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Reactive power pricing strategy

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Outline

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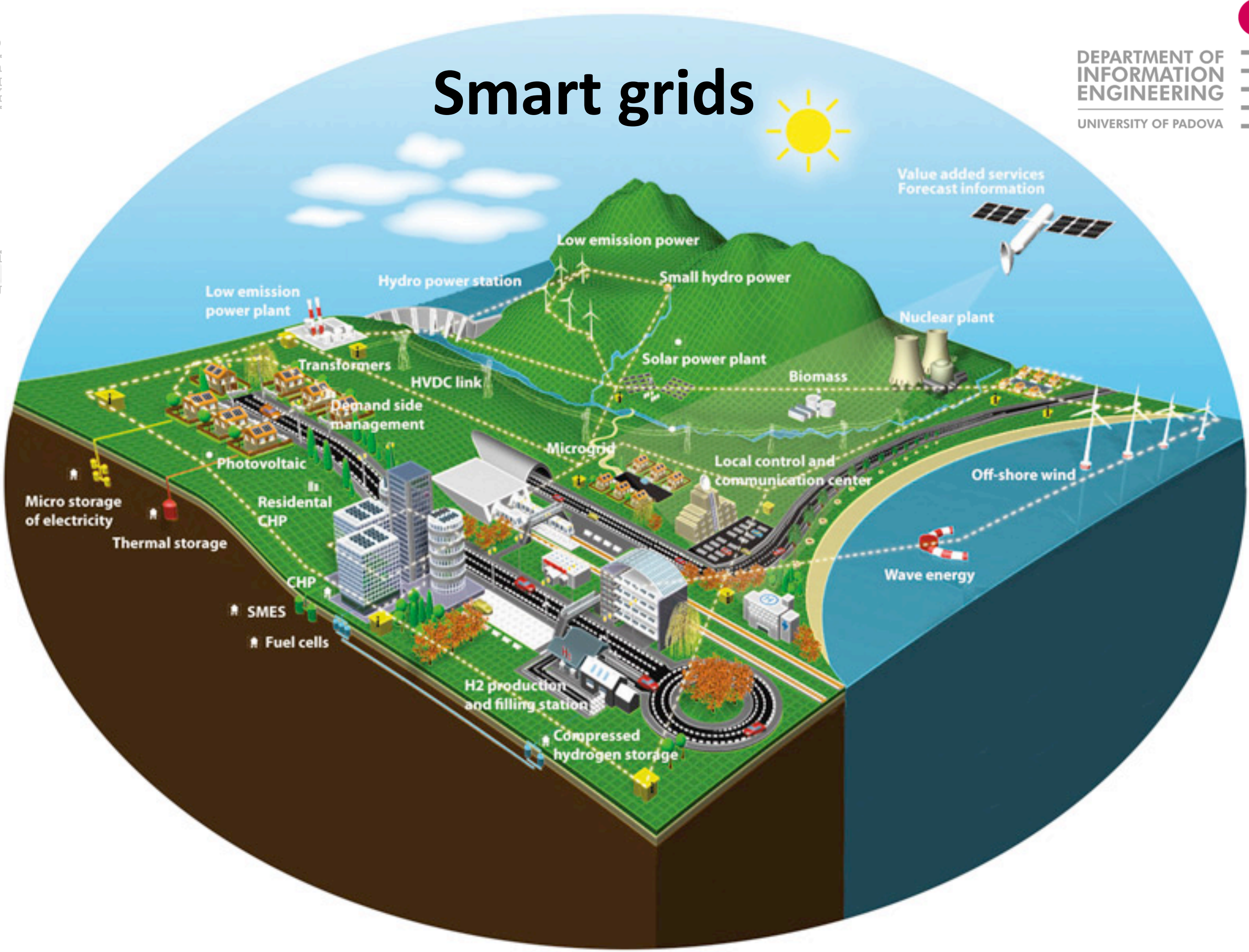
- Smart Grids
  - Model
  - Reactive power compensation
- Pricing Tools
  - Lagrange multipliers (centralized)
  - Gradient estimation (distributed)
- Pricing Strategy
  - Shadow prices
  - Storage strategy

Smart grids



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Smart grids

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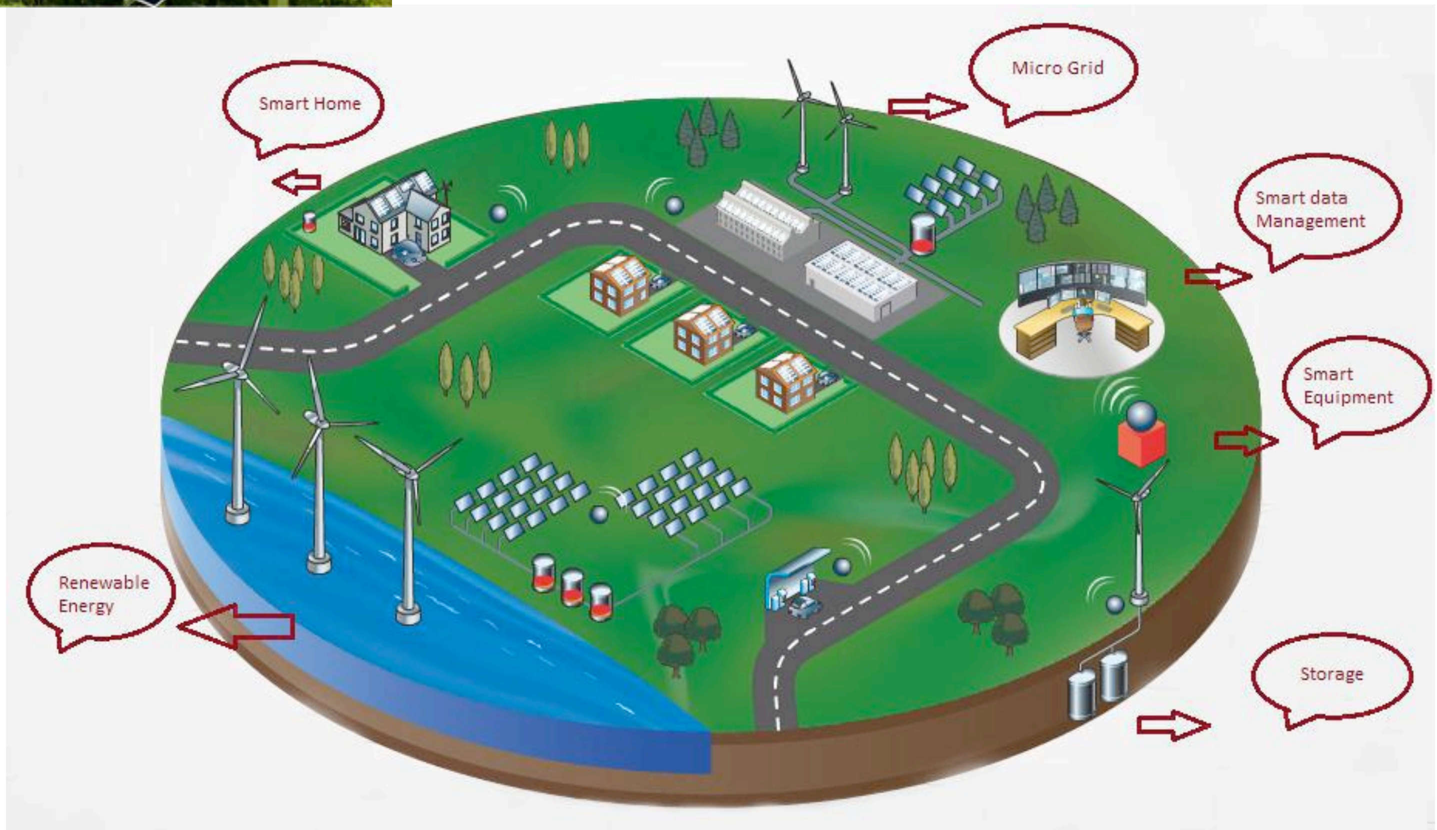

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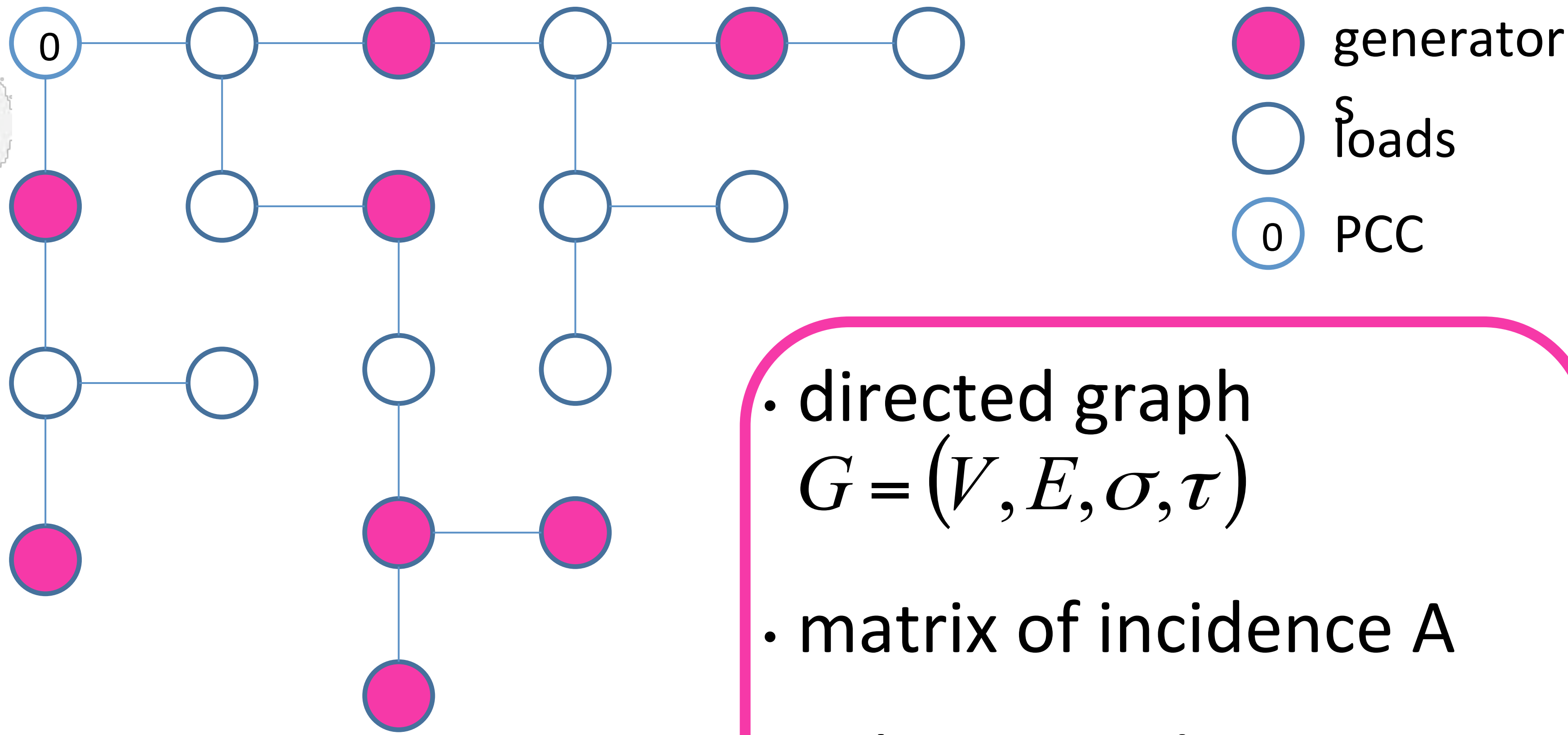
Smart grids - Model



- directed graph
  $G = (V, E, \sigma, \tau)$
- matrix of incidence A
- voltages and currents
  $y(t) = |y|\sqrt{2}\sin(\omega_0 t + \angle y)$

Smart grids - Model

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Initial model equations

- (KLC) and (KVL) laws

$$A^T \xi + i = 0$$
$$Au + Z\xi = 0$$

- PCC → constant voltage generator

$$u_0 = U_0$$

- generic node → exponential model

$$u_v \bar{i}_v = s_v \left| \frac{u_v}{U_0} \right|^{\eta_v}, \forall v \in V \setminus \{0\}$$

Approximate model(1)

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Laplacian matrix, $L = \mathrm{A}^T \mathrm{Z}^{-1} \mathrm{A}$

Green matrix, X satisfying:

$$\begin{cases} XL = I - 1 1_0^T \\ X 1_0 = 0 \end{cases}$$

Solution for the currents:

$i = Lu$

Approximate model(2)

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The system can be rewritten in the following form:

$$\begin{cases} u = Xi + U_0 1 \\ u_v \bar{i}_v = s_v \left| \dfrac{u_v}{U_0} \right|^{\eta_v}, \forall v \in V \setminus \{0\} \\ 1^T i = 0 \end{cases}$$

Smart grids - Compensation

Reactive power flows contribute to:

- power losses on the lines
- voltage drop
- grid instability

Problem of optimal reactive power compensation

Bolognani, S., and Zampieri, S. (2011)
Distributed control strategy for optimal reactive power compensation in smart microgrids.

Smart grids - Compensation

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$$J' = \bar{i}^{T} \operatorname{Re}(x) i = \frac{1}{|U_0|^2} p^T \operatorname{Re}(x) p + \frac{1}{|U_0|^2} q^T \operatorname{Re}(x) q + \frac{1}{|U_0|^2} \tilde{J}(U_0, s)$$

where it has been used the Taylor approximation of $i(U_0)$ for large U_0

Being $\tilde{J}(U_0, s)$ infinitesimal, we have an ***uncoupling phenomenon***

QUADRATIC LINEARLY CONSTRAINED PROBLEM

Smart grids - Compensation

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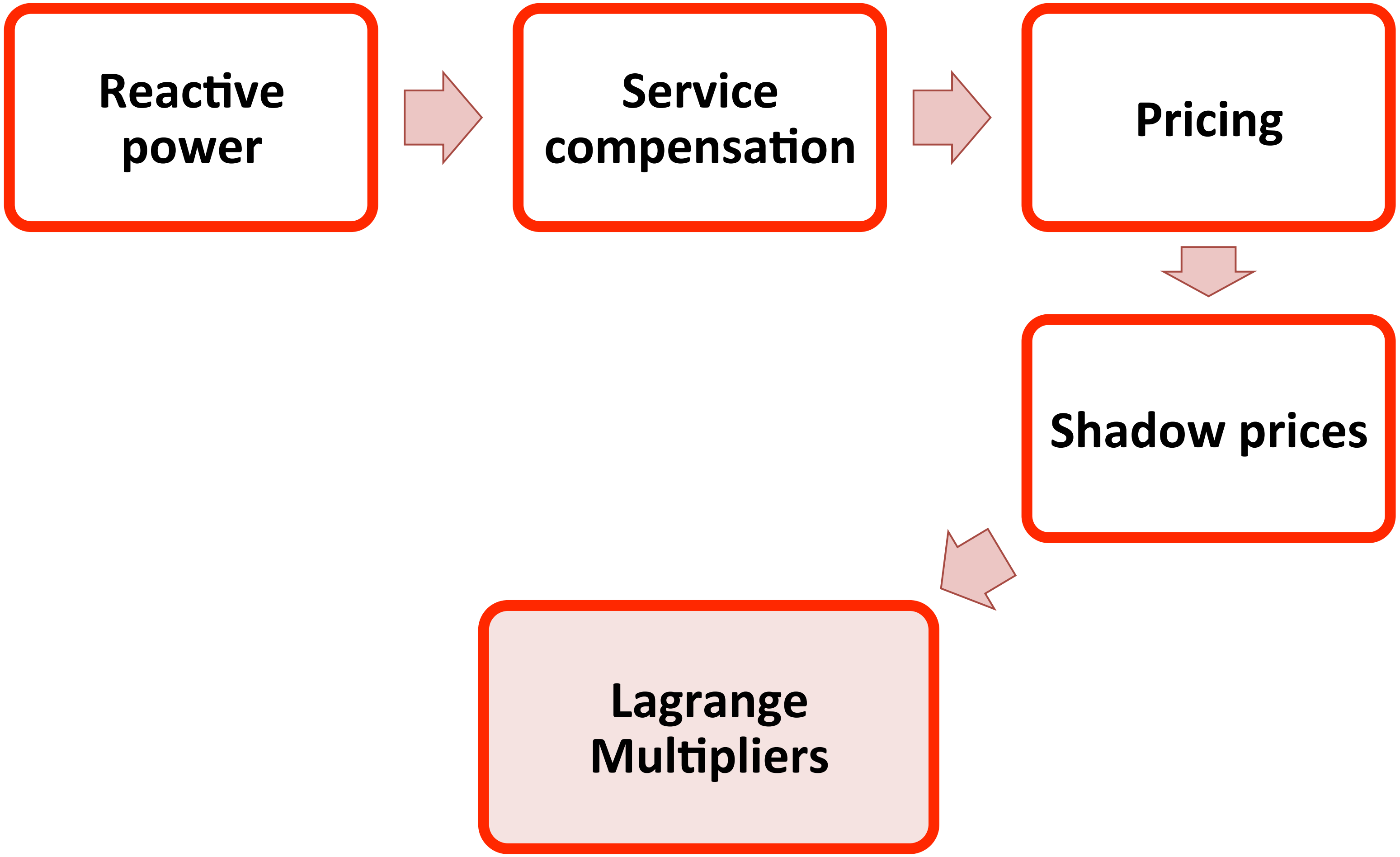
QUADRATIC LINEARLY CONSTRAINED PROBLEM

$$\min_q \ J(q), \quad where \ J(q) = \frac{1}{2} q^T \operatorname{Re}(x) q,$$

$$subject \ to \quad 1^T q = 0$$

$$q_v = \operatorname{Im}(s_v) \quad \forall \, v \in V \setminus C$$


Reactive power
Service compensation
Pricing
Shadow prices
Lagrange Multipliers

Lagrange Multipliers-Theory(1)

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Duality theory

- Primal $\min f_0(x) \text{ s.t } f_i(x) \leq 0,\ \text{h}_i(x) = 0$
- Lagrangian $g(\lambda,\nu) = \inf L(x,\lambda,\nu)$
- Dual $\max g(\lambda,\nu)\ s.t\ \lambda \geq 0$

Primal+dual ➡ λ^*, ν^*

Lagrange Multipliers-Theory(2)

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$$\lambda^*_i = -\frac{\partial p^*(0,0)}{\partial u_i}$$

p^* Primal solution

u_i Constraint perturbation

Optimal Lagrange multipliers → **LOCAL SENSITIVITIES**

Lagrange Multipliers-Application(1)

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- Objective function $J(q)$
- Constraints on loads and compensators

$$q = Q_l \ , q \leq Q_c$$

- Lagrangian

$$L(q,\lambda) = J(q) + \lambda^T (q - Q)$$

- Solution

$$\lambda_l^* = -\frac{2}{|u_0^2|} D^{-1} Q_l$$

Lagrange Multipliers-Application(2)

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Solution requires global knowledge of network topology

→ Centralized algorithm

- Centralized unit
- High computational cost with large number of nodes
- Broadband communication system
- Change in network topology

Lagrange multipliers-Graphic

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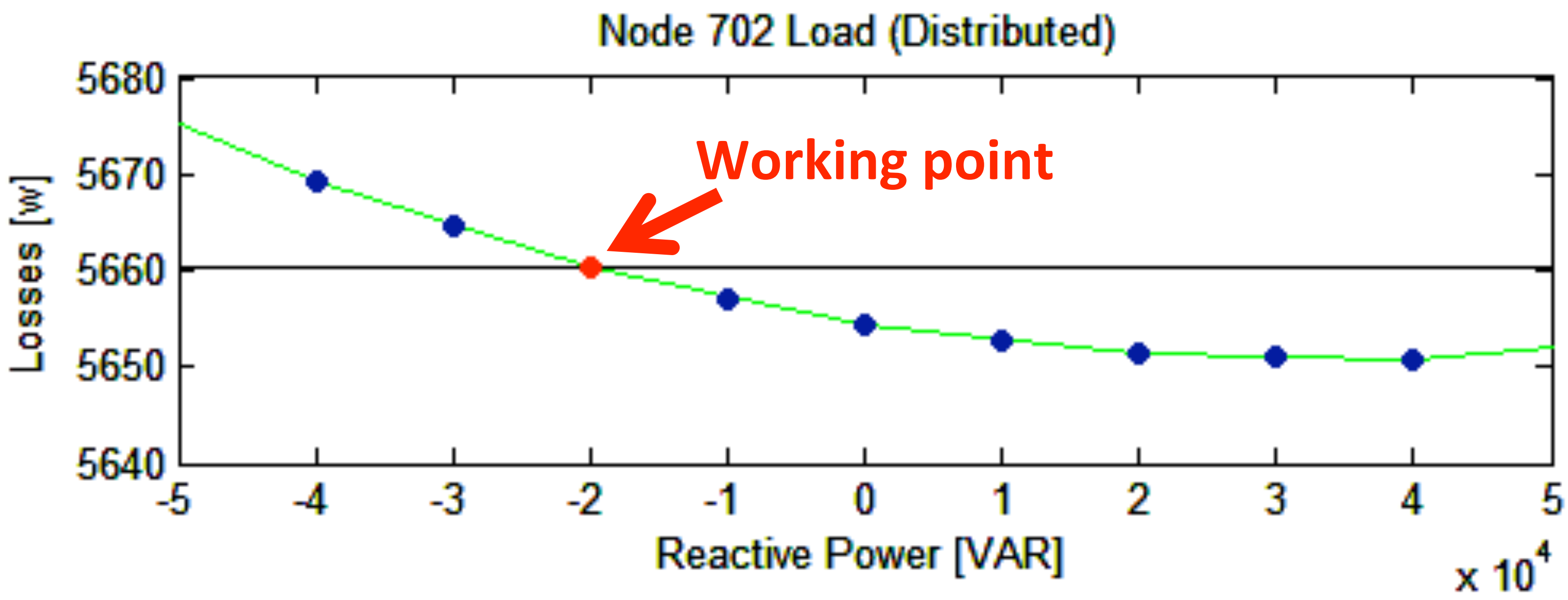
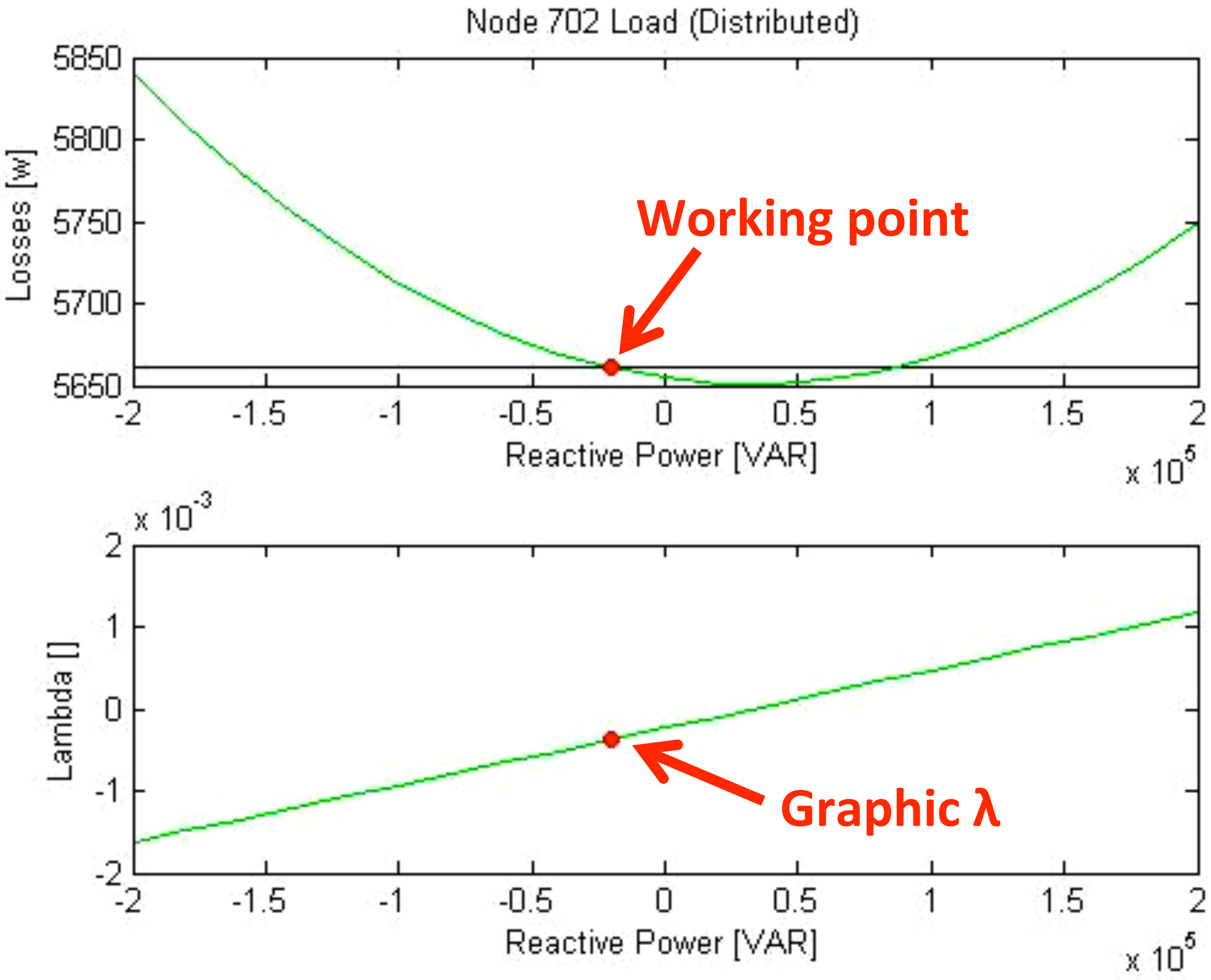
Exploiting the perturbation theory of duality we identify the losses parabola




Node 702 Load (Distributed)
Losses [w]
5850
5800
5750
5700
5650
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Reactive Power [VAR]
x 10^5
Working point
x 10^-3
Lambda []
2
1
0
-1
-2
Reactive Power [VAR]
x 10^5
Graphic λ

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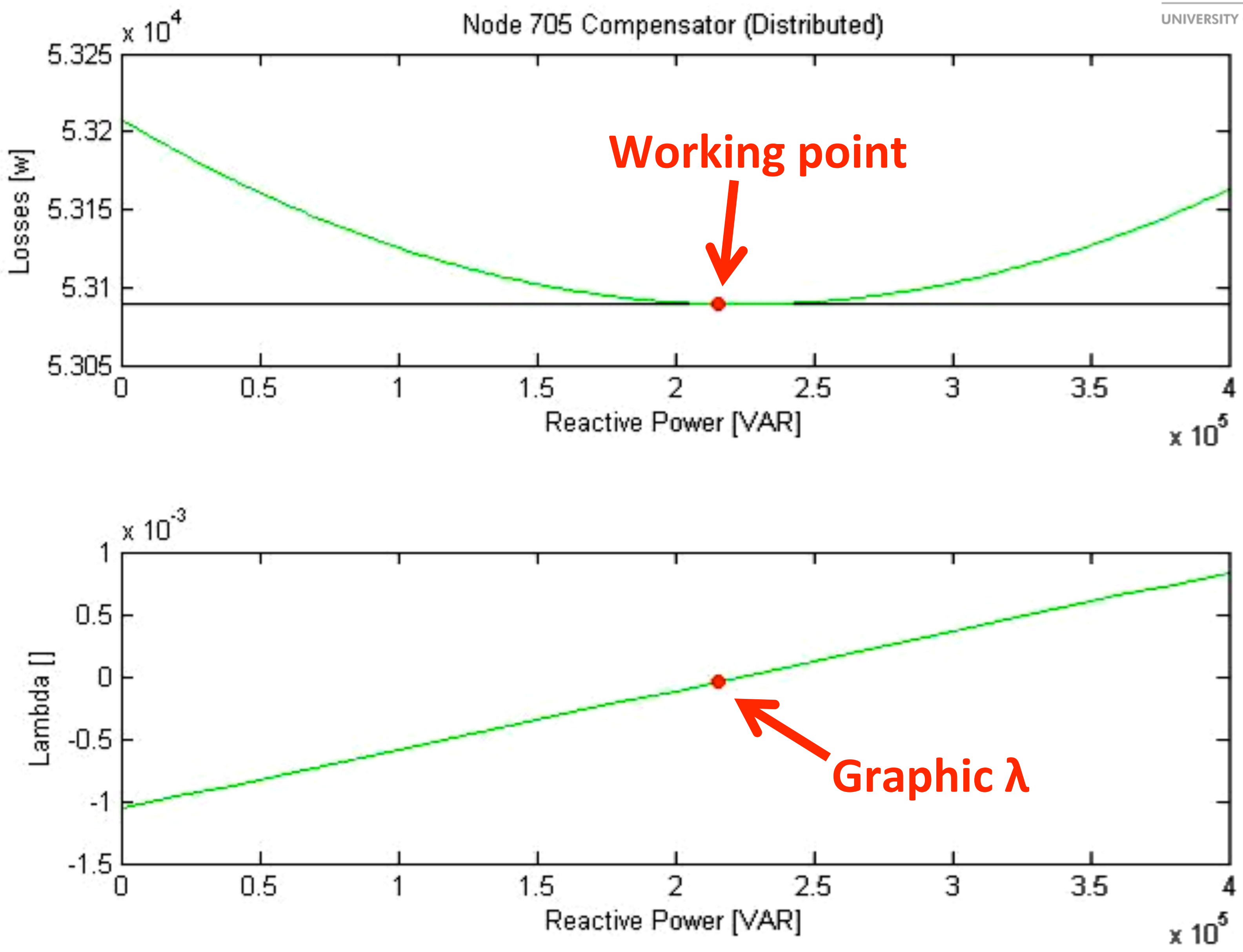

Gradient estimation

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Beginning from the Lagrangian with constraints on both

loads and cor

$$q(\lambda) = \frac{|u_0^2|}{2} Re(\mathbf{X})^{-1} \lambda$$

Solving for λ:

$$\lambda(q) = -\frac{2Re(\mathbf{X})}{|u_0^2|} q$$

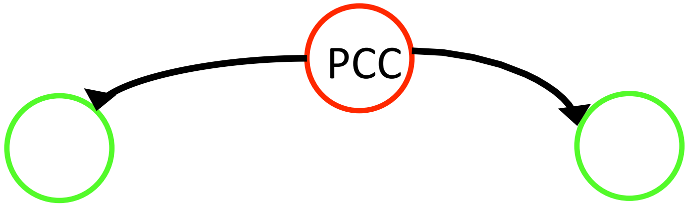
$Re(\mathbf{X})q$ → $Re(\mathbf{X})q$

timation

Gradient estimation (1)

- Using PCC voltage

Each node receives information from the PCC

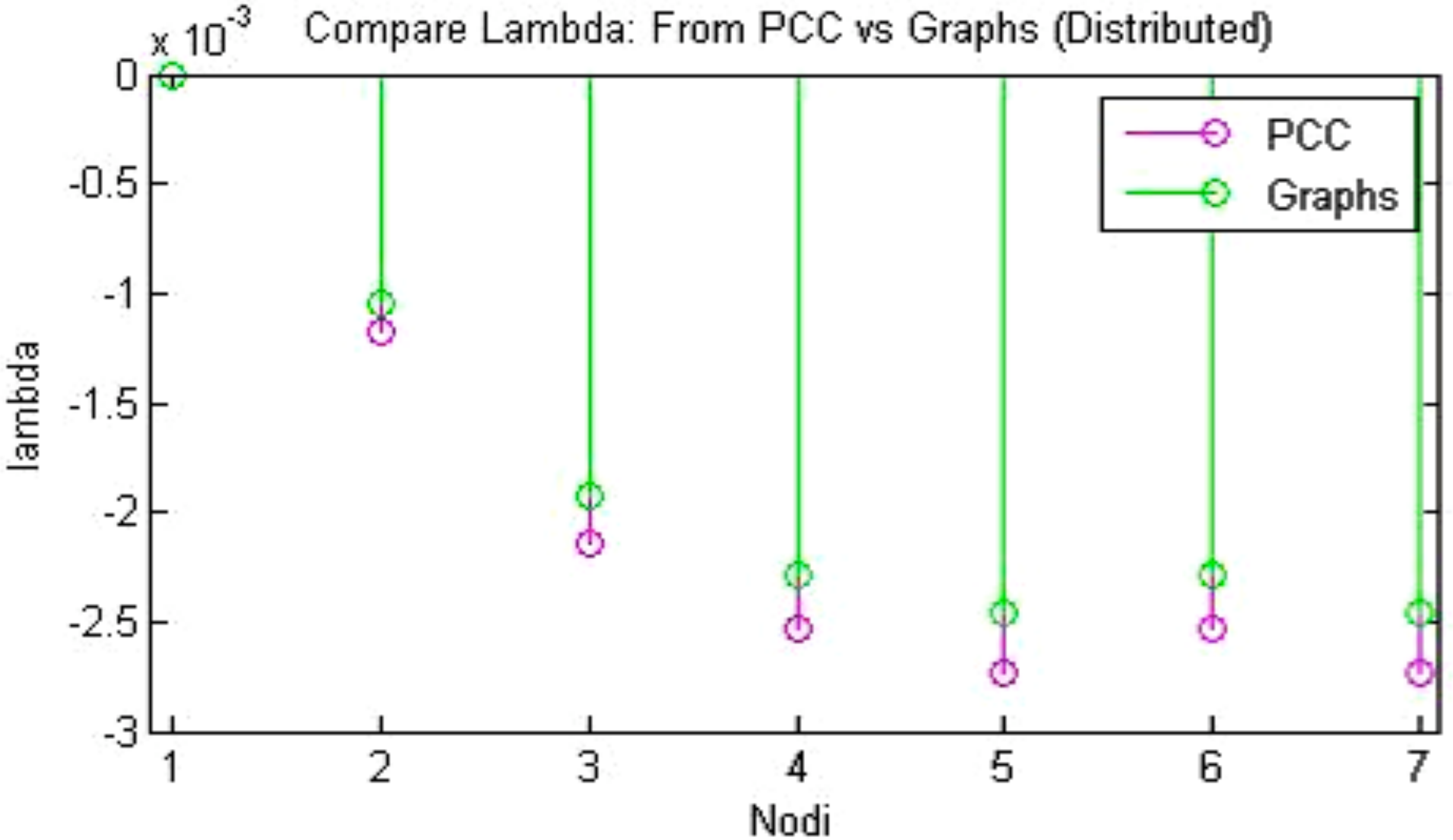


$$\lambda(u) = 2\cos\theta \, \mathrm{Im}\left[\frac{e^{-j\theta}(u - u_0 1)}{u_0}\right]$$

Θ phase shift

- constant
- between node and PCC

→ overestimate results


Compare Lambda: From PCC vs Graphs (Distributed)
x 10^-3
0
-0.5
-1
-1.5
-2
-2.5
-3
lambda
1
2
3
4
5
6
7
Nodi
PCC
Graphs

Gradient estimation (2)

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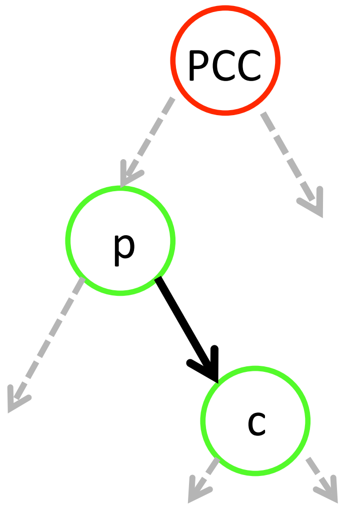
- Distributed

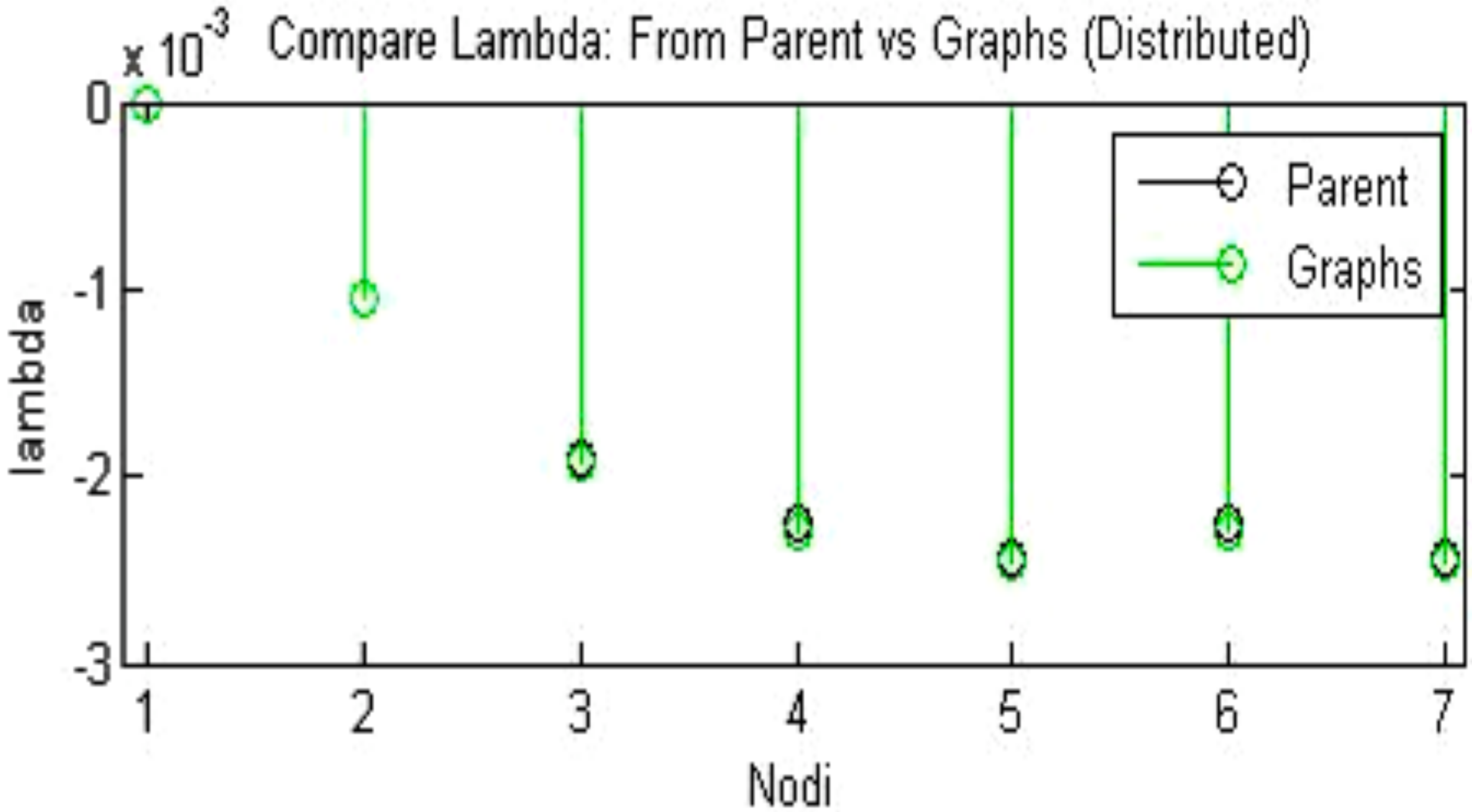
Two nodes exchange information between each other



$$\lambda_c = \lambda_p + \frac{\cos\theta^2}{|u_0^2|}\mathrm{Im}[e^{-j\theta}(\bar{u}_p + \bar{u}_c)(u_p - u_c)]$$

Θ phase shift

- Between the two nodes results → underestimate


x 10^-3
Compare Lambda: From Parent vs Graphs (Distributed)
0
-1
-2
-3
lambda
Parent
Graphs
1
2
3
4
5
6
7
Nodi

Validation test for λ estimation

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Real λ analysis

Estimate λ analysis

Test Grid

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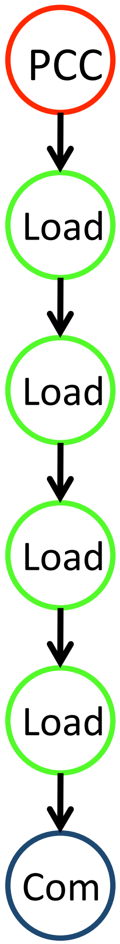
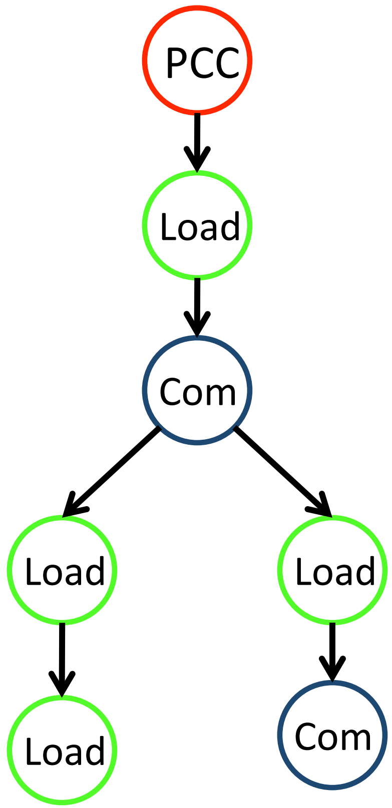
Aligned nodes



Symmetric bifurcation



1° observation

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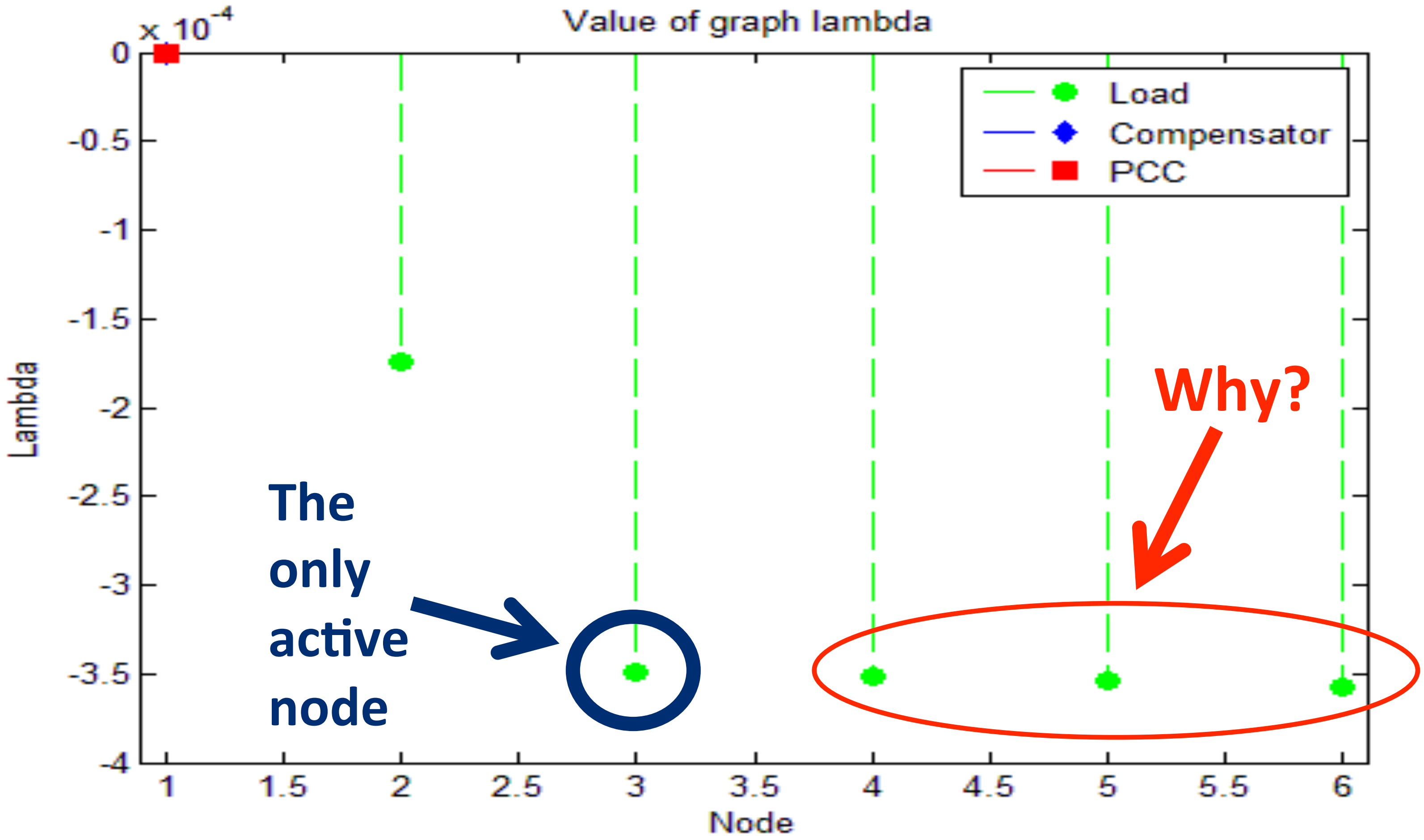
Branch with no power flowing ($\lambda \neq 0$)



2° observation

Linear increasing of λ in a branch with constant power flowing

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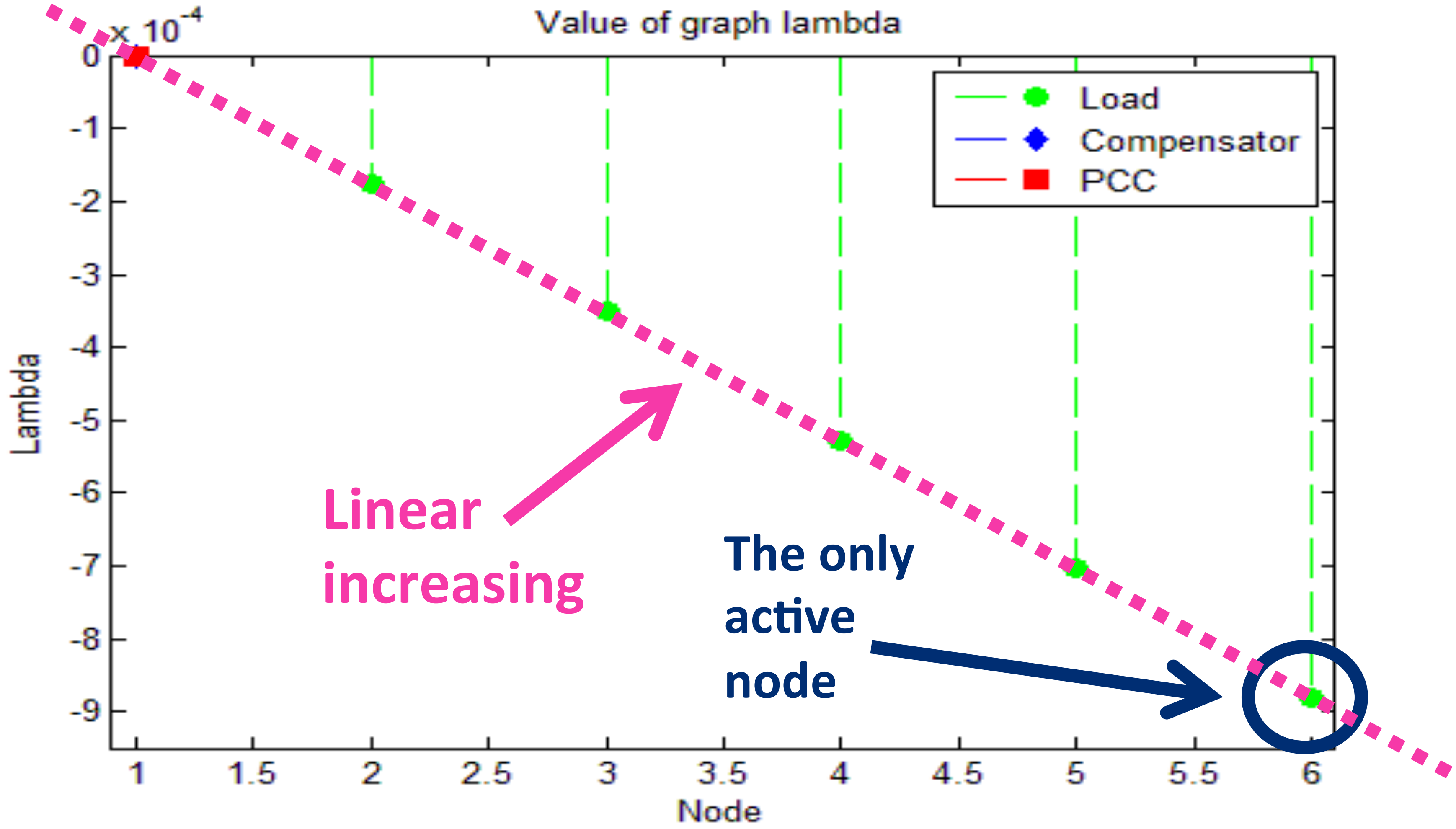
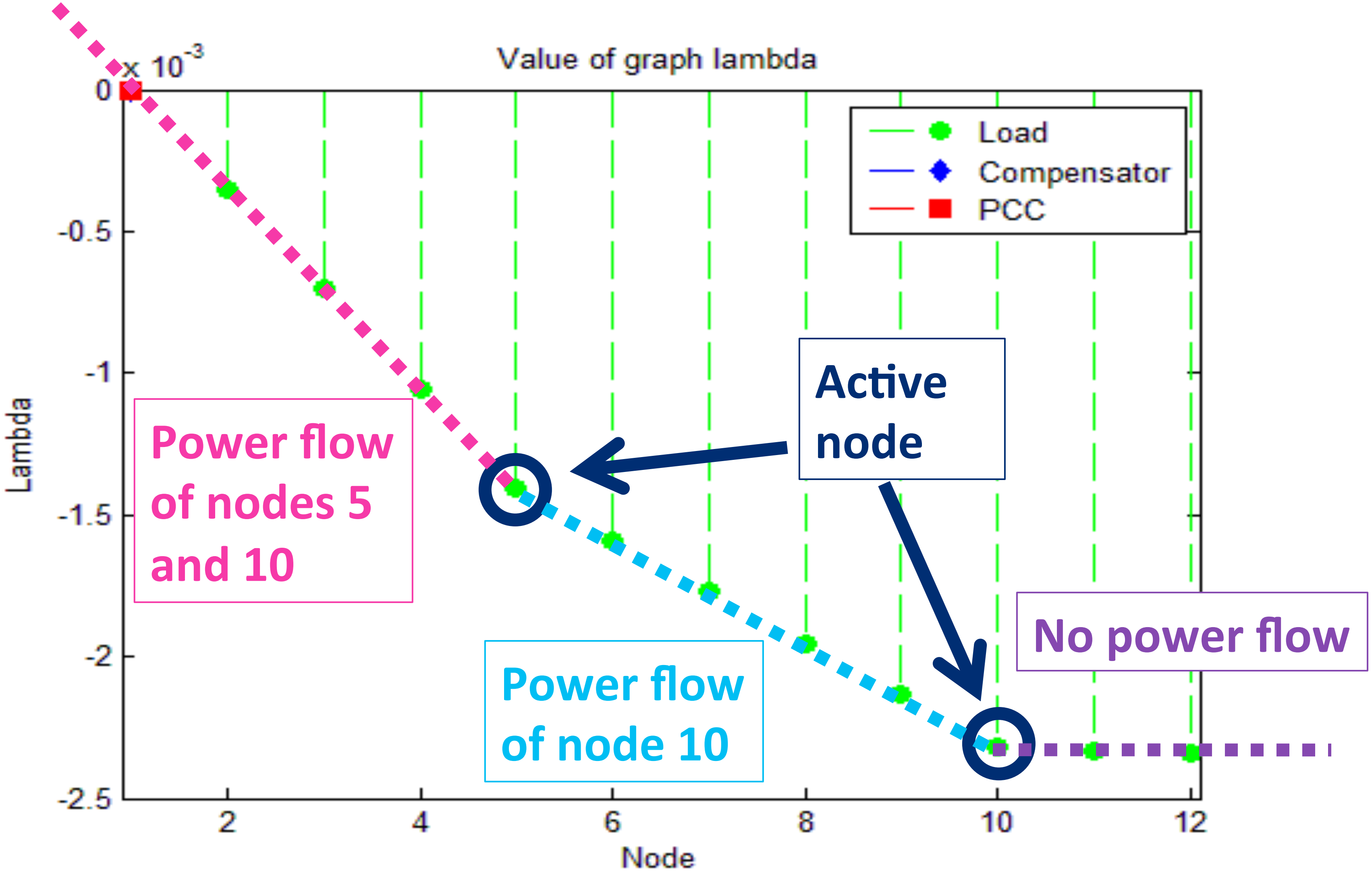


3° observation

Superposition of the various effects



Performance of the λ estimation

Simple grid

Different absorptions

Many compensators

-Not homogeneus grid

-Depth tree

Hypothesis of no homogeneity

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If we have different impedance phase angles:

$$\mathbf{X} = e^{j\theta} X$$

then the assumption done on Re(**X**) isn't true and we can't extract the phase angles from **X**

Anyway ,the homogeneus hypothesis is a realistic assumption in the smart microgrid

Depth tree

The error increases with the increase of depth tree. This is due to two errors:

$$\lambda_c = \lambda_p + \frac{\cos\theta^2}{|u_0^2|}\,\mathrm{Im}[e^{-j\theta}(\bar{u}_p + \bar{u}_c)(u_p - u_c)]$$

from parent λ — **from upgrade**

Nevertheless :

- we can consider compensator information
- generally, we have 12-20 levels of depth

Parent-child communication strategies

Ordered
- Right upgrades
- Time: # levels -1

Randomized
- Wrong upgrades
- Time: ≥ # nodes-1

Centralized
- Only voltage needed
- Time: 1 step

Shadow prices interpretation

λ_i tells us approximately how much more profit the process could make for a small increase in availability of resource q_i

Lagrange Multiplier = Shadow Price

However, the cost function $f_0(x)$ doesn't consider all the benefits related to reactive power

~~**Lagrange Multiplier = Shadow Price**~~

Storage strategy

$f_0(x)$ doesn't consider:

- voltage constraints
- congestion constraints

We define a storage strategy in order to evaluate what is truly convenient for a compensator:

Sell or Store active power?

Compensator limitations

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A compensator provides an apparent power A that is function of the active power P and of the reactive power Q.

These quantities are linked by Boucherot theorem:

$$\overline{A}^2 = \overline{P}^2 + \overline{Q}^2$$

Where bars indicate the available active power or the reactive power required.

The inverter installed on the compensator is characterized by an apparent power limitation M. Thus, we have two cases:

- $M \geq \overline{A}$ → No limitation
- $M < \overline{A}$ → Limitation is active

We must decide whether to sell or to store active power

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What to do in case of limitation?

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Time t=0

At the initial time we suppose to provide the whole active power and produce a quantity Q(t=0) of reactive power

The corresponding inverter limitation is:

$$M(t_0)^2 = P(t_0)^2 + Q(t_0)^2 = \overline{P}^2 + Q(t_0)^2$$

Where the reactive power tends to zero.

Time t=1

Suppose to increase the reactive power to:

$$Q(t_1) = Q(t_0) + dQ$$

Due to active limitation, we must decrease the active power sold and store a certain quantity:

$$dS(t_1)^2 = Q(t_1)^2 - Q(t_0)^2 \Rightarrow dS(t_1) = dQ\sqrt{\left(1 + 2\frac{Q(t_0)}{dQ}\right)}$$

For any production increment of reactive power we can consider an equal storage quantity:

$$dS = dQ$$

Cost function

storage isn't free

considering the storage inefficiency η

$$dC = \eta dS\,\$_P = \eta dQ\,\$_P$$

Where $\$_P$ is the unit price of active power

Gain function using λ

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For an infinitesimal variation of reactive power we assume a linear increasing of active power

$$dP = \lambda dQ$$

From which we obtain the gain function:

$$dG = \lambda dQ \,\$_P$$

Where $\$_P$ is the unit price of active power

Profits inequality

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To have profits we need to comply with:

$$dG \geq dC$$

Solving the inequality we obtain:

$$1 \leq \frac{\lambda}{\eta}$$

→ **Never true**

- $\lambda < 0.05$
- $\eta = 0.1–0.07$

We have a confirmation that λ hasn't all the necessary information for pricing

Gain function with Q price

Assuming to know the correct reactive power price $\$_Q$, we can define the gain:

$$dG = dQ\,\$_Q$$

Solving the profit inequality, we obtain:

$$\$_Q \geq \eta\,\$_P \cong 0.1\,\$_P$$

It is plausible that in some cases reactive power importance is very high even higher than the active power one

Conclusions

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$\lambda \neq$ shadow price for

- Voltage limits
- Risk of congestion

Pricing → **Storage strategy**

<u>Future developments</u>

- Define a new cost function
- Build different Lagrangians
- Abandon Lagrange theory