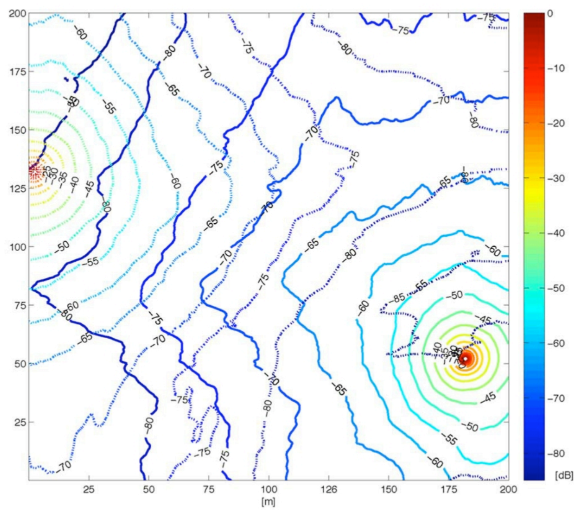


Robotic networks apps: exploration & map-building

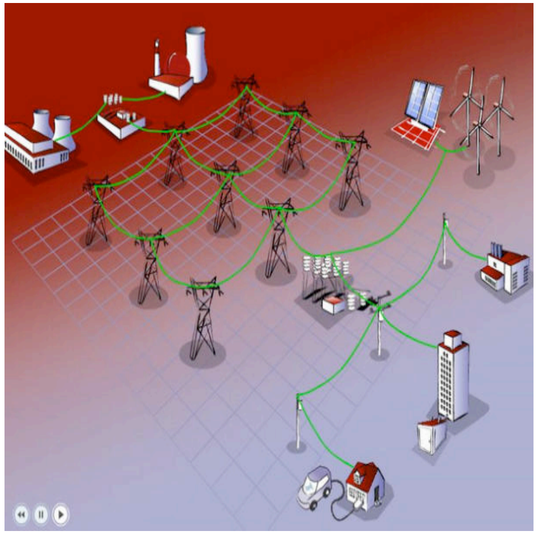


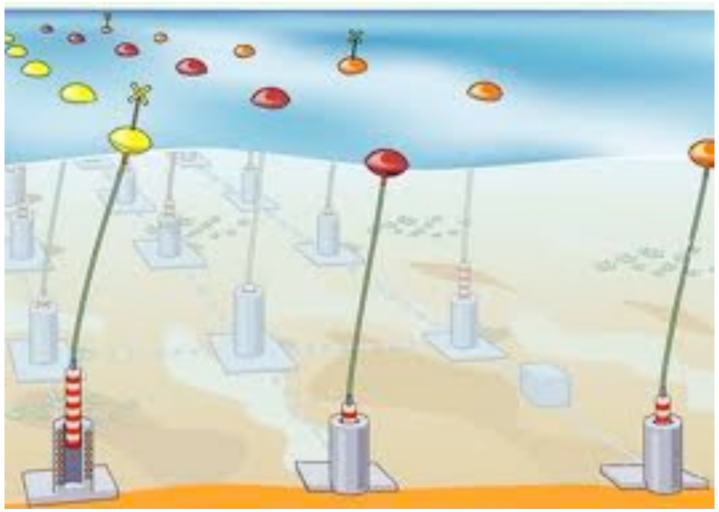


- Optimal patrolling
- Optimal coverage
- Distributed sensing
- Collaborative map-building
- Adaptive navigation



Smart Power Grids and Renewable energies

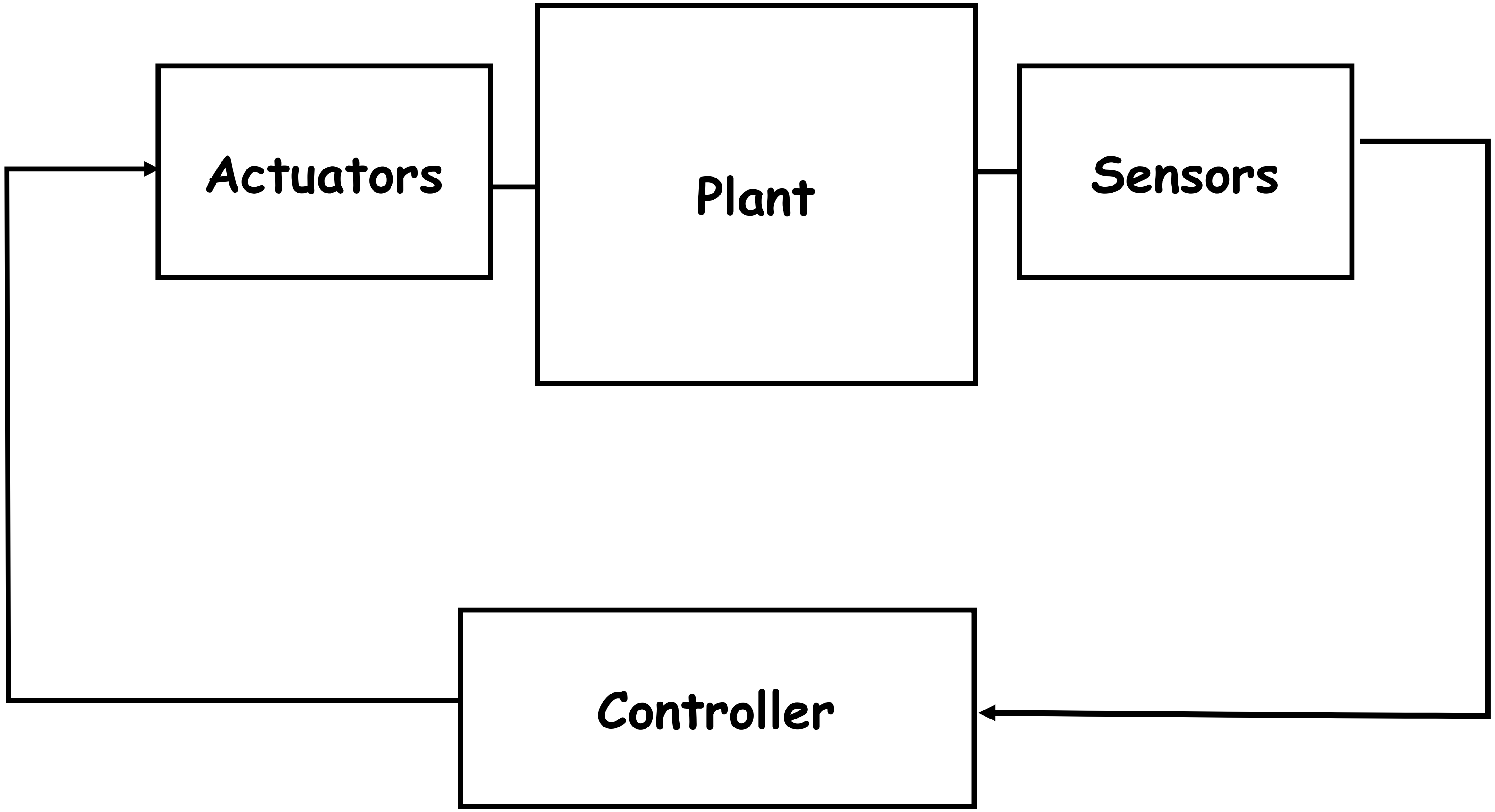




- Foreseeable future
  - Many consumers & producers
  - Cooperation vs greedy behavior
  - Network topology not known and dynamic
  - Need for distributed estimation and control

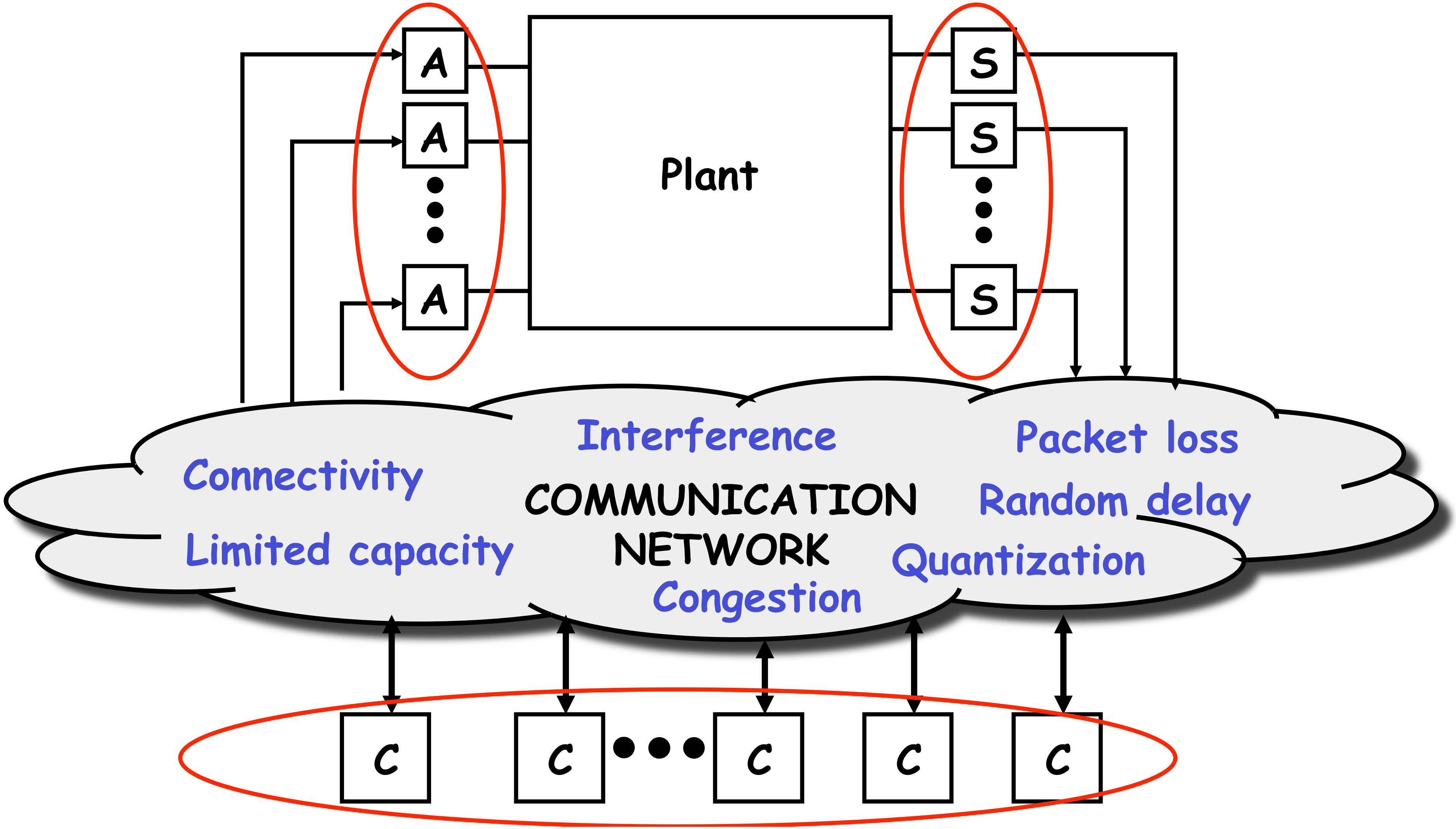
NCSs: what's new for control?

Classical architecture: Centralized structure



NCSs: what's new for control?

NCSs: Large scale distributed structure



M.A.g.I.C. Multi Agent Intelligent Control

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Outline

- Motivations and target applications
- Overview of consensus algorithms
- Application of consensus to WSN:
  - Sensor calibration
  - Least-square parameter identification
  - Time-synchronization
  - Distributed Kalman filtering
- Open vistas and conclusions

The consensus problem

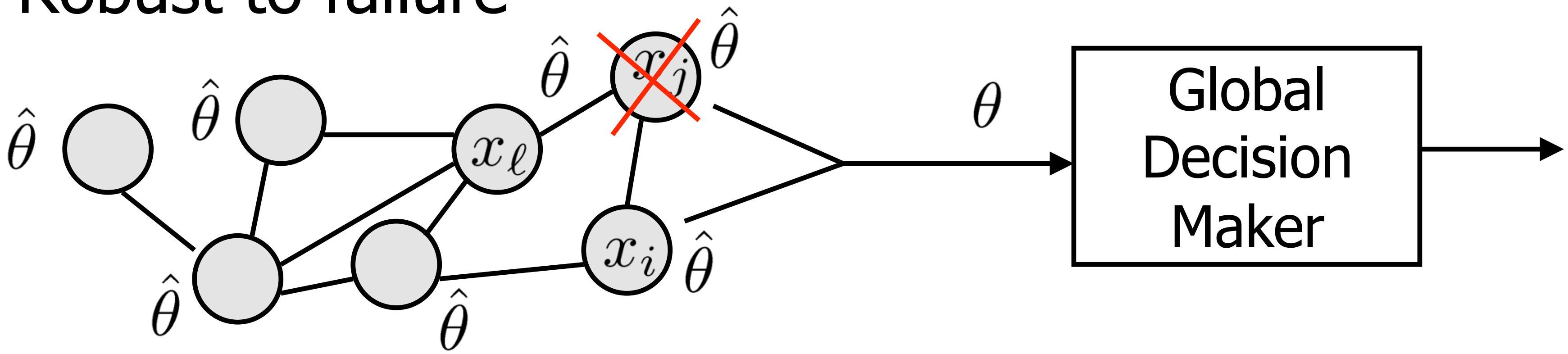
- Main idea
  - Having a set of agents to agree upon a certain value (usually global function) using only local information exchange (local interaction)
- Also known as:
  - Agreement problem (economics, signal processing, social networks)
  - Gossip algorithms (CS & communications)
  - Synchronization (statistical mechanics)
  - Rendezvous and flocking (robotics)

Main features

- Distributed computation of general functions

$$\theta = f(x_1, \ldots, x_N) = f\left(\frac{1}{N}\sum_{i=1}^{N} g_i(x_i)\right) \qquad \left(\text{ex. } \theta = \frac{1}{N}\sum_{i=1}^{N} x_i \text{ for } f = g = ident\right)$$

- Computational efficient (linear & asynchronous)
- Independent of graph topology
- Incremental (i.e. anytime)
- Robust to failure



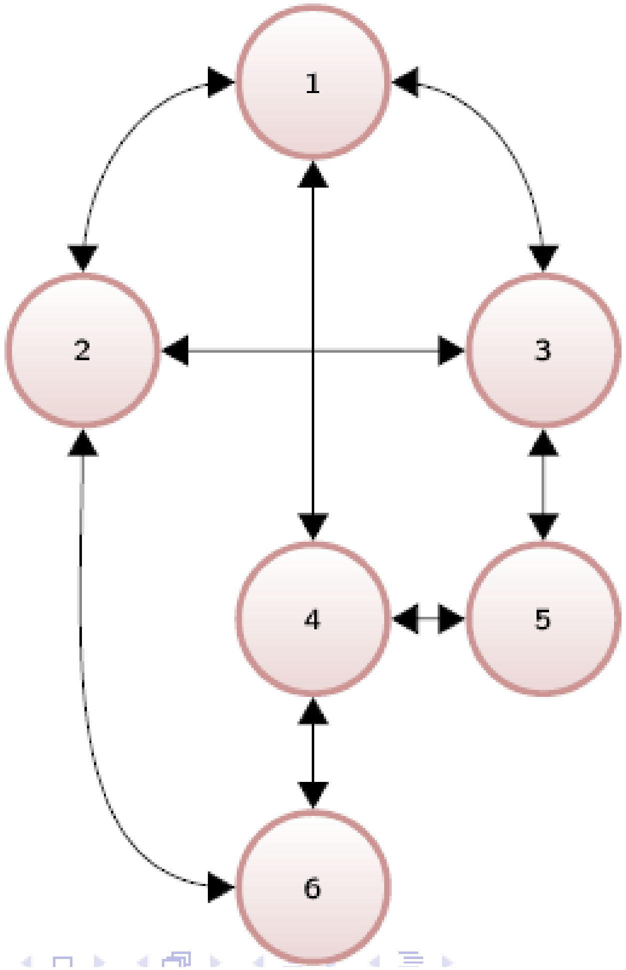
Some history (in control)

- Convergence of Markov Chains (60's) and Parallel Computation (70's)
- John Tsitsiklis "*Problems in Decentralized Decision Making and Computation*", Ph.D thesis, MIT 1984
- A. Jadbabaie, J. Lin, and A. S. Morse "*Coordination of groups of mobile autonomous agents using nearest neighbor rules*", CDC' 02 (Axelby Best Paper Award TAC)
- Time-varying topologies (worst-case)
  - **L. Moreau**, "*Consensus seeking in multi-agent systems using dynamically changing interaction topologies*," IEEE, Transactions on Automatic Control, vol 50, No. 2, 2005
  - **M. Cao, A. S. Morse, and B. D. O. Anderson**. "*Reaching a Consensus in a Dynamically Changing Environment: A Graphical Approach*." SIAM Journal on Control and Optimization, Feb 2008
- Randomized topologies
  - **S. Boyd, A. Ghosh, B. Prabhakar, D. Shah** "*Randomized Gossip Algorithms*", TIT 2006
  - **F. Fagnani, S. Zampieri**, "*Randomized consensus algorithms over large scale networks*", JSAC 08
- Applications:
  - Vehicle coordination: Jadbabaie, Francis's group, Tanner, …
  - Kalman Filtering: Olfati Saber-Murray, Alighanbari-How, Carli-Chiuso-Schenato-Zampieri
  - Generalized means: Giarre',Cortes
  - Time-synchronization: Solis-P.R. Kumar,Osvlado-Spagnolini, Carli-Chiuso-Schenato-Zampieri
  - WSN sensor calibration and parameter identification: Bolognani-DelFavero-Schenato-Varagnolo

Consensus formulation (1/2)

Network of

- N agents
- Communication graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$
- i-th node neighbors: $\mathcal{N}(i)$
- Every node stores a variable: node i stores x_i.

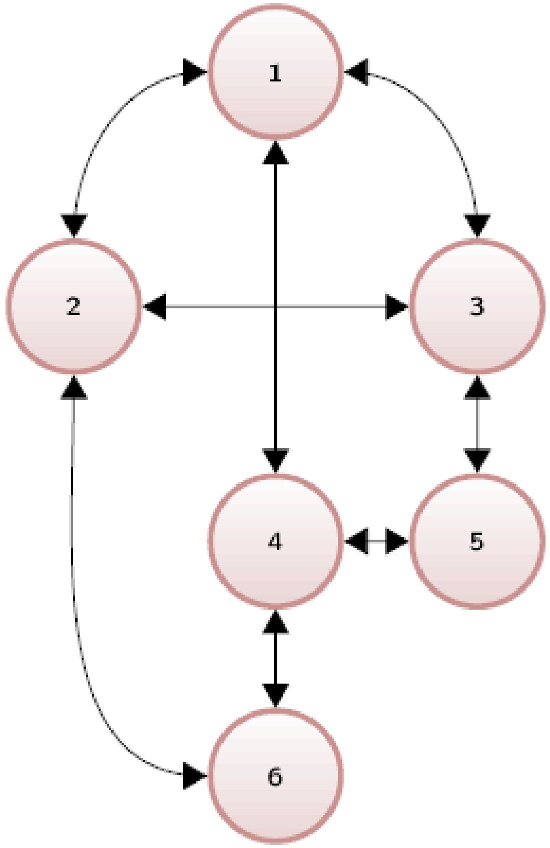


Consensus formulation (2/2)

Definition (Recursive Distributed Algorithm adapted to the graph $\mathcal{G}$)

Any recursive algorithm where the i node's update law of depends only on the state of i and in its neighbors $j \in \mathcal{N}(i)$

$$x_i(t+1) = f(x_i(t), x_{j_1}(t), \dots x_{j_{N_i}}(t))$$
$$\text{with } j_1, \dots, j_{N_i} \in \mathcal{N}(i)$$

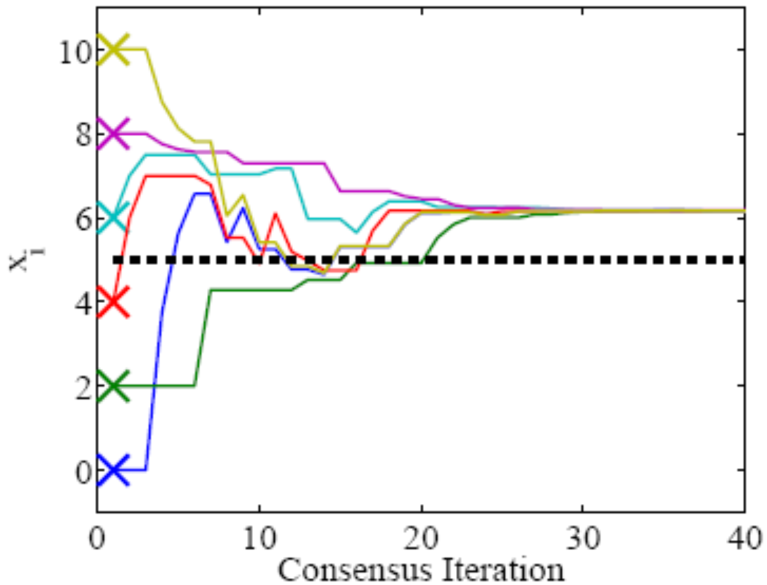


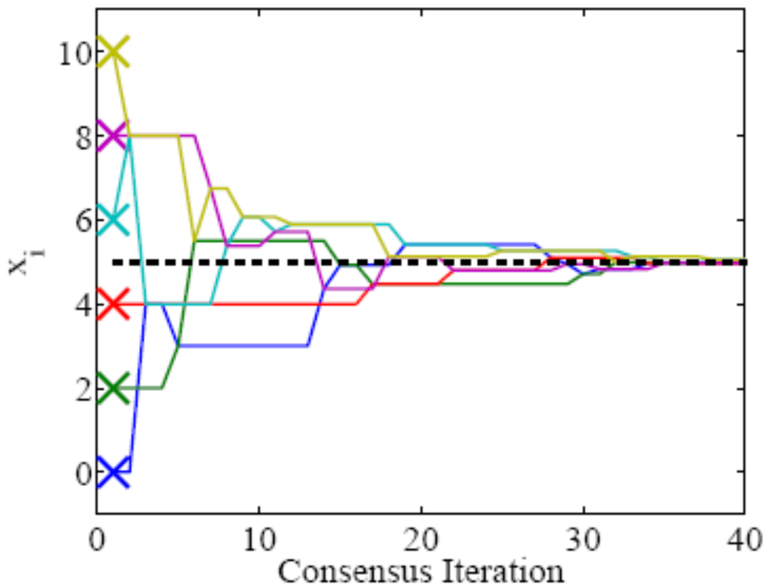
Consensus definitions

Definition

A Recursive Distributed Algorithm adapted to the graph $\mathcal{G}$ is said to asymptotically achieve consensus if

$$x_i(t) \rightarrow \alpha \qquad \forall i \in \mathcal{N}$$





Definition

A Recursive Distributed Algorithm adapted to the graph $\mathcal{G}$ is said to asymptotically achieve *average* consensus if

$$x_i(t) \rightarrow \frac{1}{N} \sum_{i \in \mathcal{N}} x_i(0) \qquad \forall i \in \mathcal{N}$$

Linear consensus

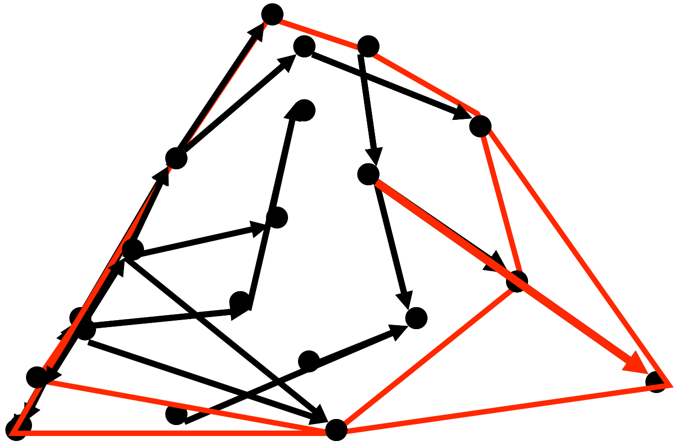
$$x_i(t+1) = p_{ii}x_i(t) + \sum_{j \in \mathcal{N}(i)} p_{ij}x_j(t)$$

$$x(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_N(t) \end{bmatrix} \qquad x(t+1) = P(t)x(t)$$

Say $\mathcal{G}_P$ Graph associated to P, $P_{i,j} \neq 0 \iff (i,j) \in \mathcal{E}_P$,

$$\mathcal{G}_P \subseteq \mathcal{G} \qquad (\mathcal{N} \equiv \mathcal{N}_P, \quad \mathcal{E} \subseteq \mathcal{E}_P)$$

A robotics example: the rendezvous problem



$$x_i(t+1) = x_i(t) + u_i(t)$$
$$x_i(t+1) = p_{ii}x_i(t) + \sum_{j \in N(i)} p_{ij}x_j$$

Convex hull always shrinks.
If communication graph sufficiently connected, then shrinks to a point

Stochastic matrix

Definition (Stochastic Matrix)

If $P_{i,j} \geq 0$ and $\sum_j P_{i,j} = 1 \ \forall i$, than P is said to be stochastic

$$P\mathbb{1} = \mathbb{1} \qquad \mathbb{1} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Remark

If P is stochastic the linear algorithm can be written in both forms:

$$x_i(t+1) = p_{ii}x_i(t) + \sum_{j \in \mathcal{N}(i)} p_{ij}x_j(t)$$

$$x_i(t+1) = x_i(t) + \sum_{j \in \mathcal{N}(i)} p_{ij}\left(x_j(t) - x_i(t)\right)$$

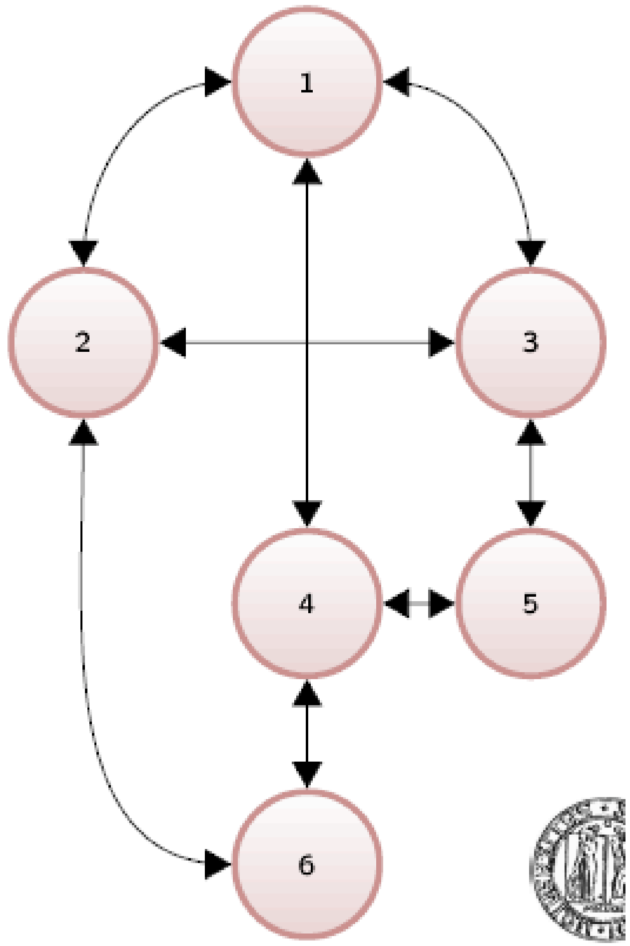
Constant matrix P

Synchronous Communication:

At each time all nodes communicate according to the communication graph

$P(t)=P$:

$$P=\begin{bmatrix} 3/6 & 1/6 & 1/6 & 1/6 & 0 & 0 \\ 1/6 & 3/6 & 1/6 & 0 & 0 & 1/6 \\ 1/6 & 1/6 & 3/6 & 0 & 1/6 & 0 \\ 1/6 & 0 & 0 & 3/6 & 1/6 & 1/6 \\ 0 & 0 & 1/6 & 1/6 & 4/6 & 0 \\ 0 & 1/6 & 0 & 1/6 & 0 & 4/6 \end{bmatrix}$$



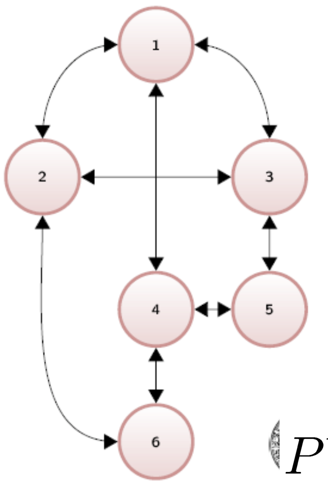
Convergence results

Theorem

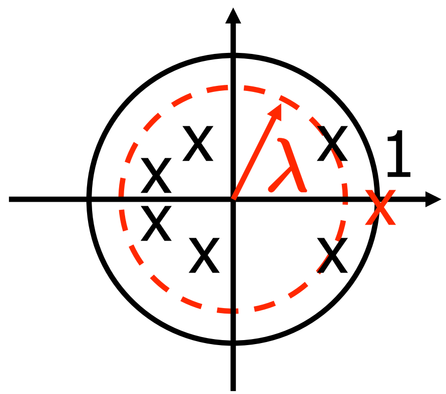
$P(t) = P$ *stochastic.*

- *If P such that $\mathcal{G}_P \subseteq \mathcal{G}$ is rooted then the algorithm achieves consensus*
- *If also P^T is stochastic (P doubly stochastic), then average consensus is achieved*

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & p_{1,3} & p_{1,4} & 0 & 0 \\ p_{2,1} & p_{2,2} & p_{2,3} & 0 & 0 & p_{2,6} \\ p_{3,1} & p_{3,2} & p_{3,3} & 0 & p_{3,5} & 0 \\ p_{4,1} & 0 & 0 & p_{4,4} & p_{4,5} & p_{4,6} \\ 0 & 0 & p_{5,4} & p_{5,3} & p_{5,5} & 0 \\ 0 & p_{6,2} & 0 & p_{6,4} & 0 & p_{6,6} \end{bmatrix}$$



eigenvalues of P:



$P^t \xrightarrow{\lambda^t} \mathbb{1}\rho^T$, ρ is left eigenvector of 1

$\sum_i \rho_i = 1, \rho_i \geq 0, \quad (\rho_i > 0$ if strong. conn.)

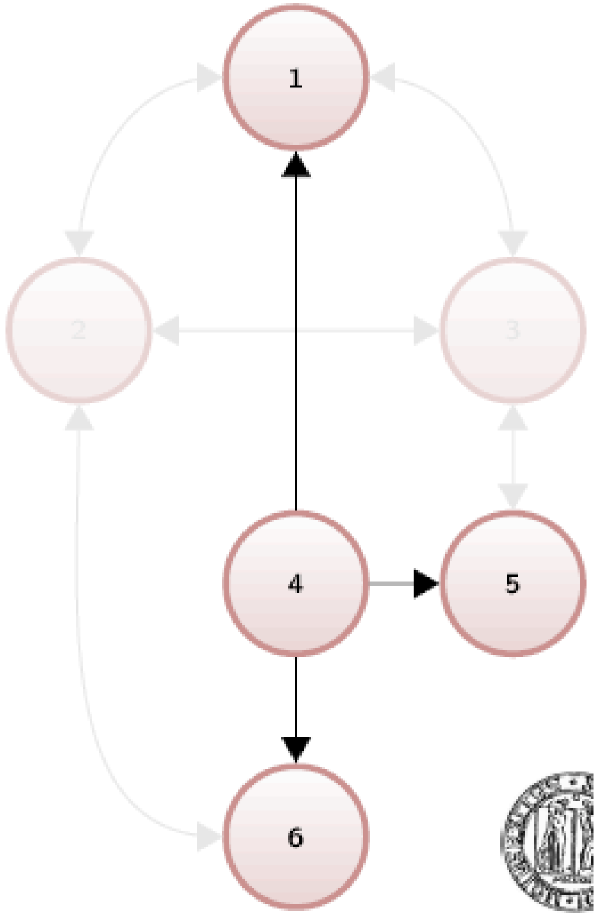
$\rho = \frac{1}{N}\mathbb{1}$ if P doubly stochastic

Time varying P(t): broadcast

Broadcast:

At each time one node randomly wakes up and broadcasts its information to all its neighbors.

$$P(t) = \begin{bmatrix} 3/4 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/4 & 0 & 3/4 \end{bmatrix}$$

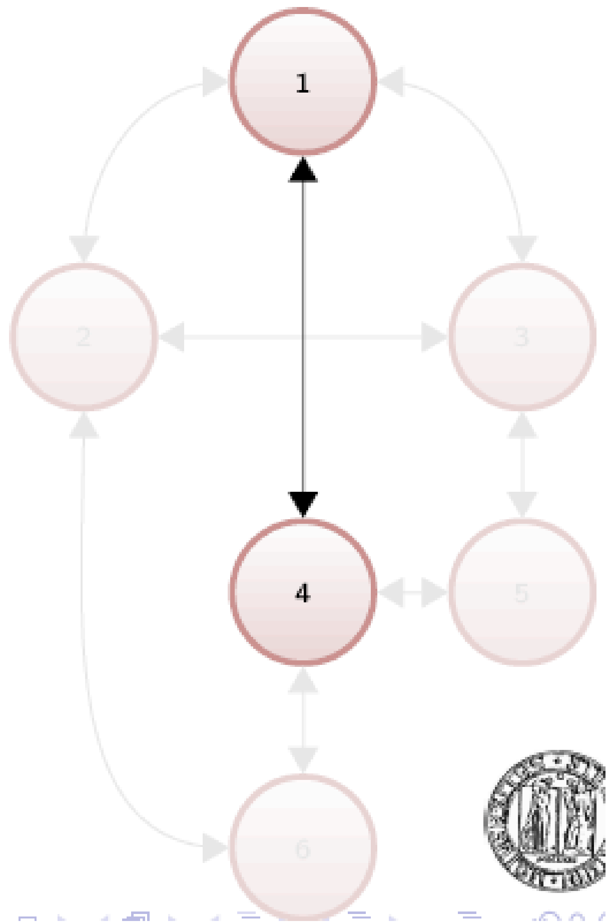


Time varying P(t): symmetric gossip

Symmetric Gossip:

At each time one node randomly wakes up and chose randomly a its neighbor. Those two nodes exchange information

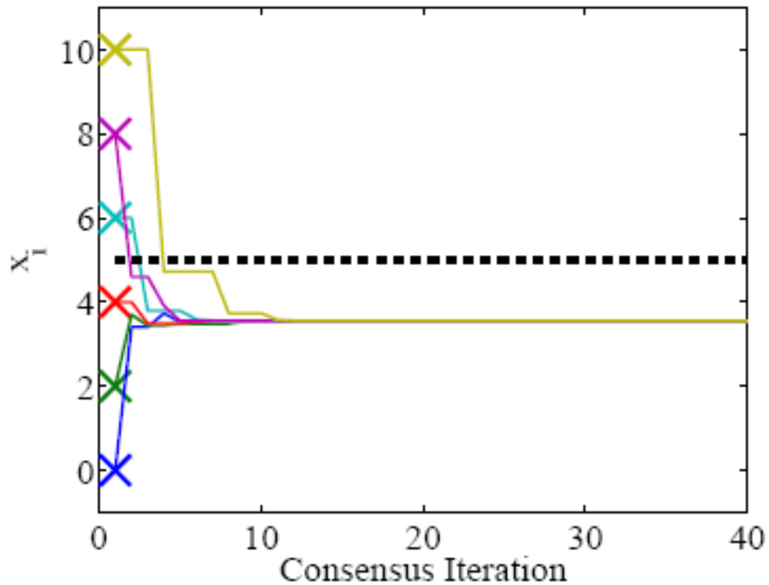
$$P(t) = \begin{bmatrix} 1/2 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

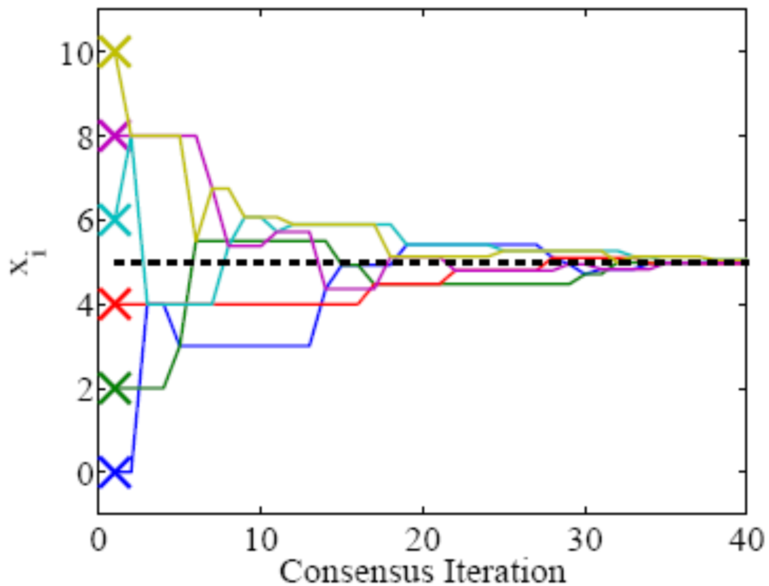


Consensus strategies for WSAN

Broadcast

- 1 message broadcasted, $|\mathcal{N}(i)|$ estimate updated
- Does not guarantee average consensus





Symmetric Gossip

- At least 3 messages exchanged, 2 estimate updated
- Guarantee average consensus

Convergence results: P=P(t) deterministic

Theorem

Suppose that $P_{ii}(t) > 0, \forall i, \forall t$ and that there exixts K such that $\mathcal{G}_\ell = \mathcal{G}_{P((\ell+1)K)} \cup \ldots \cup \mathcal{G}_{P(\ell K)}$ is rooted at some node j for all ℓ then

- the sequence $\{P(t)\}$ achieves consensus
- if also $P^T(t)$ are stochastic for all t, then the sequence $\{P(t)\}$ achieves *average* consensus

Remark:

Estimates of rate of convergence are very conservative (worst case)

L. Moreau, "*Consensus seeking in multi-agent systems using dynamically changing interaction topologies*," IEEE, Transactions on Automatic Control, vol 50, No. 2, 2005

M. Cao, A. S. Morse, and B. D. O. Anderson. "*Reaching a Consensus in a Dynamically Changing Environment: A Graphical Approach*." SIAM Journal on Control and Optimization, Feb 2008

Convergence results: P=P(t) randomized

Theorem

Suppose $\{P(t)\}$ is a sequence of i.i.d. stochastic random matrices. Suppose moreover $\mathcal{G}_{P(t)} \subseteq \mathcal{G}$ $\forall t$ and call $\bar{P} = \mathbb{E}[P]$.

- *If $\mathcal{G}_{\bar{P}}$ is rooted that consensus is achieved w.p.1*
- *If also $P(t)^T$ is stochastic for every t, then average consensus is achieved w.p.1*

Remark:

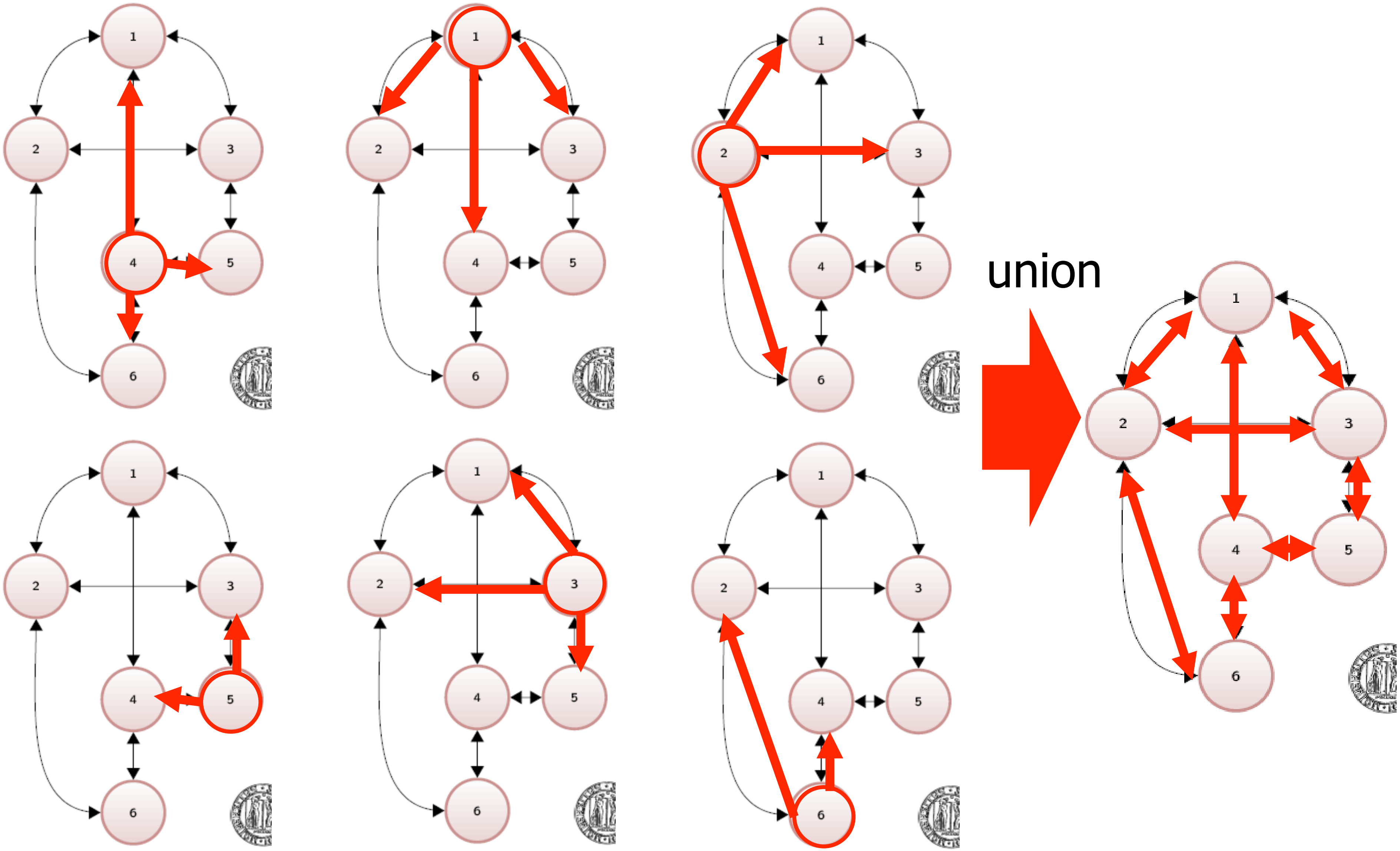
It is not sufficient $\bar{P}$ doubly stochastic to guarantee average consensus

$$x(t+1) = P(t)x(t) = P(t)P(t-1)\cdots P(0)x(0) = Q(t)x(0) \qquad (Q(t) = P^t \text{ if } P(t) = P)$$

$$Q(t) \to \mathbb{1}\rho^T, \ \mathbb{E}[\rho] = \frac{1}{N}\mathbb{1}, \boxed{Var(\rho) \sim \frac{1}{N}}$$

F. Fagnani, S. Zampieri, "*Randomized consensus algorithms over large scale networks*", IEEE Journal on Selected Areas in Communications, 2008

How to apply these results: broadcast communication



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  - Clock synchronization
  - Map-Building
  - Perimeter Patrolling
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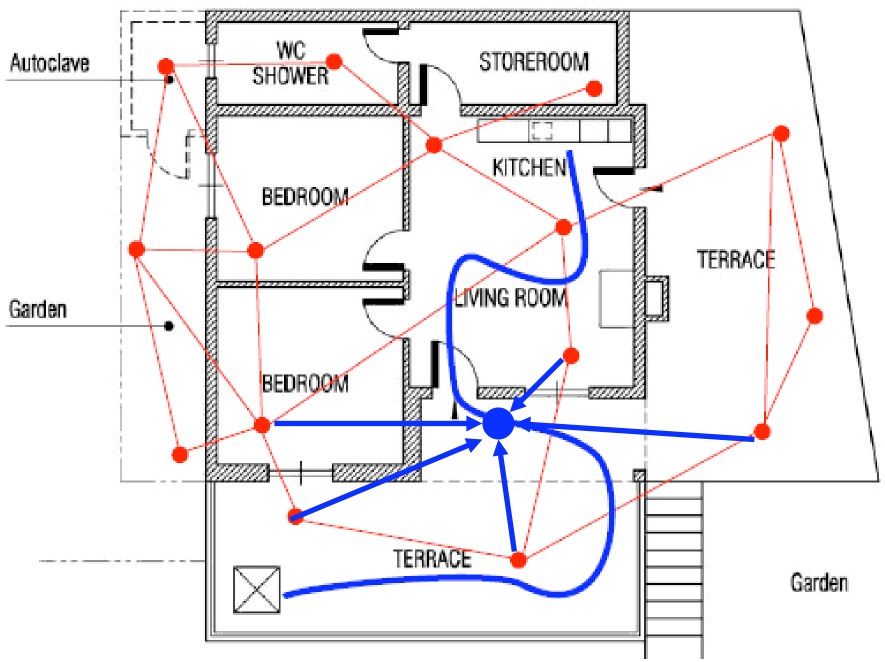
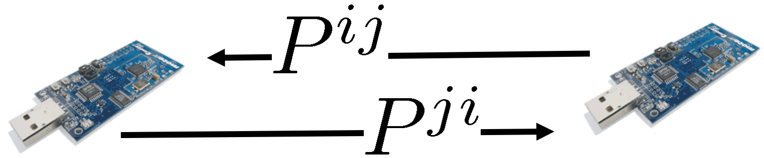
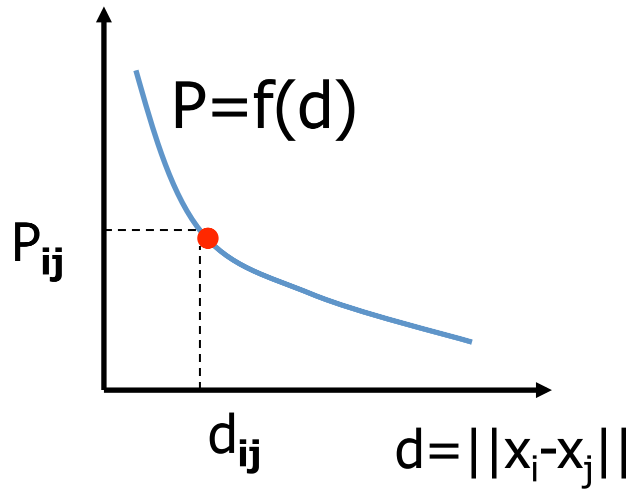
Sensor calibration issues in RF-based localization





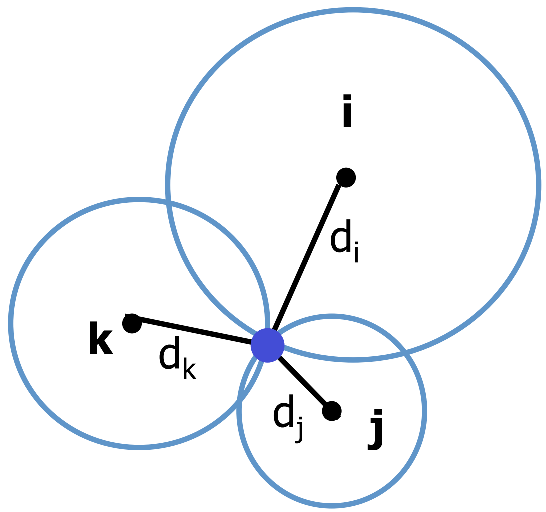


Systematic calibration errors

$$P_{rx}^{ij} = g(x_i, x_j) + o_i$$

$$P_{rx}^{ji} = g(x_j, x_i) + o_j$$

$$g(x_i, x_j) = g(x_j, x_j)$$



WSN sensor calibration

Ideally:

- Estimate o_i: $\hat{o}_i$
- Use $\hat{o}_i$ to compensate the offset: $o_i - \hat{o}_i = 0$

Remember the previous example

$$P_{rx}^{ij} = g(x_i, x_j) + o_i - \hat{o}_i$$

$$P_{rx}^{ij} - P_{rx}^{ji} = o_i - o_j$$

Calibration as consensus problem

Remark

If P is stochastic the linear algorithm can be written in both forms:

$$x_i(t+1) = p_{ii} x_i(t) + \sum_{j \in \mathcal{N}(i)} p_{ij} x_j(t)$$

$$x_i(t+1) = x_i(t) + \sum_{j \in \mathcal{N}(i)} p_{ij} \left(x_j(t) - x_i(t) \right) \qquad o_i - \hat{o}_i(t) = x(t)$$

$$o_i - \hat{o}_i(t+1) = o_i(t) - \hat{o}_i(t) + \tfrac{1}{2}\left((o_i - \hat{o}_i(t)) - (o_j - \hat{o}_j(t)) \right)$$

$$o_i - \hat{o}_i(t+1) = o_i(t) - \hat{o}_i(t) + \tfrac{1}{2}\left((o_i + g(x_i, x_j) - \hat{o}_i(t)) - (o_j + g(x_i, x_j) - \hat{o}_j(t)) \right)$$

$$\hat{o}_i(t+1) = \hat{o}_i(t) - \tfrac{1}{2}\left(P^{ij} - P^{ji} - \hat{o}_i(t) + \hat{o}_j(t) \right)$$

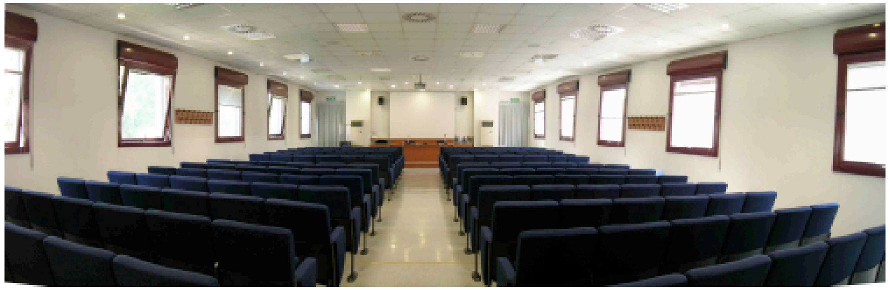
update equation

$$\hat{o}_i(t) \to o_i - \tfrac{1}{N} \textstyle\sum_i o_i = o_i - \alpha \approx o_i$$

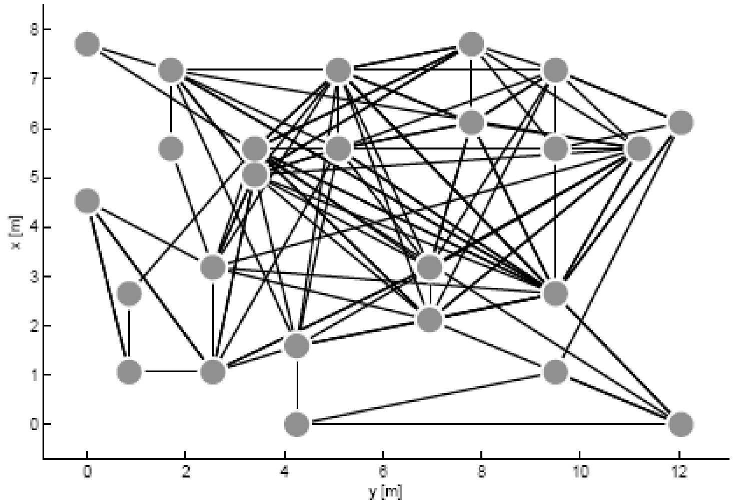
Steady state

Experimental Testbed

25 TMote-Sky nodes with Chipcon CC2420 RF Transceiver randomly placed inside a single conference room:



Network topology and nodes displacement:

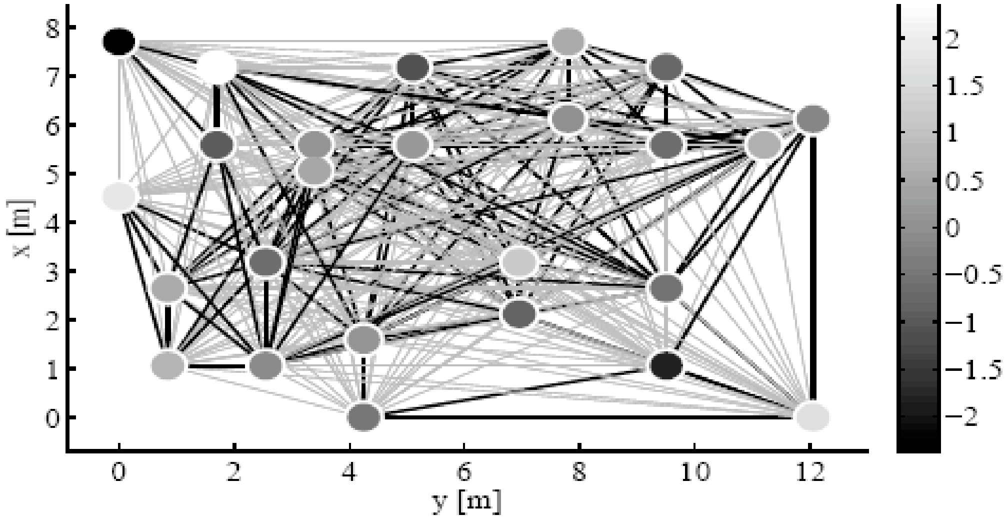


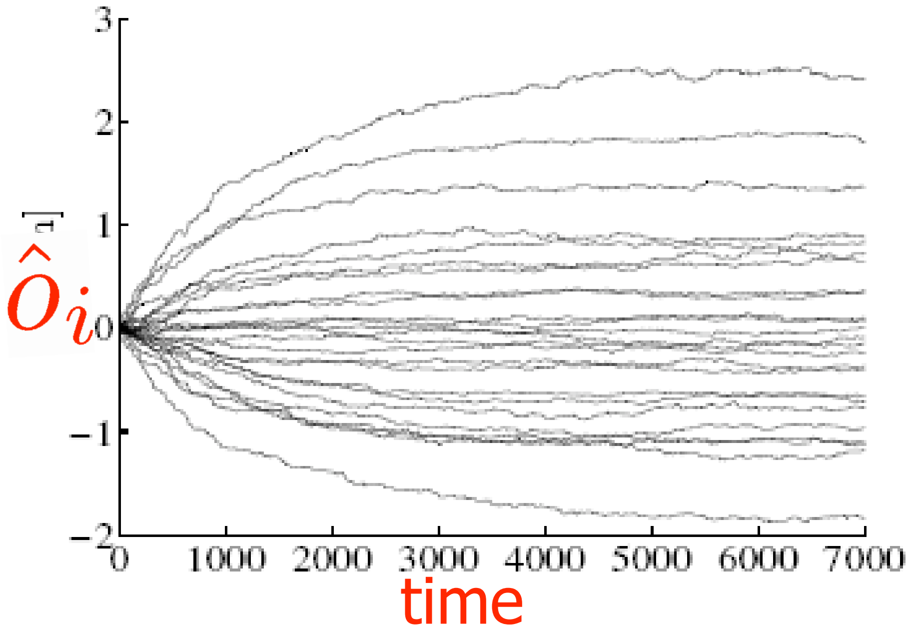
Kept just the links that safely carried the 75% of the sent messages over them

Experimental results

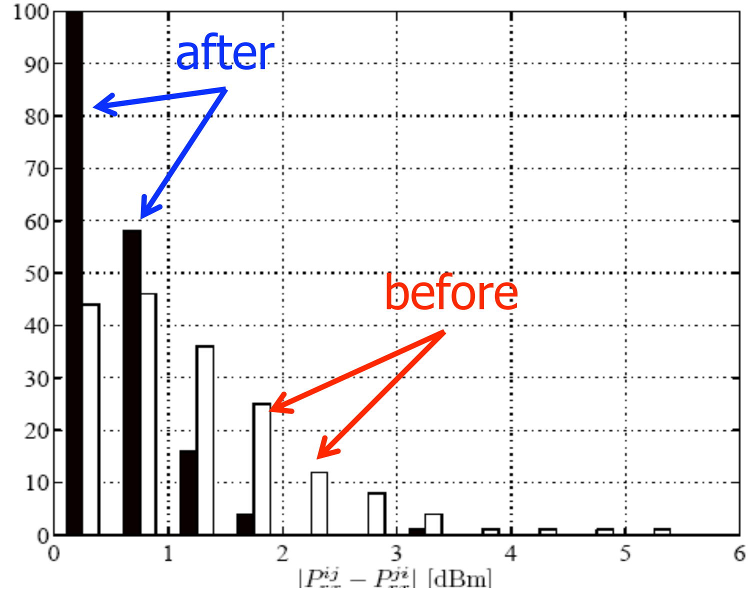
Links divided in 2 categories:

- Training links (black)
- Validation links (gray)





Error distribution

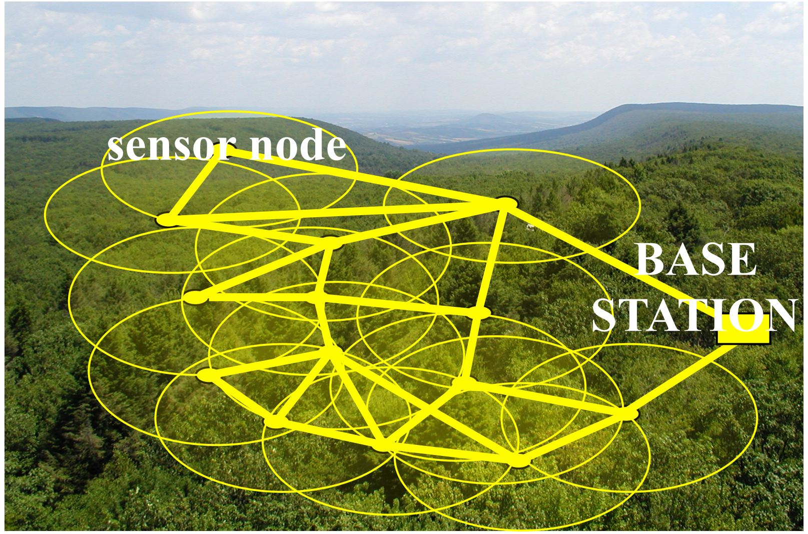


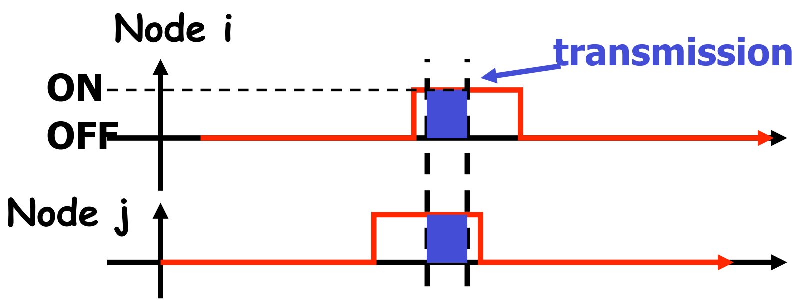
Clock Synchronization in WSN

Cromotherapy

Synchronized sequence of
RGB colors on wireless lamps

Low Power TDMA communication for battery powered nodes





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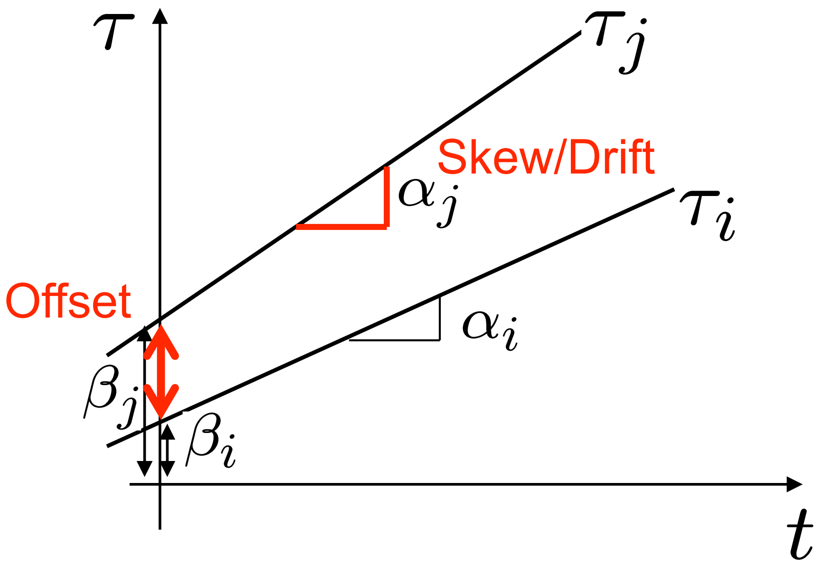
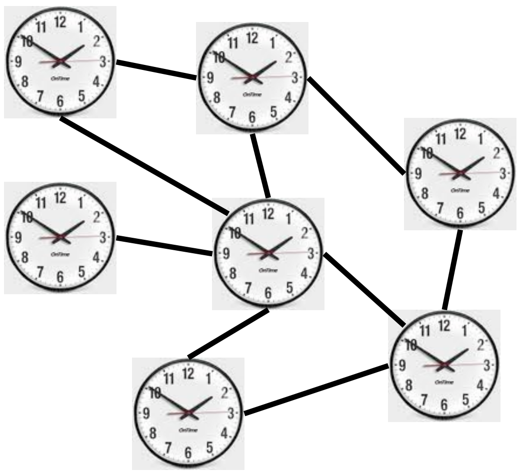
Clock Synchronization (1/2)





Local clocks

$$\tau_i(t) = \alpha_i t + \beta_i \quad i = 1, \dots, N$$

Virtual reference clock

$$\tau^*(t) = \alpha^* t + \beta^*$$

Local clock estimate

$$\hat{\tau}_j(t) = \hat{\alpha}_j \tau_i + \hat{o}_j \quad i = 1, \dots, N$$

$$\hat{\tau}_j(t) = \underbrace{\hat{\alpha}_j \alpha_j}_{\alpha^*} t + \underbrace{\hat{\alpha}_i \beta_i + \hat{o}_j}_{\beta^*}$$

GOAL: find $(\hat{\alpha}_j, \hat{o}_j)$ such that

$\lim_{t \to \infty} \hat{\tau}_i(t) = \tau^*(t), \forall i = 1, .., N$

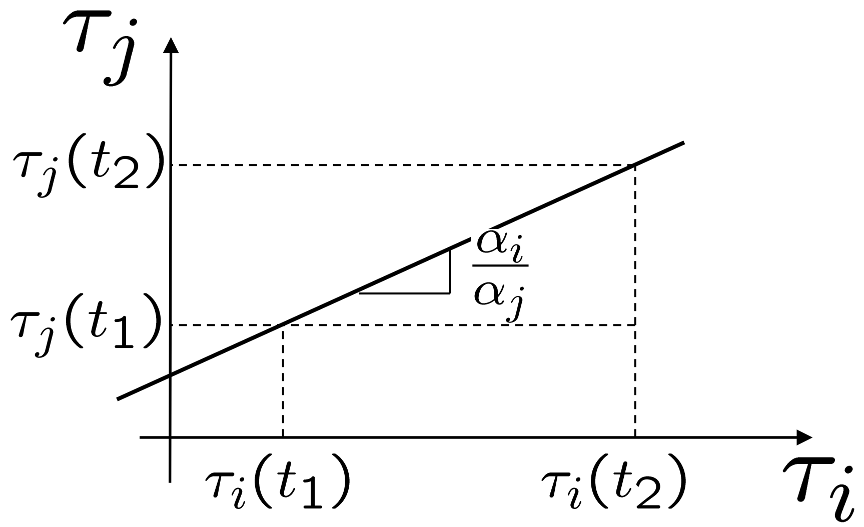
Strategy:

1) set $x_j^\alpha = \alpha_j \hat{\alpha}_j$ and $x_j^\beta = \hat{o}_j + \hat{\alpha}_j \beta_j$ write consensus

2) find update equations for $\hat{\alpha}_j(t)$ and $\hat{o}_j(t)$

3) $\alpha_i \hat{\alpha}_i(t) \to \frac{1}{N} \sum_{i=1}^{N} \alpha_i$ and $\hat{o}_j(t) + \hat{\alpha}_j(t) \beta_j \to \beta^*$

Clock Synchronization (2/2)



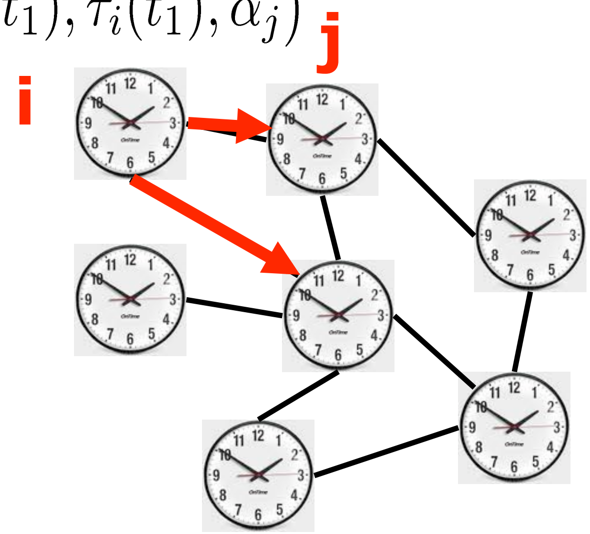
$$\hat{\tau}_j(t) = \overbrace{\hat{\alpha}_j \alpha_j}^{x_j^\alpha} t + \overbrace{\hat{\alpha}_i \beta_i + \hat{o}_j}^{x_j^\beta}$$

$$x_j^\alpha(t^+) = \frac{1}{2} x_j^\alpha(t) + \frac{1}{2} x_i^\alpha(t)$$

$$\hat{\alpha}_j(t^+)\alpha_j = \frac{1}{2}\hat{\alpha}_j(t)\alpha_j + \frac{1}{2}\hat{\alpha}_i(t)\alpha_i$$

$$\hat{\alpha}_j(t^+) = \frac{1}{2}\hat{\alpha}_j(t) + \frac{1}{2}\hat{\alpha}_i(t)\frac{\alpha_i}{\alpha_j}$$

$\left(\tau_i(t_1), \hat{\tau}_i(t_1), \hat{\alpha}_j\right)$



Drift compensation

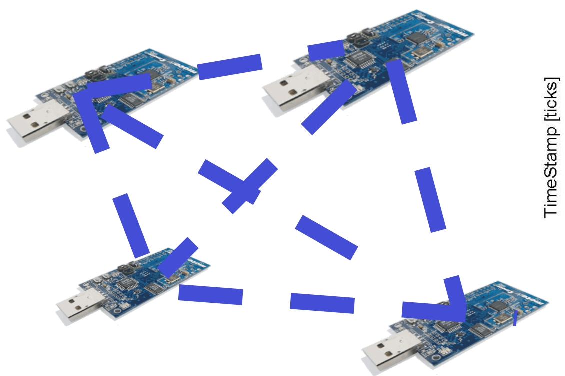
$$\hat{\alpha}_j(t^+) = \frac{1}{2}\hat{\alpha}_j(t) + \frac{1}{2}\hat{\alpha}_i(t)\frac{\tau_i(t_2) - \tau_i(t_1)}{\tau_j(t_2) - \tau_j t_1)}$$

Offset compensation

$$\hat{o}_i^+ = \hat{o}_i + \frac{1}{2}(\hat{\tau}_j - \hat{\tau}_i)$$

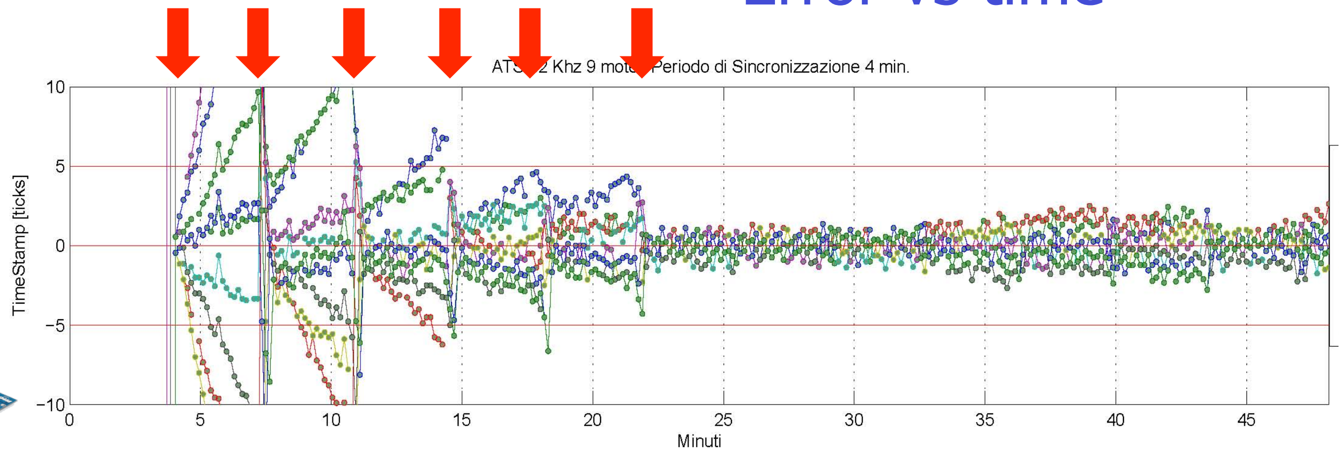
Clock Synch in WSN: experiments

Tmote Sky nodes

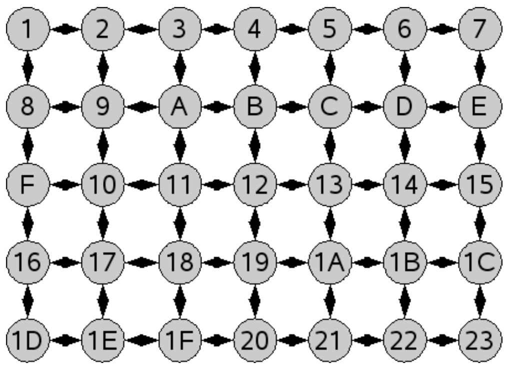


Synch instants

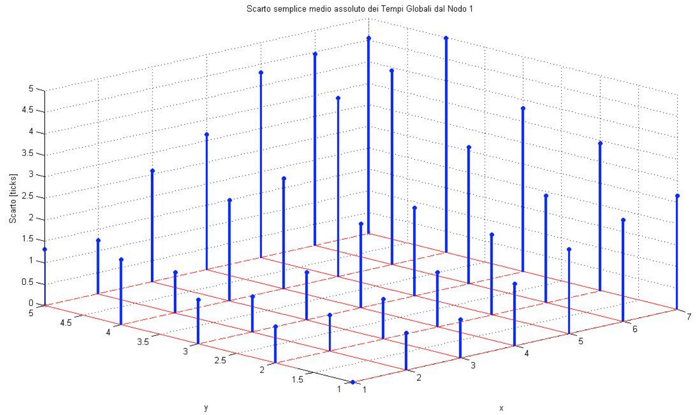
Error vs time



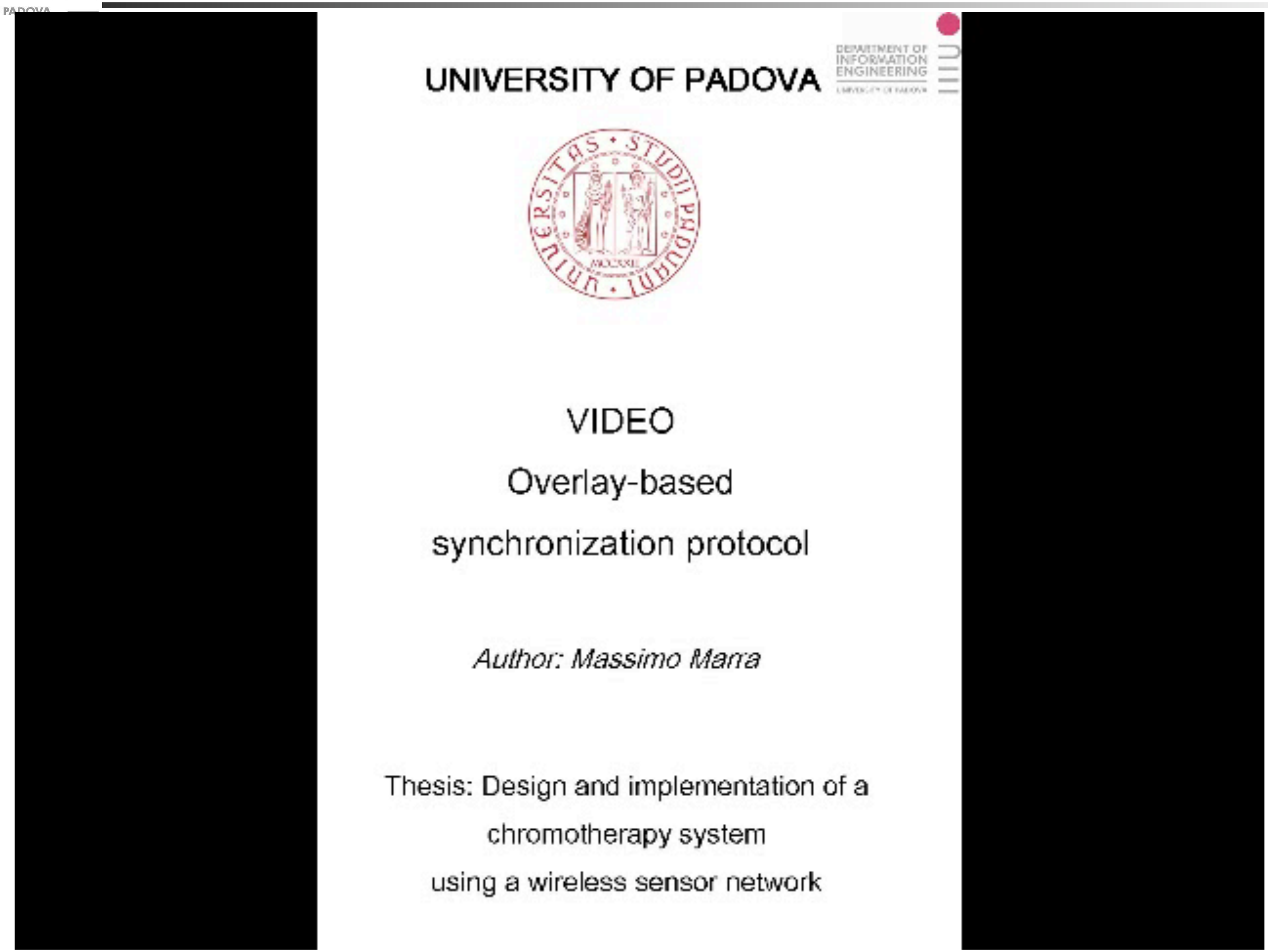
7x5 grid (10 hops)



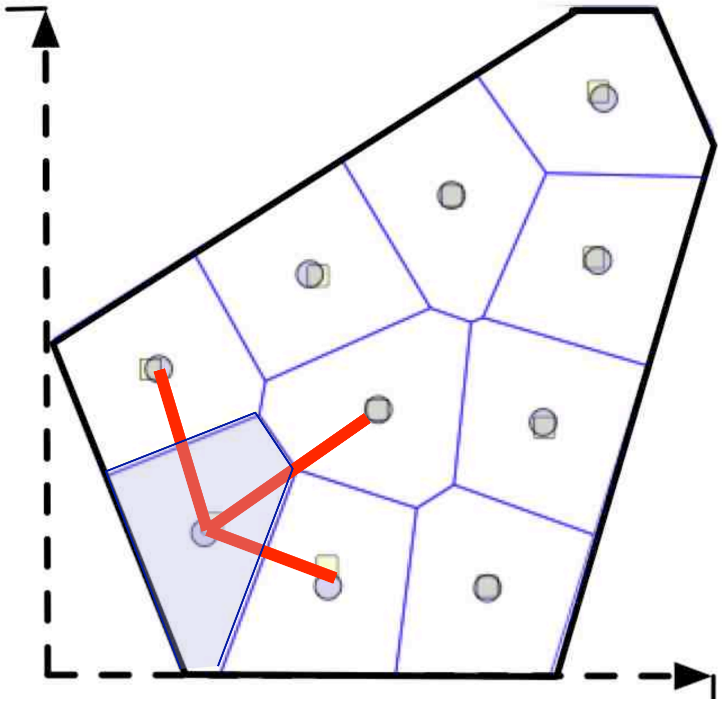
Error vs distance



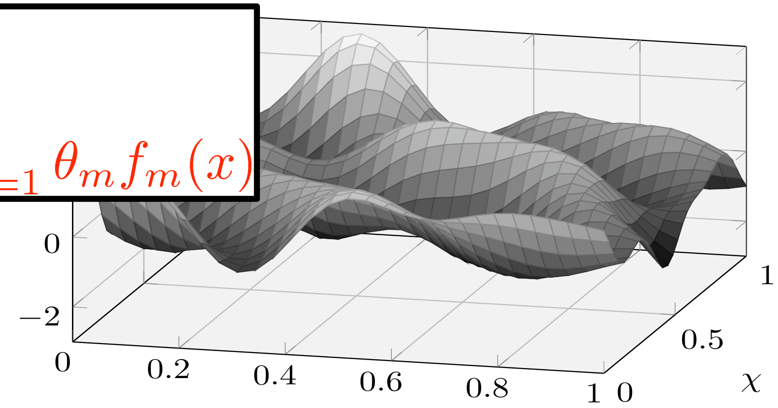
Clock Synch in WSN: video



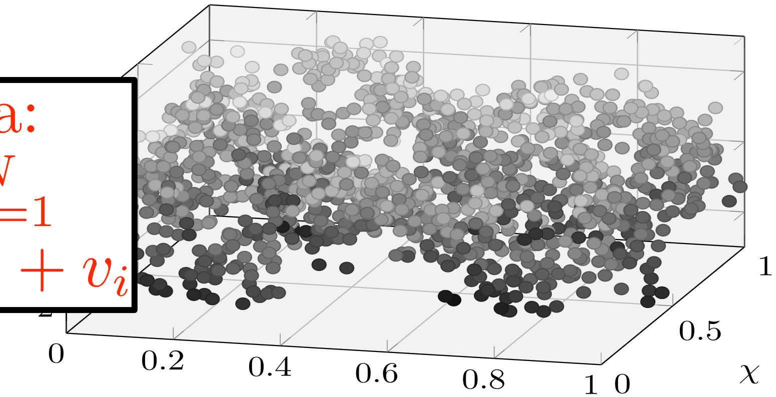
Map-building in robotic networks



Parametric Model:
$f(x) = \sum_{m=1}^{M} \theta_m f_m(x)$



Noisy data:
$\{(x_i, y_i)\}_{i=1}^{N}$
$y_i = f(x_i) + v_i$



- Issues:
  - Each robot collects local data
  - Local communication with robot
  - Patrolled area dynamically change

J. Choi, S. Oh, R. Horowitz, "*Distributed learning and cooperative control for multi-agent systems*", Automatica, 2009
Mac Schwager, Daniela Rus and Jean-Jacques Slotine *"Decentralized, Adaptive Coverage Control for Networked Robots"*, Int. Jour. Robotics research, 2009

Map-building as least-squares regression

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Estimate

$$f(x) = \sum_{m=1}^{M} \theta_m f_m(x)$$

with unknown parameters $\theta_1, \dots, \theta_M$ from noisy measurements

$$y_i = \sum_{m=1}^{M} \theta_m f_m(x_i) + v_i, \quad i = 1, \dots, N$$

By stacking all measurements

$$\begin{bmatrix} y(x_1) \\ y(x_2) \\ \vdots \end{bmatrix} = \begin{bmatrix} f_1(x_1) & \dots & f_M(x_1) \\ \vdots & \vdots & \vdots \\ f_1(x_N) & \dots & f_M(x_N) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_M \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix}$$

F_i

or equivalently:

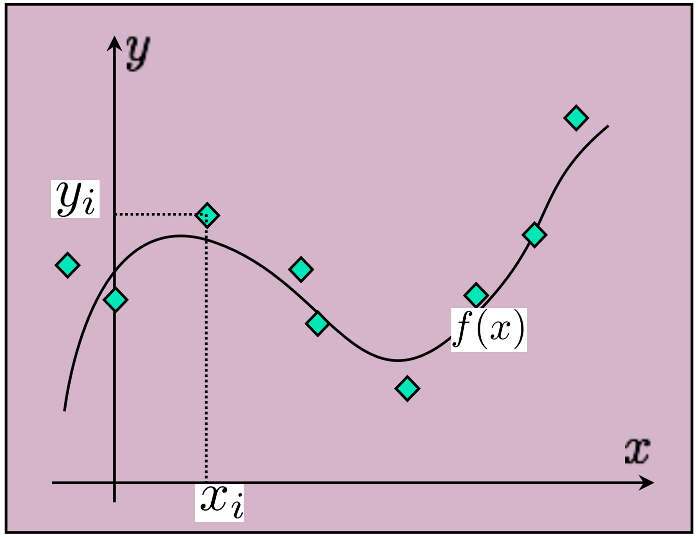
$$y = F\theta + v$$

Goal:

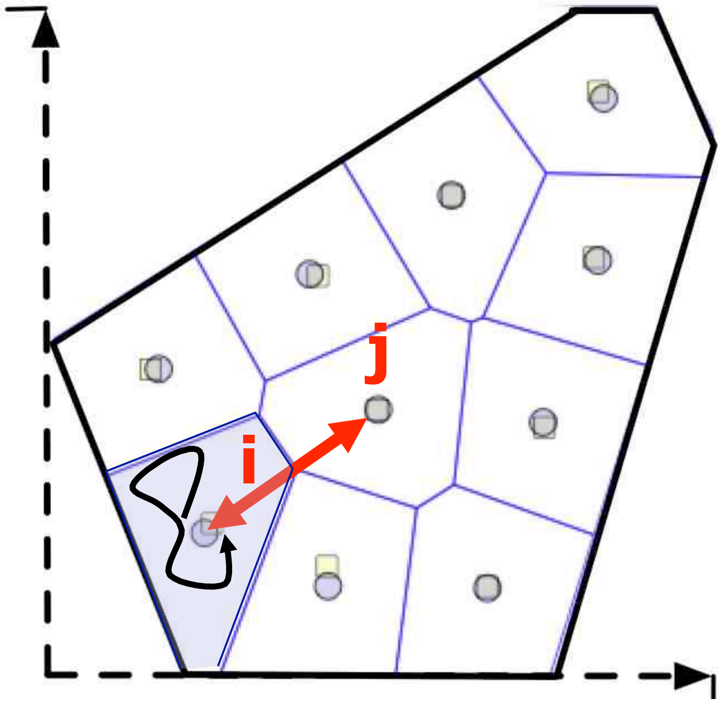
$$\hat{\theta} = \text{argmin}_\theta \sum_{i=1}^{N} v_i^2 = \text{argmin}_\theta ||F\theta - b||^2 = (F^T F)^{-1} F^T y$$

can be written as

$$\hat{\theta} = (\sum_{i=1}^{N} F_i F_i^T)^{-1} (\sum_{i=1}^{N} F_i y_i) = (\frac{1}{N} \sum_{i=1}^{N} F_i F_i^T)^{-1} (\frac{1}{N} \sum_{i=1}^{N} F_i y_i)$$



Consensus-based Map-building



Strategy for each robot i:

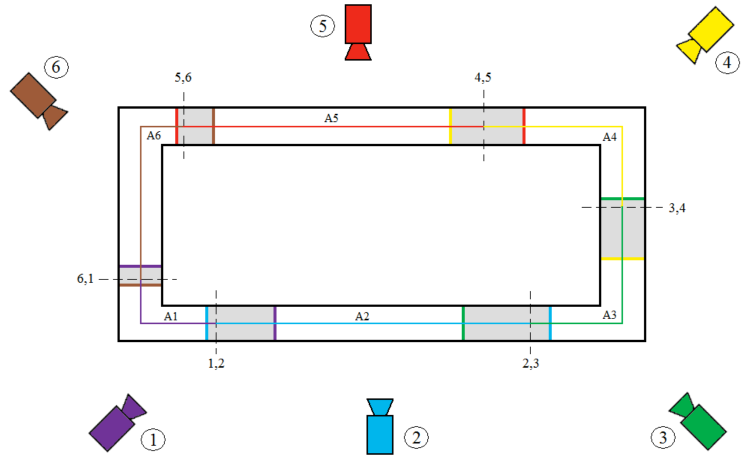
1) Initialize statistics:
$$Z_0^i = 0 \in R^{M\times M}$$
$$z_0^i = 0 \in R^M$$
2) Collect data and build local statistics:
$$Z_{t+1}^i = Z_i^t + F_t^i {F_t^i}^T$$
$$z_{t+1}^i = z_i^t + F_t^i y_t^i$$
3) Choose neighbor j and do gossip consensus:
$$Z_{t+1}^j = Z_{t+1}^i = \frac{1}{2}Z_t^i + \frac{1}{2}Z_t^j$$
$$z_{t+1}^j = z_{t+1}^i = \frac{1}{2}z_i^t + \frac{1}{2}z_j^t$$
4) Estimate map:
$$\hat{\theta}_t^i = (Z_t^i)^{-1} z_t^i$$
5) Repeat steps 2,3,4 (non necessarely in oder)

$$F_t^i := \begin{bmatrix} f_1(x_i(t)) \\ f_2(x_i(t)) \\ \vdots \\ f_M(x_i(t)) \end{bmatrix}$$

- Pros:
  - Asynchronous
  - Communication graph can change
- Cons:
  - Exchange of O(M²) data
  - Parametric model ←→ curse of dimensionality

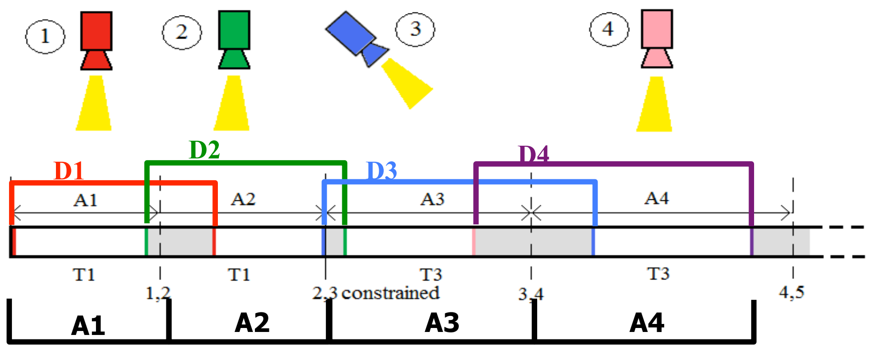
Perimeter Patrolling

- Scenario:
  - Perimeter surveillance: 1-D scenario
  - Camera position are fixed
  - 1 d.o.f. cameras: pan movements only
- Contraints
  - Limited mobility range: D_i
  - Limited pan speed: v_i
- Objective:
  - Determine A_i to minimize probability of undetected events



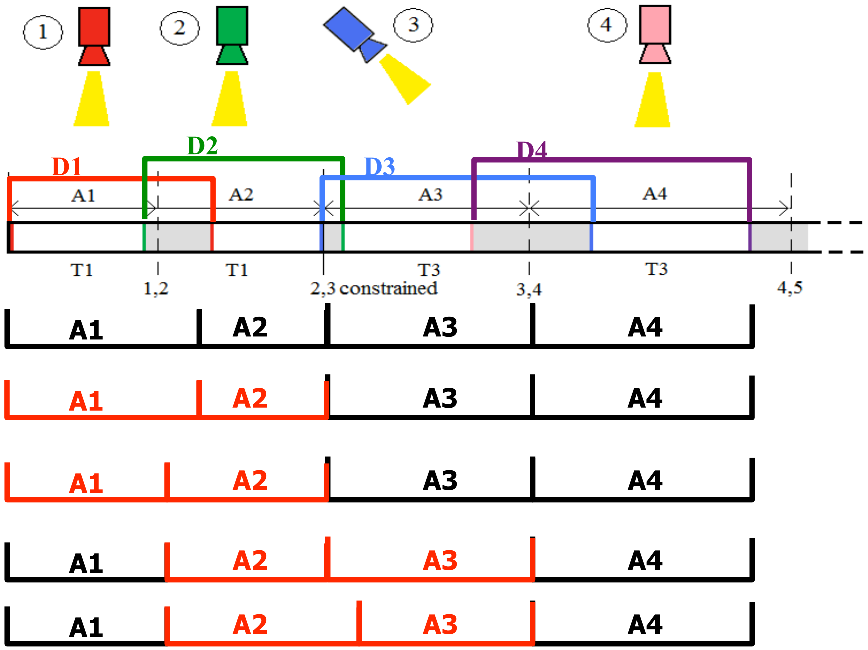
$$\text{Time-of-last-visit: } T_i = \frac{2|A_i|}{v_i}$$

$$T^* := \min_{A_1,\dots,A_N} \max_i \left\{ \frac{|A_1|}{v_1}, \dots, \frac{|A_N|}{v_N} \right\}$$



$$T^* = \frac{|A_1|}{v_1} = \dots = \frac{|A_N|}{v_N}, \text{ (if no mobility constraits } D_i\text{)}$$

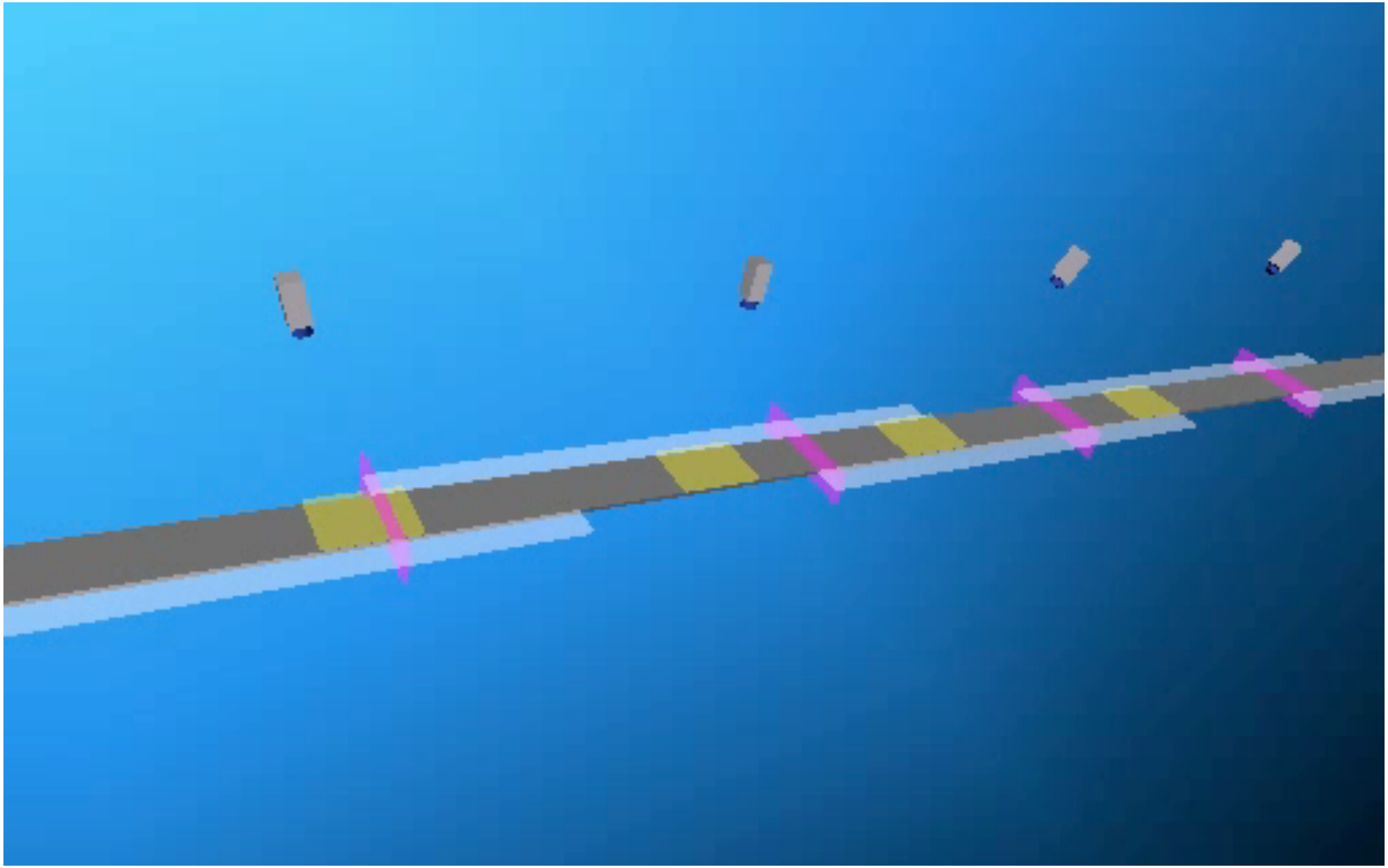
Perimeter Patrolling: asynchronous gossip consensus



Why using an asymptotic algorithm for a simple 1D problem where location of camera is fixed and known?

Just compute the centralized solution once at the beginning

Perimeter Patrolling: video

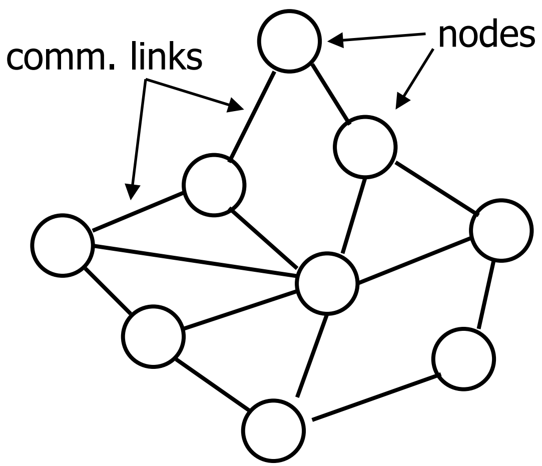


Outline

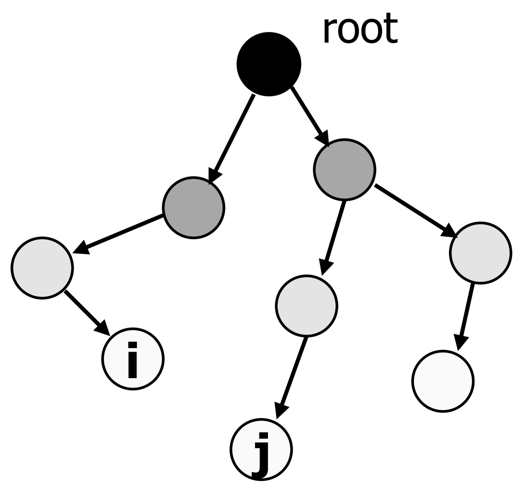
- Motivations and target applications
- Overview of consensus algorithms
- Application of consensus to WSN:
  - Sensor calibration
  - Clock synchronization
  - Map-Building
  - Perimeter Patrolling
- Open vistas and conclusions

Soft Hierarchical Control

Time synchronization example:



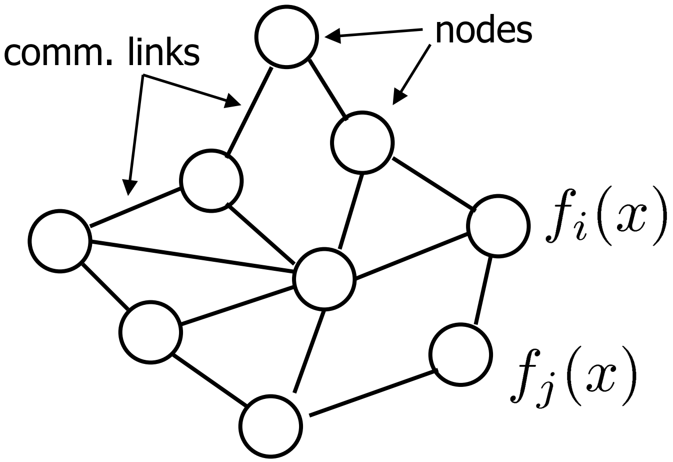
P_{dist} symmetric:
slow convergence but robust



P_{hier} asymmetric:
fast convergence but fragile to node failure

$$P_{soft} = \alpha P_{dist} + (1 - \alpha) P_{hier}, \qquad \text{optimal } \alpha \text{ depends on failure rate}$$

Average Consensus and distributed optimization



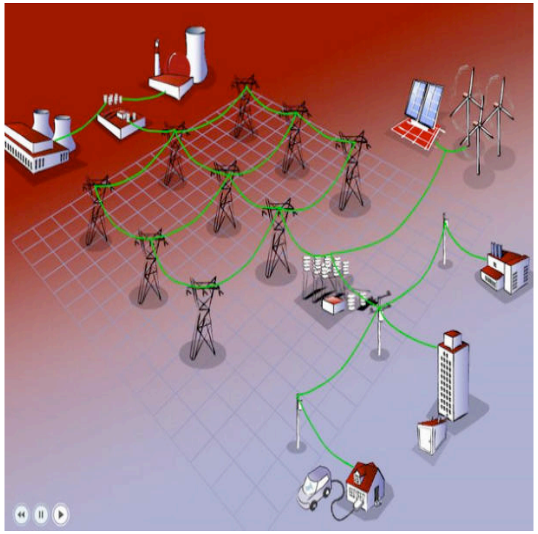
$$x^* = \text{argmin}_x \sum_{i=1}^N f_i(x)$$

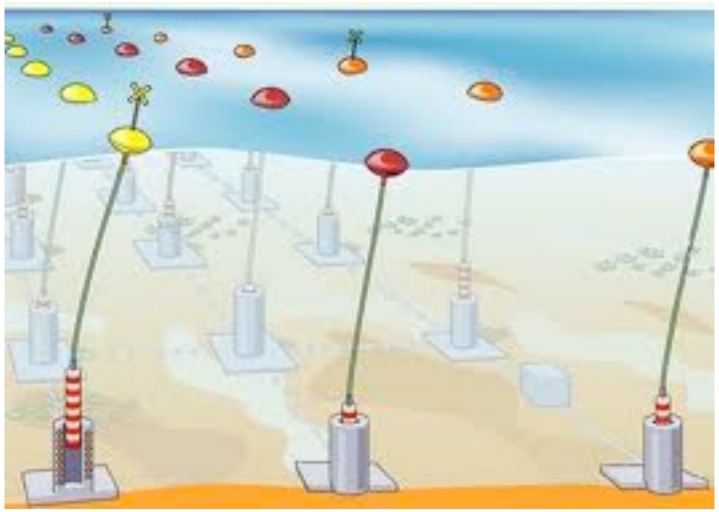
If $f_i(x) = (x - \theta_i)^2$ then $x^* = \frac{1}{N}\sum_{i=1}^N \theta_i$

If $f_i(x)$ is convex then:

1. Consensus-based subgradient methods (Nedic, Ozdaglar,...)
2. Alternating Direction Method of Multipliers (Bersekas, Boyd,Giannakis,...)
3. Newton-Raphson consensus (our approach)
4. others ?

Smart Power Grids and Renewable energies



- Foreseeable future
  - Many consumers & producers
  - Cooperation vs greedy behavior
  - Network topology not known and dynamic
  - Need for distributed estimation and control

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Conclusions

- Consensus is successful tool for multi-agent applications
- Many (important) details swept under the carpet
- Multisciplinary research is key for success (Communication, CS, Software Engineering, …)
- Distributed vs hierarchical not well understood in large scale systems
- Smart energy grids: a lot of hype right now but …. great control opportunity if we stick to reality

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