Advances in System Identification: Gaussian Regression and Robot Inverse Dynamic Learning

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Ph.D. Defense

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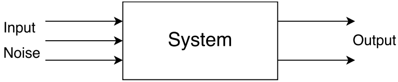
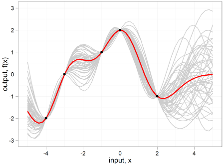
Introduction

System Identification:



Methodology:



GP regression

Targets:

- Methodological aspects
- Applicability to real systems

Applications:

PhD Overview

I Enhance Methodology: Gaussian Process Regression

- Comparison of Classical methods GP regression.

  G. Prando, D. Romeres, G. Pillonetto and A. Chiuso. Classical vs. Bayesian methods for linear system identification: point estimators and confidence sets. 15th European Control Conference, ECC 2016.

- Enforce stability in nonparametric prediction error identification methods

  D. Romeres, G. Pillonetto and A. Chiuso. Identification of stable models via nonparametric prediction error methods. 14th European Control Conference, ECC 2015.

- Online GP regression

  D. Romeres, G. Prando, G. Pillonetto and A. Chiuso. On-line Bayesian System Identification. 15th European Control Conference, ECC 2016.

  G. Prando, D. Romeres and A. Chiuso. Online Identification of Time-Varying Systems: a Bayesian approach. 55th IEEE Conference on Decision and Control, CDC 2016.

II Application: Robotic Inverse Dynamic Learning

- Learning the Inertia Parameters of a Robotic Dynamical Model

Online semiparametric learning for inverse dynamics modeling

D. Romeres, M. Zorzi, R. Camoriano and A. Chiuso. Online semiparametric learning for inverse dynamics modeling. 55th IEEE Conference on Decision and Control, CDC 2016.

Motivations

Physics-based models (Parametric models)

☺ Global approximation

☹ Subject to assumptions (it is not possible to model everything!)

Data-Driven models (Nonparametric models)

☺ Flexibility

☹ Local approximation (poor generalization to different data)

Semiparametric models

☺ Combine the strengths of Parametric & Nonparametric models

Why Online Setting?

- Real-time update
- Large scale datasets
- Time variant systems

Outline

Outline

Set-up

Problem statement

Inverse Dynamics $g: \underbrace{q_t^\top \; \dot{q}_t^\top \; \ddot{q}_t^\top}_{\text{joint pos vel acc}} \longrightarrow \underbrace{y_t}_{\text{joint torques \& forces}}$

Data: Output $\{y_t\}_{t=1,..,N}$,

Input $\{x_t\}_{t=1,..,N}$, $x_t = [q_t^\top \; \dot{q}_t^\top \; \ddot{q}_t^\top]^\top \in \mathbb{R}^{3n_{\text{dof}}}$

Model: $y_t = g(x_t) + e_t \qquad e_t \sim \mathcal{N}(0, \sigma^2 I_{n_{\text{dof}}})$

Goal: Estimate the inverse dynamics g

- How to model g?
- How to update the estimate of g online?

iCub

iCub is a full-body humanoid robot.

Outline

Parametric Model

Rigid Body Dynamics (RBD)

$$y_t = M(q_t)\ddot{q}_t + C(q_t, \dot{q}_t)\dot{q}_t + G(q_t)$$

M inertia matrix

C Coriolis and centripetal forces

G gravity forces

Linear Inverse Dynamics Model

$$y_t = \psi^\top(x_t)\,\pi, \qquad x_t = [q_t^\top \; \dot{q}_t^\top \; \ddot{q}_t^\top]^\top \in \mathbb{R}^{3n_{\text{dof}}}$$

π are the Inertial Parameters

Problem: need of assumptions (e.g. links rigidity, simple friction models,...).

How to account for the non linear dynamical effects?

Nonparametric Gaussian Model

Prior Distribution

$p(g|\eta) \sim \mathcal{N}(0, K_\eta)$

- K_η: covariance matrix $K_\eta(x_s, x_t) = cov(g(x_s), g(x_t)) = \lambda \exp\left(-0.5 \frac{\|x_s - x_t\|^2}{\tau}\right)$
- η: set of hyper-parameters
    - Marginal likelihood maximization (ML)
    - Cross Validation (CV)

Minimum Variance Estimate

With η fixed and with Gaussian innovation, the posterior distribution

$$p(g|Y,X) = \frac{p(Y,X|g)p(g|\eta)}{p(Y,X)}$$

is Gaussian. Hence, the *minimum variance estimator* is known in closed form:

$$\hat{g}(\cdot) := \mathbb{E}[g|Y,X,\eta] = K_{\hat{\eta}}(\cdot,X)\left(K_{\hat{\eta}}(X,X) + \sigma^2 I_N\right)^{-1} Y$$

How to overcome the local properties of this model?

Semiparametric Models

$$y_t = \underbrace{\Psi^\top(x_t)\pi + f(x_t)}_{g(x_t)} + e_t, \quad e_t \sim \mathcal{N}(0, \sigma^2 I_{n_{\text{dof}}})$$

	P	NP	SP	SPK
π	deterministic		deterministic	$\mathcal{N}(0, \gamma I)$
f		$\mathcal{N}(0, K_G)$	$\mathcal{N}(0, K_G)$	$\mathcal{N}(0, K_G)$

General Model

All the models can be written as Nonparametric Gaussian Model e.g.

$$\text{SP:} \quad g(x_t) \sim \mathcal{N}(\Psi^\top(x_t)\pi, K_G(x_t, \cdot))$$

$$\text{SPK:} \quad g(x_t) \sim \mathcal{N}(0, \gamma\Psi^\top(x_t)\Psi(\cdot) + K_G(x_t, \cdot))$$

Predictor when a new point x^* arrives:

$$\hat{g}(x^*) = \mu(x^*) + K(x^*, X)\left(K(X, X) + \sigma^2 I_N\right)^{-1}(Y - \mu(X))$$

Online Setting

Kernel Approximation by Random Features:

$$K_G(x_t, x_s) = \int_{\mathbb{R}^m} p(\omega) e^{i \frac{\omega^\top (x_t - x_s)}{\tau}} \mathrm{d}\omega, \quad p(\omega) = \frac{1}{(\sqrt{2\pi})^m} e^{-\frac{\|\omega\|^2}{2}}.$$

$$\approx \frac{1}{d} \sum_{k=1}^{d} e^{i \frac{\omega_k^\top (x_t - x_s)}{\tau}} = \phi_\tau(x_t)^\top \phi_\tau(x_s), \quad \omega_k \sim p(\omega),\ \phi_\tau(x_s) \in \mathbb{R}^{2d}$$

A. Rahimi and B. Recht. Random Features for Large-Scale Kernel Machines. Advances in neural information processing systems, 2007.

Problem approximation e.g. in NP (SP and SPK handled similarly):

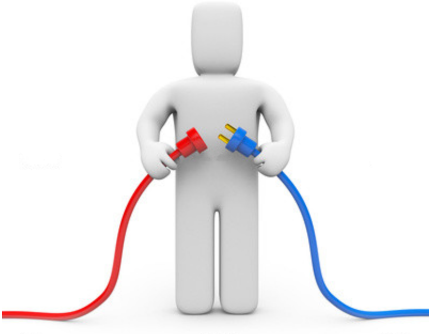
$$\hat{g}_T = \underset{g \in \mathscr{H}}{\operatorname{argmin}} \frac{1}{\sigma^2} \sum_{t=1}^{T} \|y_t - g(x_t)\|^2 + \lambda \|g\|^2_{\mathscr{H}}$$

$$\hat{\theta}_T = \underset{\theta \in \mathbb{R}^{2d}}{\operatorname{argmin}} \frac{1}{\sigma^2} \sum_{t=1}^{T} \|y_t - \phi_\tau^\top(x_t)\theta\|^2 + \|\theta\|^2_{\Sigma_0^{-1}}$$

Online Update

Recursive Regularized Least Squares

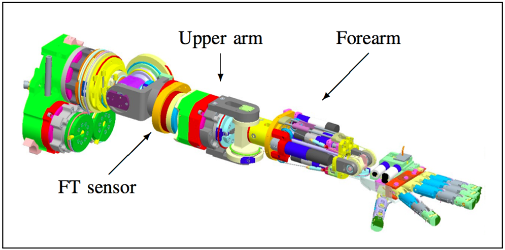
...putting all together



- P, NP, SP, SPK $\longrightarrow$ Nonparametric Gaussian model
- Kernel approximation $\longrightarrow$ Random features
- Nonparametric Gaussian model $\longrightarrow$ approximated to a linear model
- Online updates $\longrightarrow$ Recursive least squares

Outline

iCub's Left Arm



iCub - Left arm components

- 3 shoulder joints
- 1 elbow joint
- Force-Torque (FT) sensor

Data type:

- Input: $x_t = [q_t^1, \dot{q}_t^1, \ddot{q}_t^1, \dots, q_t^4, \dot{q}_t^4, \ddot{q}_t^4]^\top$
- Output: $y_t = [f_t^x, f_t^y, f_t^z, \tau_t^x, \tau_t^y, \tau_t^z]^\top$

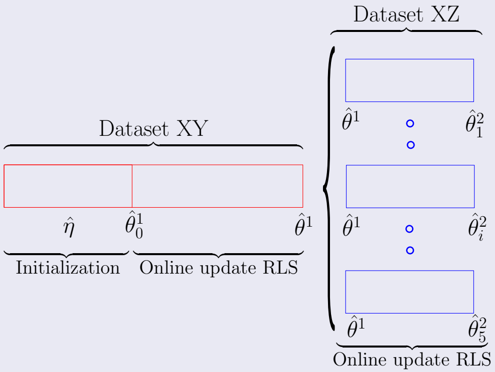
Online Learning Scenario

Recordings

Dataset 1: XY-plane circles, radius $= 10cm$, speed $= 6m/s$, 8 minutes of recordings, $F_s = 20Hz$, 10000 points.

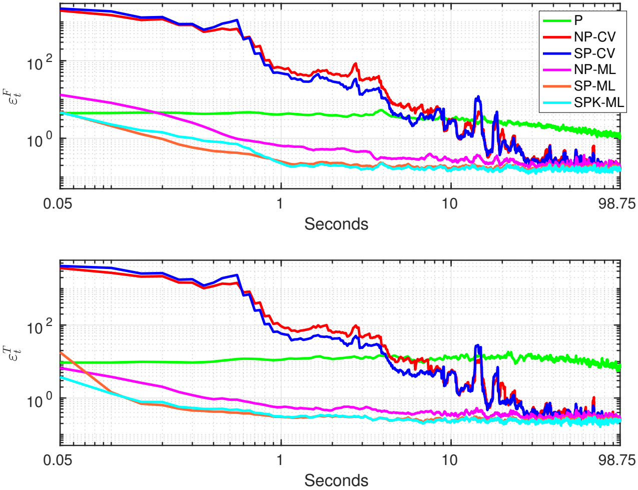
Dataset 2: XZ-plane circles, radius $= 10cm$, speed $= 6m/s$, 8 minutes of recordings, $F_s = 20Hz$, 10000 points.

Experiment



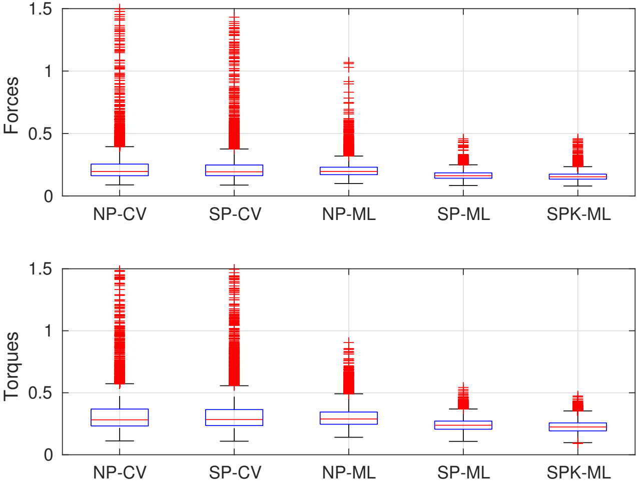
Performance

Prediction Error: $\varepsilon_t = \frac{\sum_{s=1}^{25}(y_{t+s} - \hat{y}_{t+s|t})^2}{\sum_{s=1}^{25}(y_{t+s})^2}$, $\varepsilon_t^F, \varepsilon_t^\tau$ average over Forces & Torques



Performance in steady state

Steady state (t $\geq$ 30 s) for ε_t^F, ε_t^τ



Outline

Differentiation Free Method

Input locations suggested by the physics

$$x_t = [q_t^i, \dot{q}_t^i, \ddot{q}_t^i]^\top i = 1, \dots, 4$$

Issue

In practise usually joint velocities and accelerations are not measured

$\rightarrow$ numerical differentiation from the measured joint positions

$\rightarrow$ necessity of low pass filter, smoothing filter

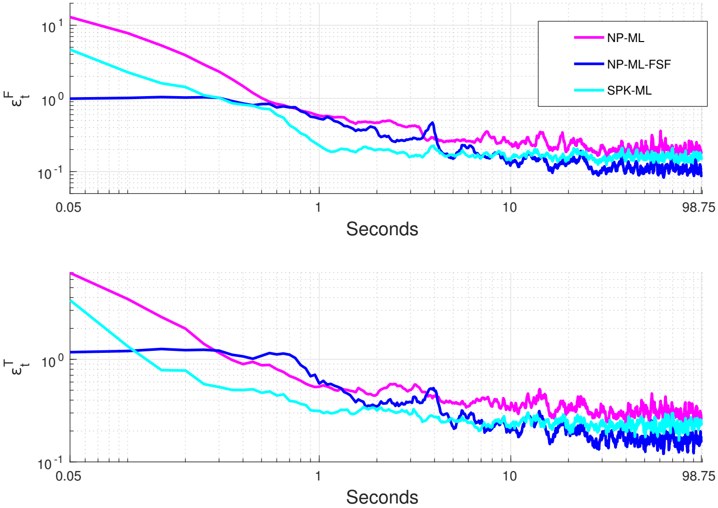
Input locations with Features Structure Free (FSF)

$$\text{Replace} \quad x_t = \begin{bmatrix} q_t \\ \dot{q}_t \\ \ddot{q}_t \end{bmatrix}, \quad \text{with} \quad x_t = \begin{bmatrix} 1 & 0 & \dots \\ & \rho_1^\top & \\ & \rho_2^\top & \end{bmatrix} \underbrace{\begin{bmatrix} q_t \\ q_{t-1} \\ \vdots \\ q_{t-K} \end{bmatrix}}_{q}$$

Hyperparameters to estimate: $\rho_1^\top, \rho_2^\top \in \mathbb{R}^{K+1}$

Performance

Relative Prediction Error at 25 steps ahead: $\varepsilon_t^F, \varepsilon_t^\tau$ average over forces & torques



Conclusion

Conclusion

- Semiparametric combines the strengths of Physical & Data-Driven models
- Formalization of Online estimation with semiparametric models
- ML criterion outperforms CV to tune the hyperparameters
- Differentiation free method is a promising research direction

Future Work

- Enforce physical meaning to parameters estimated in Semiparametric models
- Apply different Online techniques
- Extend differentiation free methods to semiparametric models

Thank you for your attention!

List of Publications

- D. Romeres, G. Pillonetto and A. Chiuso. Identification of stable models via nonparametric prediction error methods. 14th European Control Conference, ECC 2015.
- G. Prando, D. Romeres, G. Pillonetto and A. Chiuso. Classical vs. Bayesian methods for linear system identification: point estimators and confidence sets. 15th European Control Conference, ECC 2016.
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- D. Romeres, M. Zorzi, R. Camoriano and A. Chiuso. Online semiparametric learning for inverse dynamics modeling. 55th IEEE Conference on Decision and Control, CDC 2016.

List of Publications

- Diego Romeres, Florian Dörfler and Francesco Bullo. Novel Results on Slow Coherency in Power Networks. European Control Conference, ECC 2013. **Finalist Best Student Paper Award**

- Ulrich Münz, Diego Romeres. Region of Attraction of Power Systems. NecSys 2013.

- **Patent.** Ulrich Münz, Diego Romeres. Method and apparatus for a load step robust unit commitment of power generation units in a power supply system. **Siemens**

- Saverio Bolognani, Andrea Carron, Alberto Di Vittorio, Diego Romeres, and Luca Schenato. Distributed multi-hop reactive power compensation in smart micro-grids subject to saturation constraints. IEEE Control on Decision and Conference, CDC 2012.