1.1 Course Introduction

System control has a long tradition, probably since the XVII century some ideas of control were ready, but not yet understood. Classical control (frequency-domain design and PIDs) was developed manly between the '30s and '40s, and then later in 60's State Space Representation (time-domain design and Linear Quadratic Control) actually came in. A different approach started in control, that was called Modern Control Theory and is still applied today. In the late 60's computers started to appear and the controllers, that initially were analog devices, started being digital devices. As so, Control Systems are part of the new technological wave refferred as *Cyber-Physical Systems (CPS)*.

The goal we want to achieve is to design the controller in such a way that the feedback-based controlled system meets the required specifications, namely:

- **stability**: system signals do not diverge

- **performance**: rise and settling time, overshoot, maximum tolerable error...

- **robustness**: rejection of external disturbances, robustness with respect to uncertainty of system's parameters and non-ideality.
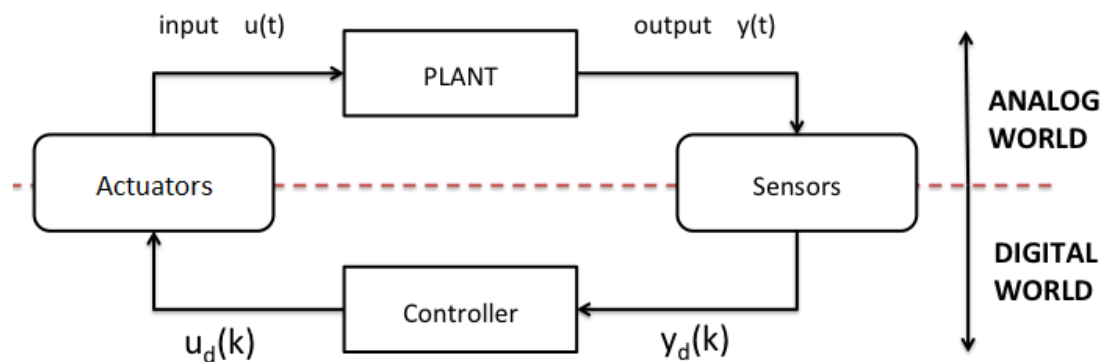

Figure 1.1. Block diagram for a controlled system

Figure 1.1 describes a classical block diagram for a controlled system which is represented by the interaction of two dynamic linear systems closed in a loop. More specifically:

- *Plant*: the dynamical system that we want to control, it could be for example an air-conditioner, an engine, an airplane;

- *Sensor*: (one or more) devices that convert a physical quantity of interest into a signal compatible with the control system;

- *Controller*: that is designed to control the plant and to meet some desired performance metrics;

- *Actuator*: generates the plant input, it transforms the (digital) controller's output into a physical (continuous) signal.

For example, for a rotative electric motor, the physical dimension measured could be the position of the shaft or its velocity. Figure 1.1 is separated by an horizontal line that divides the whole system in two *worlds* or *spaces*.
On top we have the *analog* or *physical* world, with continuos variables and on bottom we have the *digital* or *cyber* world with discrete variables.
It is important to mark that the controller represent the *smartness* of the system: it is a device that based on periodic sampling of the plant provides to the whole system the ability to act autonomously also in present of environmental changes or higher level commands.
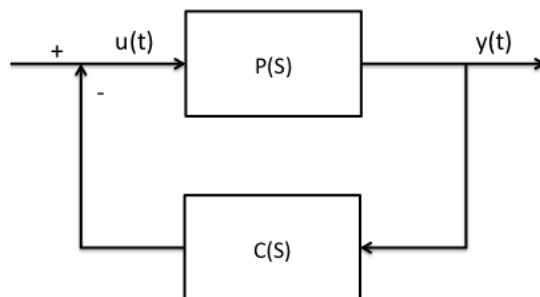

Figure 1.2.

Typically we include the plant, actuator and sensor dynamics into a single block $P(s)$ as we can see in Figure 1.2.
Actuator model $A(s)$, Phisical Plant model $P'(s)$ and Sensor model $S(s)$ are choosed in a preliminary phase: typically $A(s)$ and $S(s)$, respectively the transfer functions of actuators and sensors, are constant.

$$P(s) = A(s) \cdot P'(s) \cdot S(s)$$

All controllers today are digital, the reason is that we have more advantages like: low probability error, more flexibility and versatility.
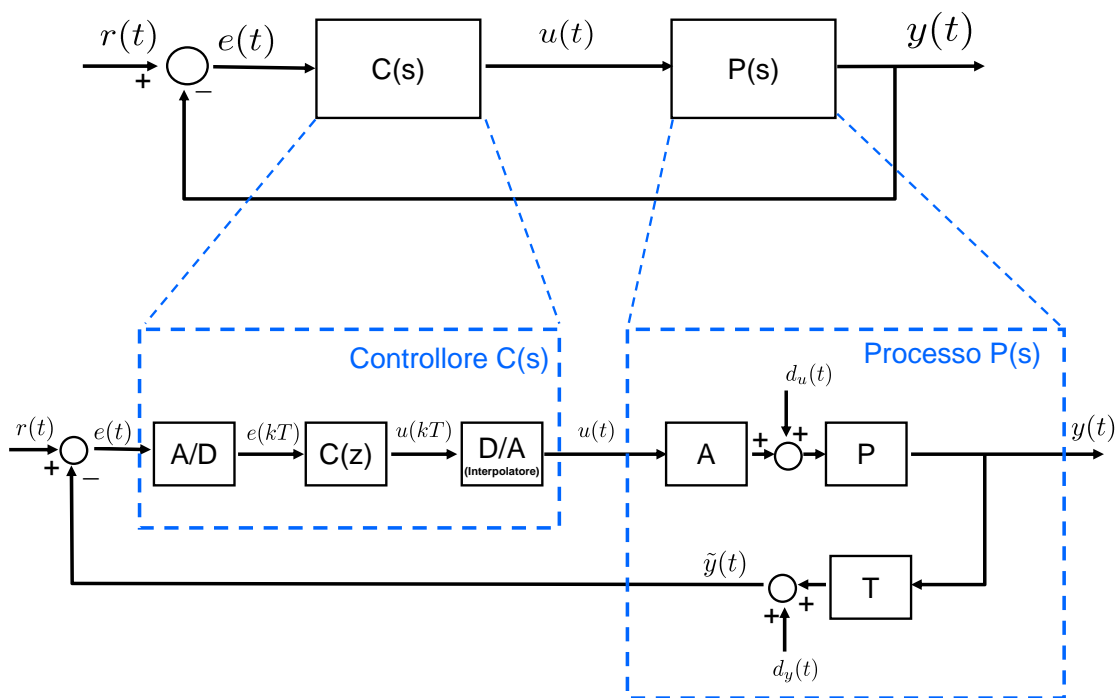Under some hypothesis we can design the controller in continuos time and then digitalize it,

Figure 1.3. Model of a classic closed-loop system

having the certainty that some specifications are met. In general, if properly designed, there is no evident performance degradation when a digital controller is used as compared to an analog controller.

However, if the controller does not act sufficiently fast as compared to the dynamic of the plant, performance could be substantially worse and even instability may appear, therefore an appropriate sampling time T has to be chosen.

However, with todays' technology we have PCs or microcontrollers that have a very high resolution in term of bits and are much faster than the time scale associated with a plant.

Another difference in the design phase is the equations that we handled in the two different approaches: differential equations and difference equations. Apart from few exceptions, most of todays's controllers are digital, so we need to convert signals to adapt them to both *worlds*. To do that, we use ADC and DAC block, as shown in Figure 1.3.

Analog to Digital converter transforms the continuos input signal into a digital signal by

sampling and quantizing. The sampling operation is made using a period T and the quantization maps continuos values to a finite set of values that depends on the number of bits used. These operations produce delays and quantization errors.

Digital to Analog converter performs the opposite operation, converting the digital signal output from the controller to another continuos signal that will be the plant's input signal. Typically, but not necessarily, it is a Zero Order Holder (ZOH) that works in sync with the ADC. A ZOH simply keeps the input values constant for a period T, like represented in Figure 1.4.
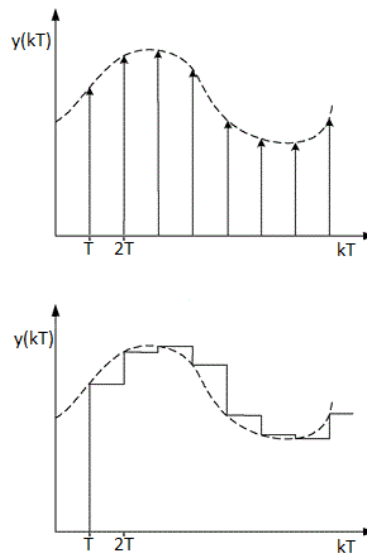
Figure 1.4. Zero Horder Hold input (top) and output (bottom)

Starting from a given transfer function $P(s)$, we will design a continous time controller $C(s)$ and, by means of a proper discretization procedure, its discrete counterpart $C(z)$, as shown in Figure 1.5. There are two different approaches to reach this goal: *classic control*, which we have seen in Automatic Control class and *modern control*, presented in System Theory class.
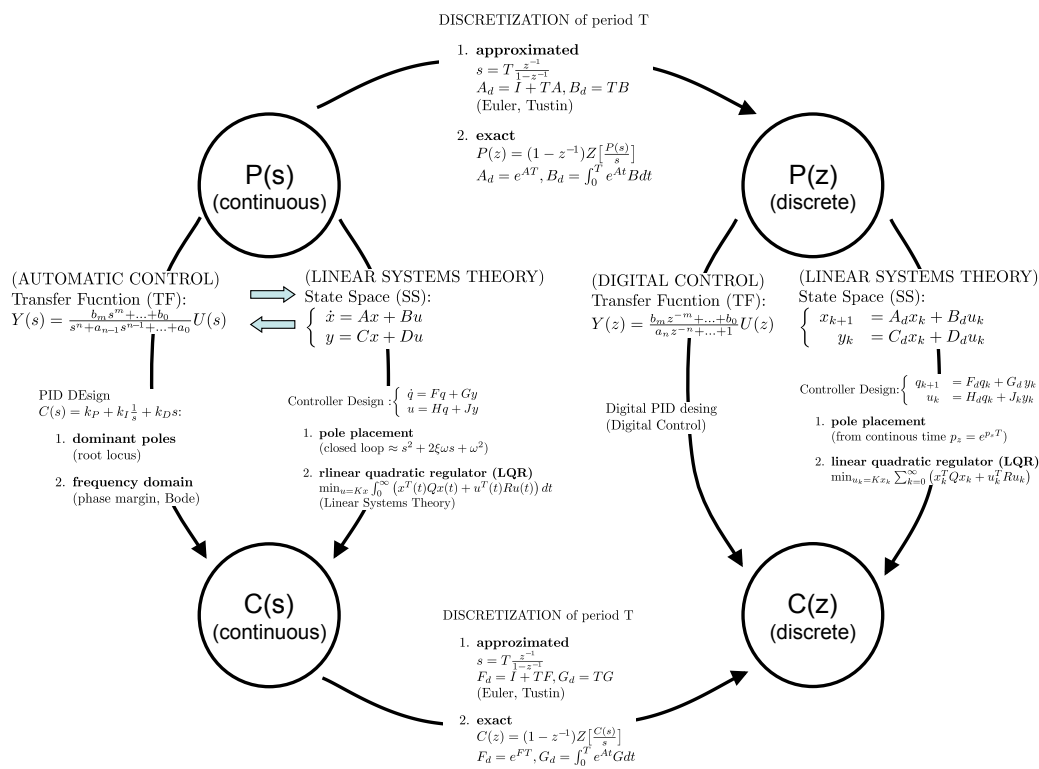
DISCRETIZATION of period T

1. **approximated**
$s = T\frac{z^{-1}}{1-z^{-1}}$
$A_d = I + TA, B_d = TB$
(Euler, Tustin)

2. **exact**
$P(z) = (1 - z^{-1})Z\left[\frac{P(s)}{s}\right]$
$A_d = e^{AT}, B_d = \int_0^T e^{At}Bdt$

P(s)
(continuous)

P(z)
(discrete)

(AUTOMATIC CONTROL)
Transfer Fucntion (TF):
$Y(s) = \frac{b_m s^m + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0}U(s)$

(LINEAR SYSTEMS THEORY)
State Space (SS):
$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$

(DIGITAL CONTROL)
Transfer Fucntion (TF):
$Y(z) = \frac{b_m z^{-m} + \ldots + b_0}{a_n z^{-n} + \ldots + 1}U(z)$

(LINEAR SYSTEMS THEORY)
State Space (SS):
$\begin{cases} x_{k+1} = A_d x_k + B_d u_k \\ y_k = C_d x_k + D_d u_k \end{cases}$

PID DEsign
$C(s) = k_P + k_I \frac{1}{s} + k_D s$:

1. **dominant poles**
(root locus)

2. **frequency domain**
(phase margin, Bode)

Controller Design : $\begin{cases} \dot{q} = Fq + Gy \\ u = Hq + Jy \end{cases}$

1. **pole placement**
(closed loop $\approx s^2 + 2\xi\omega s + \omega^2$)

2. **rlinear quadratic regulator (LQR)**
$\min_{u=Kx}\int_0^\infty \left(x^T(t)Qx(t) + u^T(t)Ru(t)\right)dt$
(Linear Systems Theory)

Digital PID desing
(Digital Control)

Controller Design: $\begin{cases} q_{k+1} = F_d q_k + G_d y_k \\ u_k = H_d q_k + J_k y_k \end{cases}$

1. **pole placement**
(from continous time $p_z = e^{p_s T}$)

2. **linear quadratic regulator (LQR)**
$\min_{u_k=Kx_k}\sum_{k=0}^\infty (x_k^T Q x_k + u_k^T R u_k)$

C(s)
(continuous)

C(z)
(discrete)

DISCRETIZATION of period T

1. **approzimated**
$s = T\frac{z^{-1}}{1-z^{-1}}$
$F_d = I + TF, G_d = TG$
(Euler, Tustin)

2. **exact**
$C(z) = (1 - z^{-1})Z\left[\frac{C(s)}{s}\right]$
$F_d = e^{FT}, G_d = \int_0^T e^{At}Gdt$

Figure 1.5. Design Procedure starting from a continuous-time plant to a digital controller