A Gentle Introduction to Dynamic Programming and the Viterbi Algorithm

Dr. Hubert Kaeslin
Microelectronics Design Center
ETH Zürich

Extra teaching material for
Digital Integrated Circuit Design,
from VLSI Architectures to CMOS Fabrication
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Quiz: Bridge traversal I

Problem: Four persons want to traverse a suspension bridge at night.

- The bridge can carry no more than two persons at a time.
- The four persons take 5, 10, 20 and 25min respectively to traverse.
- Each party must carry a torch while traversing the bridge.
- The torch lasts no longer than 60 minutes.

Can you help these people?

How did you arrive at your plan?

Can you formulate a general policy (i.e. an algorithm)?

Quiz: Bridge traversal II

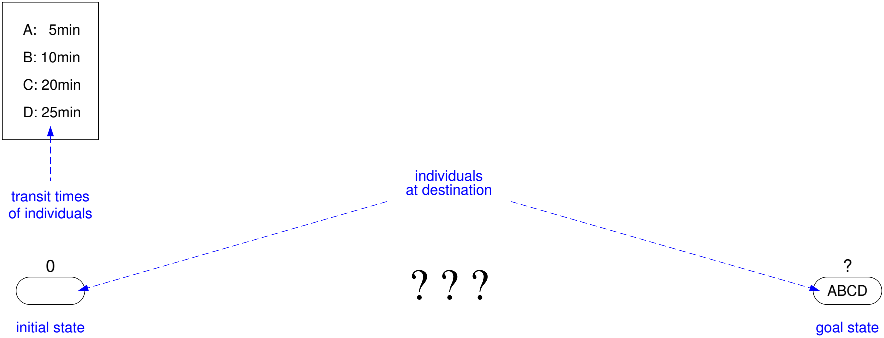


Figure: The problem stated graphically.

Quiz: Bridge traversal III

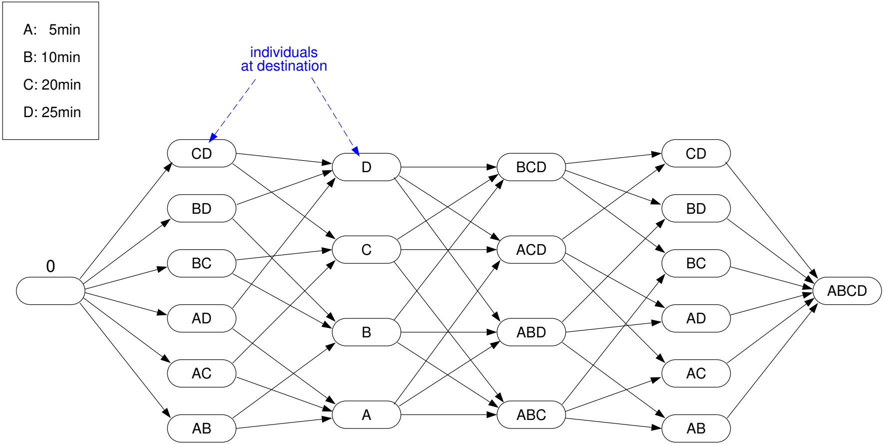


Figure: The solution space drawn as a trellis graph.

Quiz: Bridge traversal IV

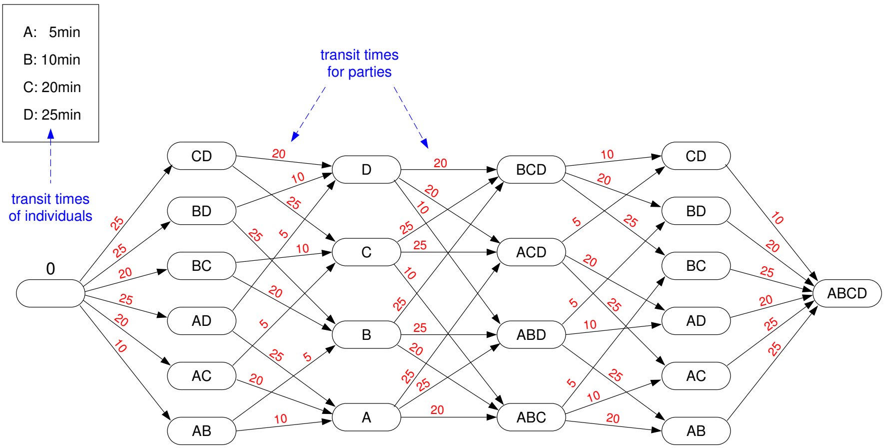


Figure: ... with branch metrics assigned to all state transitions (edges).

Quiz: Bridge traversal V

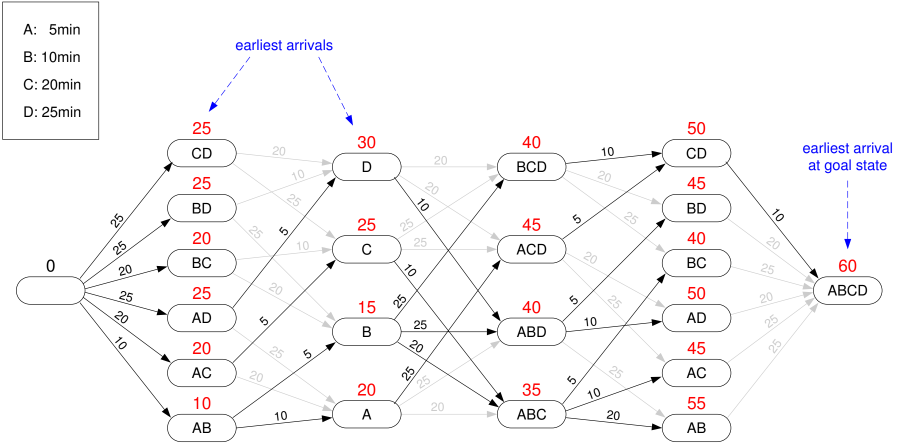


Figure: ... with minimum path lengths updated for all states (nodes).

Quiz: Bridge traversal VI

A: 5min
B: 10min
C: 20min
D: 25min

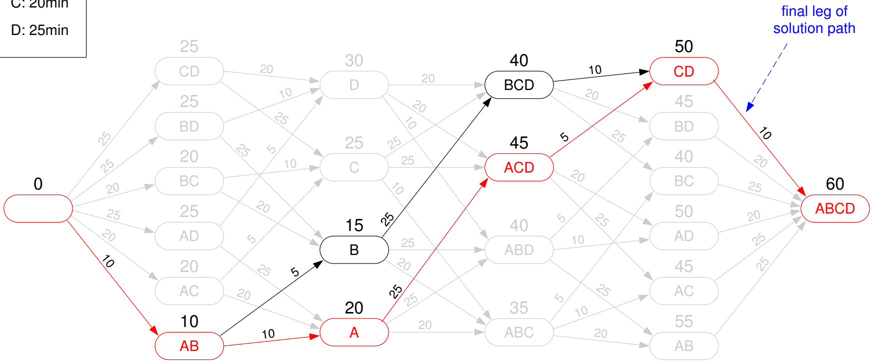
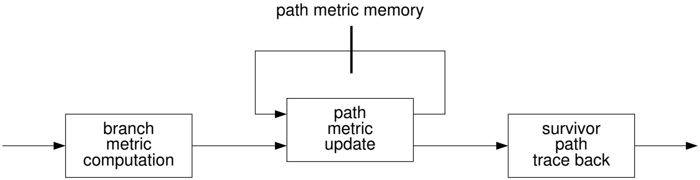


Figure: The surviving trace yields the solution (shortest path through trellis).

General policy

The procedure is known as "Dynamic Programming" and goes

1. Assign each branch a cost metric.
2. Update all path metrics by discarding those edges found to be suboptimal (add-compare-select = ACS).
3. Starting from the final state, trace back the surviving path.



Note:

- "Programming" here is a synonym for "finding an optimal plan".
- Step 2. is to be carried out recursively stage by stage.

Richard Bellman's principle of optimality

- Not all multi-stage decision problems can be solved with Dynamic Programming.
- Those that can satisfy

Bellman's principle of optimality

The globally optimum solution includes no suboptimal local decision.

In Bellman's original words from 1957:

"An optimal policy has the property that, regardless of the decisions taken to enter a particular state, the remaining decisions made for leaving that stage must constitute an optimal policy."

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"Life can only be understood backwards, but must be lived forwards."
(Sören Kierkegaard)

Applications

In numerous multistage decision problems of optimum fit such as:

- Error correction coding (Viterbi algorithm).
- Automatic labelling of speech segments (dynamic time warping).
- Video coding.
- Digital watermark detection.
- Cell library binding (as part of logic optimization).
- Flight trajectory planning.
- Genome sequencing (Smith-Waterman and Needleman-Wunsch algorithms).
- Knapsack problems.
- Stereo vision.
- ...