Control Laboratory:	**a.a. 2015/2016**

Lezione 5 — 09 March

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5.1 How to design a controller

To design a controller we start from the linear block system given in figure 5.1:

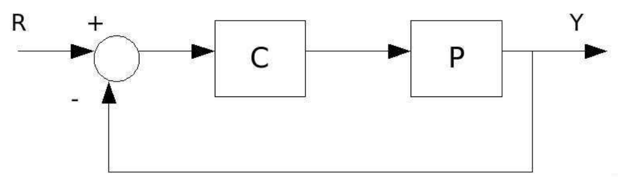

Figure 5.1. Feedback system

where we define

$G(s) = C(s)P(s)$

$W(s) = \frac{G(s)}{1+G(s)}$

In this lecture we will see what are the methods to design a controller in the frequency domain.
To do that we must pass among two steps:

- Evaluate the stability of the closed loop system, we can use two methods:
  - Routh criterion (numerator and denominator of $G(s)$)
  - Nyquist criterion (gives information about the number of the unstable poles)
- Performance metrics: we need methods to evaluate the system performance (M_s, t_r and t_s).

The first step gives us the information about the stability of the system. If it is stable we can move to the second step.

5.2 Nyquist

The Nyquist plot is the representation in the imaginary plane of the values of the transfer function at different frequencies. It can be seen as a function where we define G: $\Re \rightarrow C$ where $\omega \in \Re$ is a real number, and s is the complex variable of the codomain.

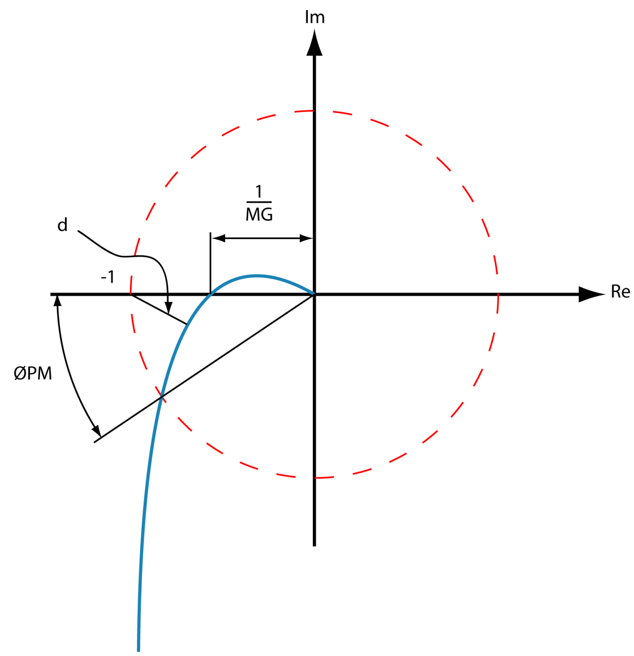

Figure 5.2. Nyquist diagram

ϕ_{pm}**: phase margin** is the amount which the phase exceed +180 degrees when $|G(j\omega)| = 1$.

$$\text{Per } \phi_{pm} \begin{cases} < 0 & \text{Unstable;} \\ > 0 & \text{Stable;} \end{cases}$$

d**: vector margin** The parameter d is the vector margin and it represent the minimum distance between the curve and the point -1.

The best parameter to evaluate the system performance is to analyze the vector margin. Despite this we use a simpler method that uses the Bode diagrams.
In some cases in which the phase margin is not very indicative, as in the case where the curve of the Nyquist plot pass multiple times for the unit circumference, we must choose the lower phase margin (figure 5.3).
The vector margin does not have these problems but it is rarely used because it is difficult to be used in the controller design and for this we use the phase margin which is easily computed from the Bode plot.

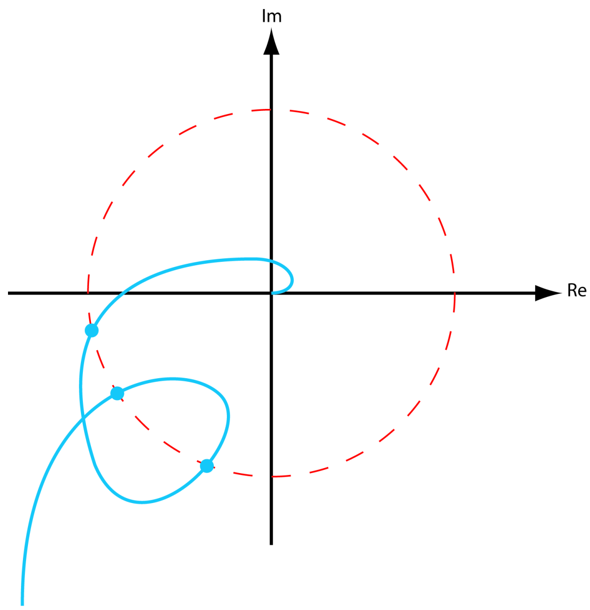

Figure 5.3. Multiple crossing

5.3 Bode

To draw the Bode's plot you can use the Nyquist diagram.
Remeber that:

$$W(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)} \tag{5.1}$$

we have:

- if $\omega \to 0, \quad W(j\omega) \to 1;$
- if $\omega \to +\infty, \quad W(j\omega) \to G(j\omega) = 0$;

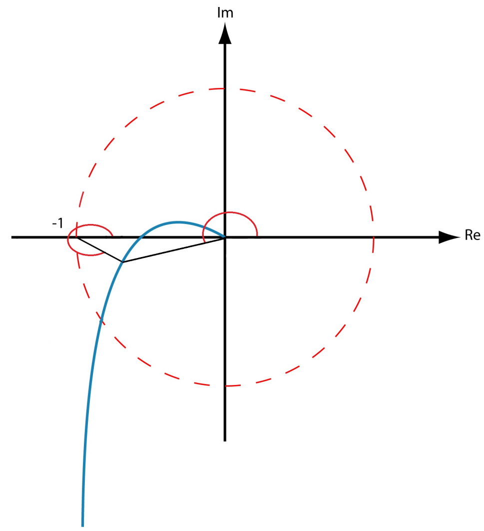

Figure 5.4.

- if $\omega \approx \omega_c$, $W(j\omega_c) \approx \frac{1}{\phi_{PM}}$; where ω_c is the crossing frequency of $G(j\omega)$

as it is showed in the (figure 5.5) for a closed loop's transfer function, approximated like a second order system with two complex poles.
The magnitude of $W(j\omega)$ for $0 < \omega < +\infty$, can be computed as the ratio:

$$\frac{|G(j\omega)|}{|1 + G(j\omega)|} \tag{5.2}$$

which is the ratio between the magnitude of the vectors in the Nyquist plot of figure 5.4.
The phase instead can be computed as the difference between the angles showed in figure 5.4:

$$\phi = \phi(G(j\omega)) - \phi(1 + G(j\omega)). \tag{5.3}$$

Observation 1: The more the Nyquist plot of $G(j\omega)$ gets closer to the point of instability -1, the more the magnitude of $W(j\omega)$ increases, showing a raising component in the Bode's plot which is called, when it reaches the maximum amplitude, resonant peak: M_r. This peak depends on the choice made for ξ and ω_n when the system is approximated by a second order system.

$$W(j\omega) \approx \frac{K\omega_n^2}{s^2 + 2s\xi\omega_n + \omega_n^2} \tag{5.4}$$

Observation 2:
If $G(j\omega) = P(j\omega)C(j\omega)$ is shaped properly by choosing $C(j\omega)$, then Bode's plots of the system approximated like a second order system looks very similar to the original up to a

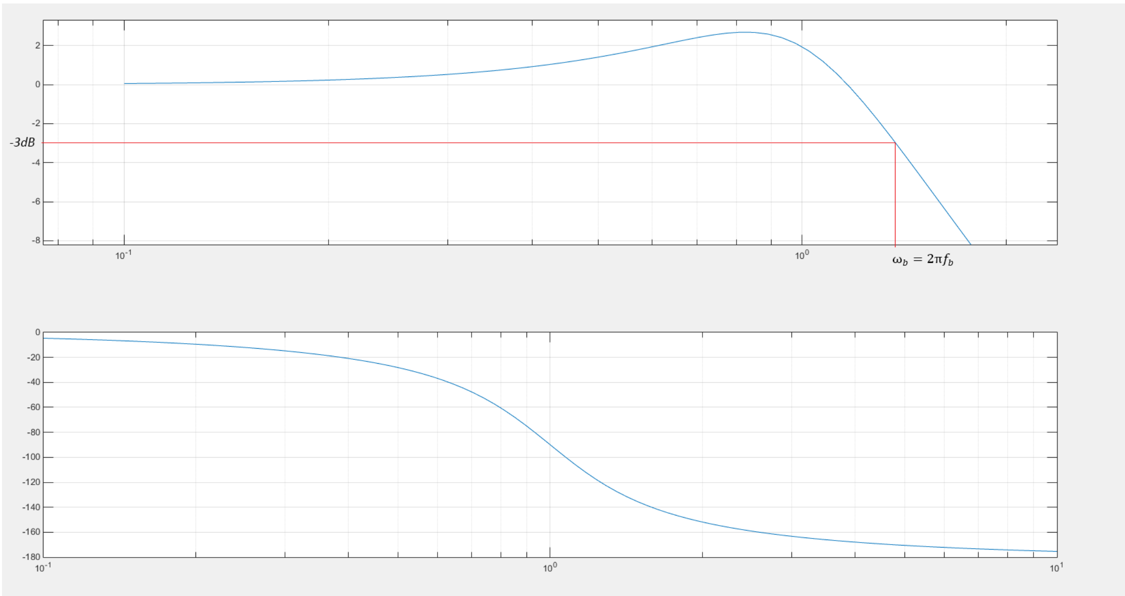

Figure 5.5. Bode diagram second order system

certain frequency $f_B = 2\pi\omega_B$ called **bandwidth** where the module value is -3db.

Observation 2.1:
The output y(t) of a closed loop system to a generic input r(t) will be similar to the output of a second order system with the same bandwidth and resonant peak M_r.

5.4 Sinusoidal input

When we apply a sinusoidal input $r(t)$ to the system we obtain

$r(t) = \sin(\omega t)$

$y(t) = |W(j\omega)| \sin(\omega t + \phi(W j\omega)))$

We can see that for $\omega >> \omega_B$ then $|W(j\omega)|$ tends to zero and consequently also y(t) tends to zero. Instead for very small ω then $|W(j\omega)|$ tends to one and y(t) is approximately equal to r(t). The system is able to track input signals whose spectral content is within the bandwidth of the closed-loop system.

5.5 Desing procedure

Design C(s) such that M_r, f_B of $W(s)$ are the same of the second order system with performance metrics (t_r, t_s, M_s, e_{ss}) $=>$ ξ, ω_n of the second order system $=>$ M_r, f_B, where e_{ss} is the steady state error to a unitary step reference input.

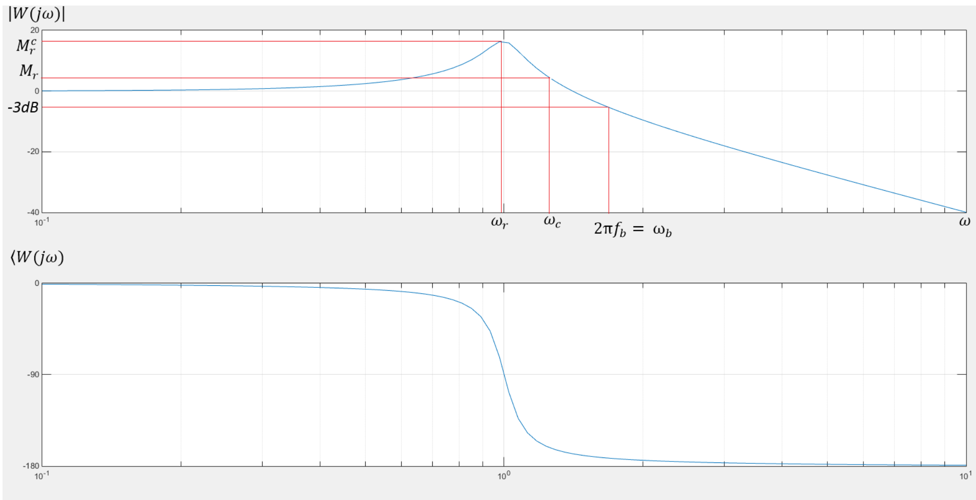

Figure 5.6. Bode diagram second order system

We know that $\omega_r \approx \omega_c$ e $M_r^c \leq M_r$

$$|W(j\omega_c)| \approx \frac{|G(j\omega_c)|}{|1 + G(j\omega_c)|} \approx \frac{1}{\phi_{PM}} \tag{5.5}$$

if $\phi_{PM} \uparrow$ then $M_r, M_s \downarrow$ and if $\omega_c \uparrow$ then t_r,$t_s \downarrow$.