Novel Results on the Factorization and Estimation of Spectral Densities

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Università degli Studi di Padova

Padova, February 22nd, 2018

My Ph.D. in a nutshell

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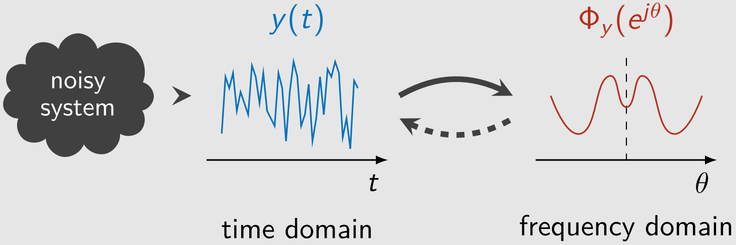

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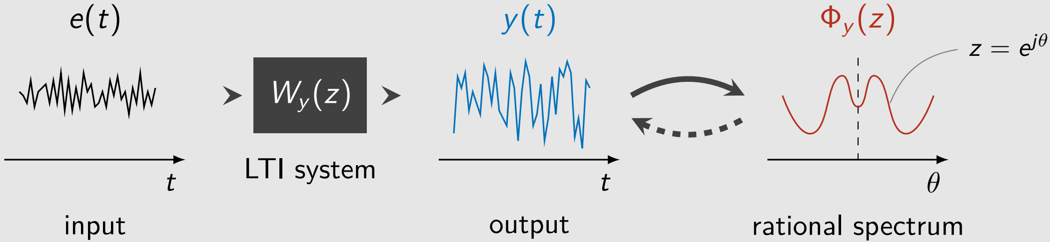

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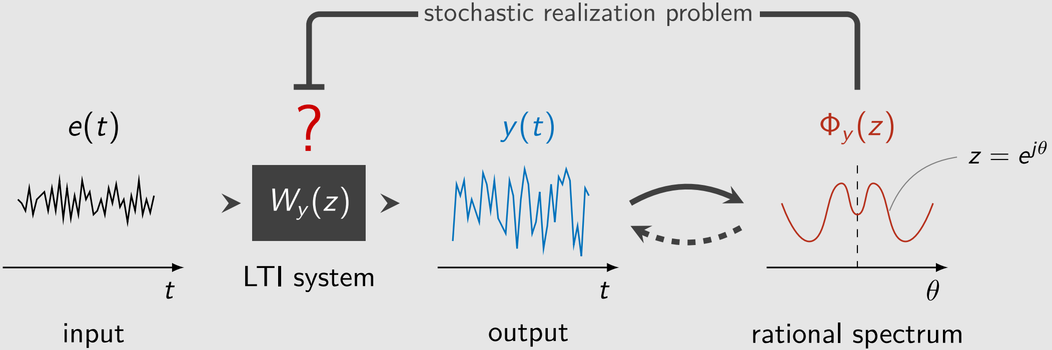

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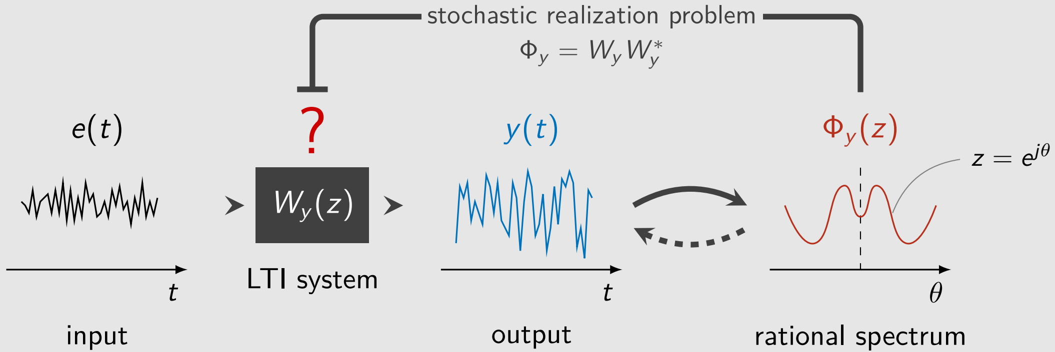

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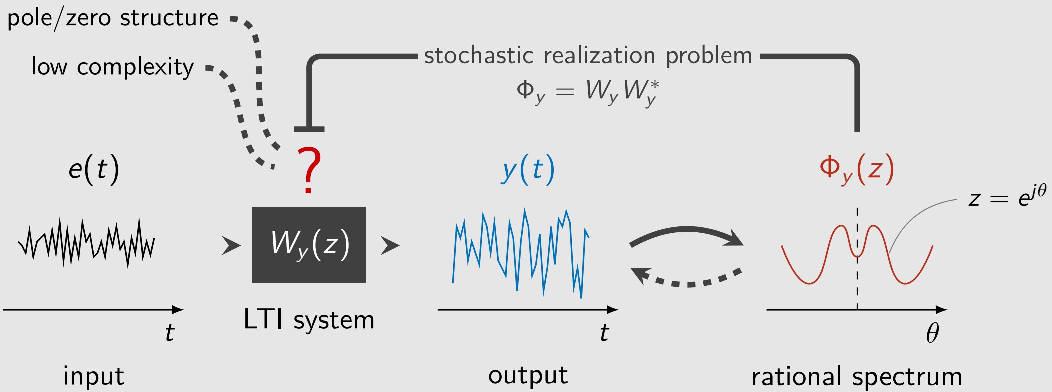

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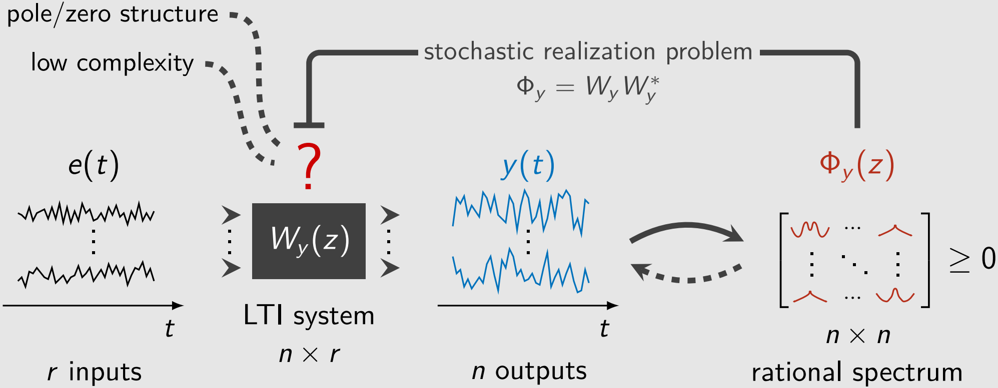

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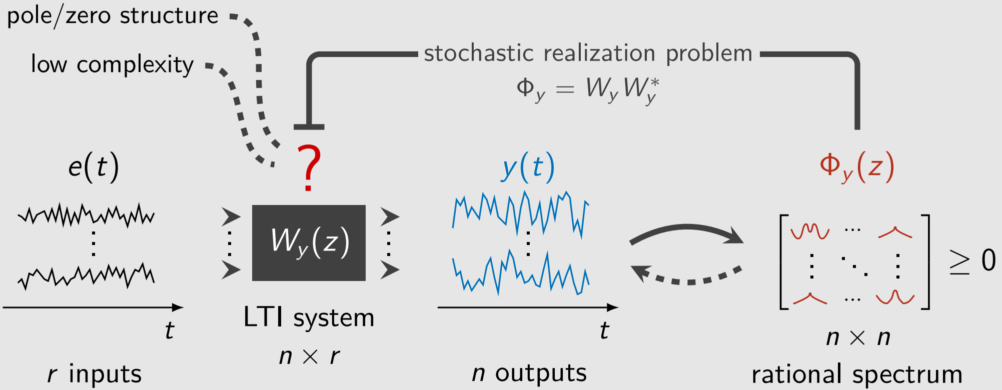

✓ Existence and computation of $W_y(z)$ [Baggio & Ferrante, IEEE TAC, 2016a]

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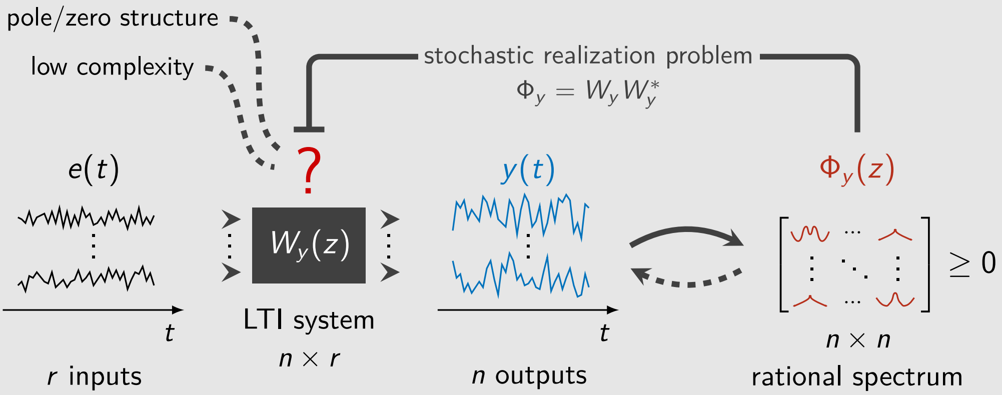

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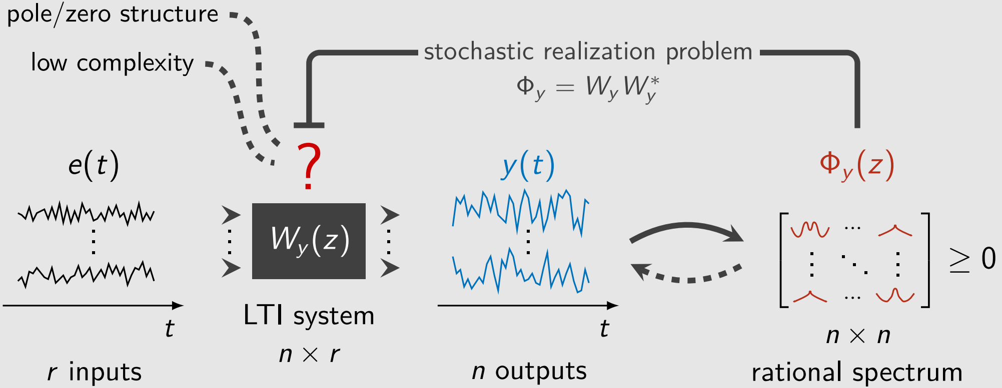

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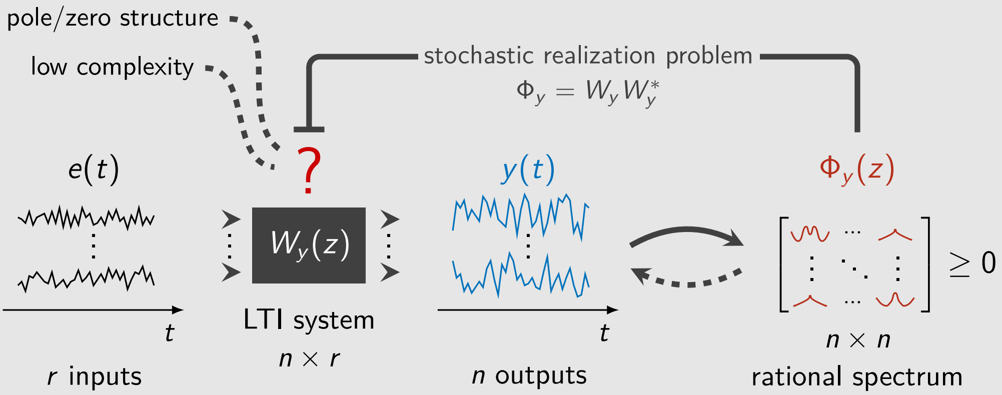

- ✓ Existence and computation ... (z) — [Baggio & Ferrante, IEEE TAC, 2016a]
- ✓ Uniqueness of min complexity $W_y(z)$ — [Baggio & Ferrante, IEEE TAC, 2016b]
- ✓ Parametrization of min complexity $W_y(z)$ — [... & Ferrante, IEEE TAC, 2018 (to appear)]

Most general setting!

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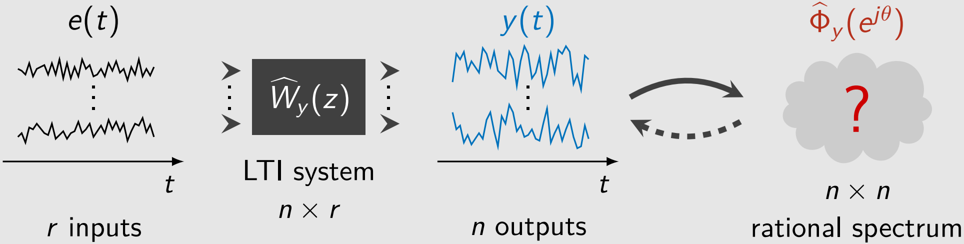

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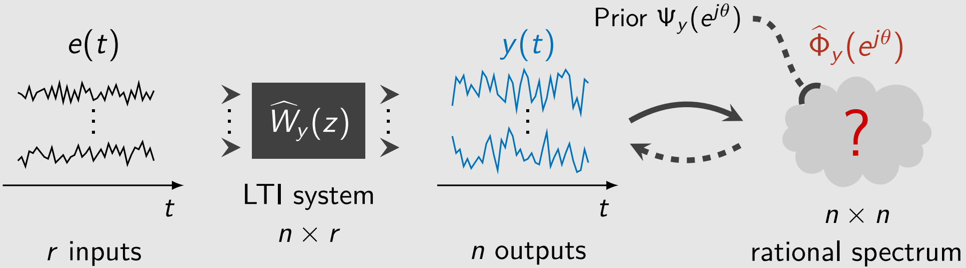

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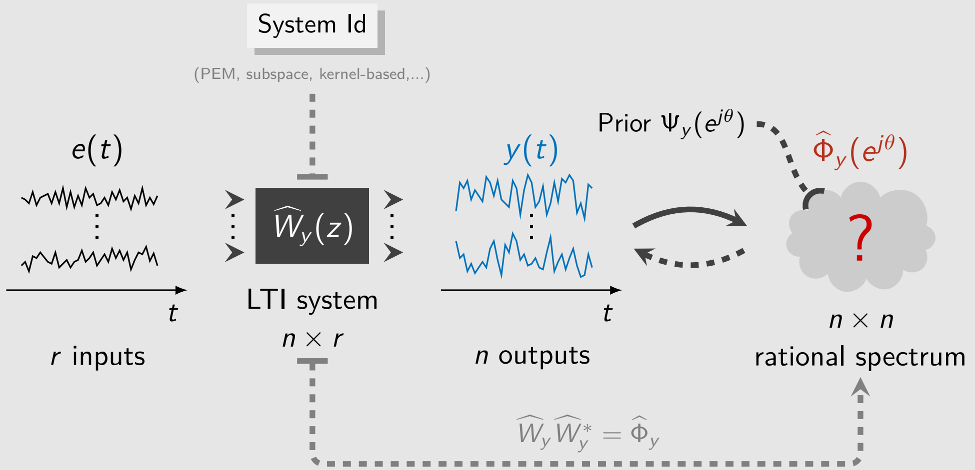

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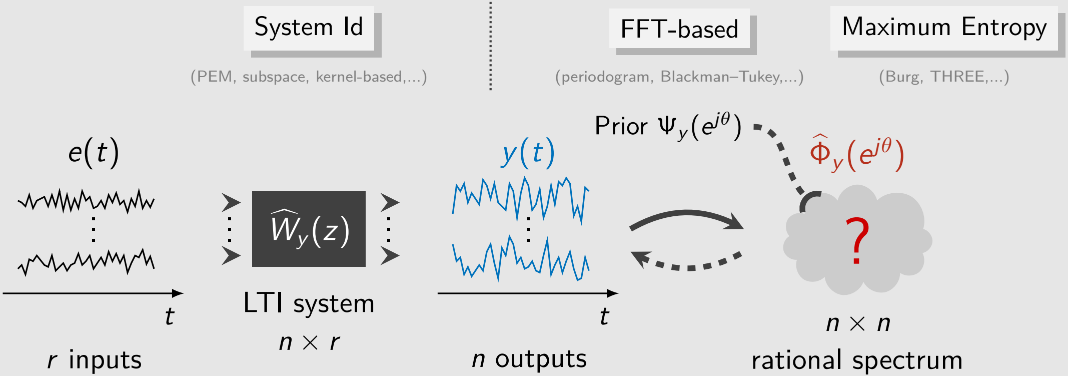

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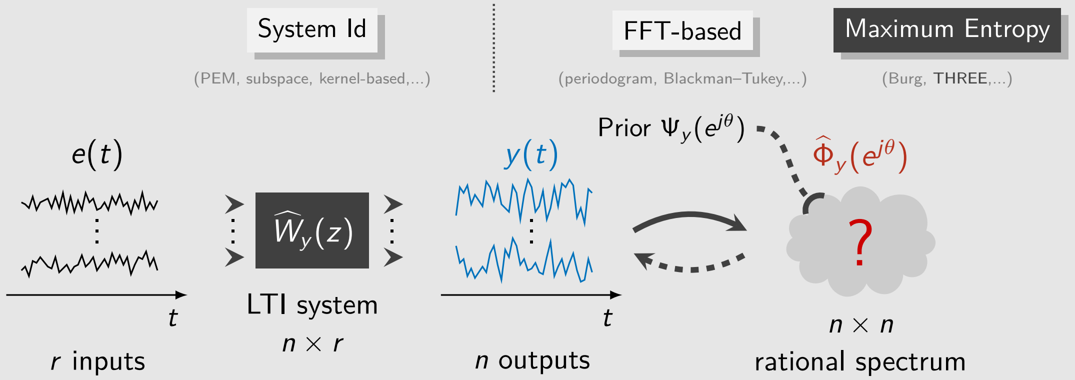

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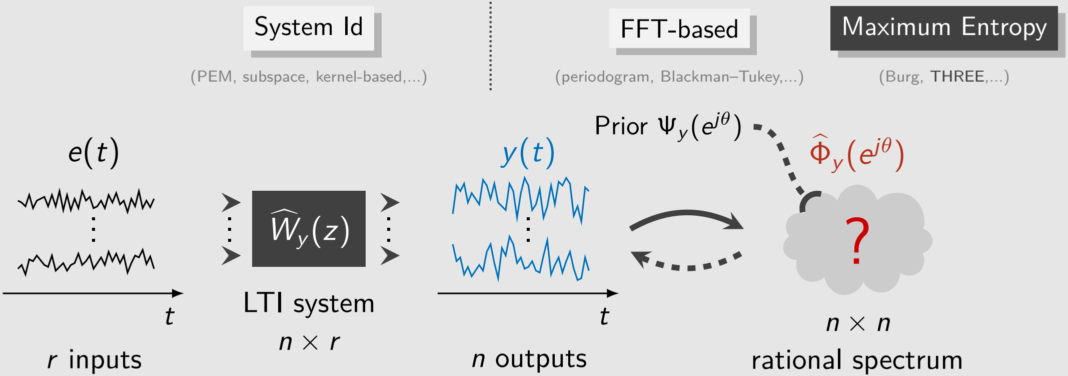

✓ Convergence analysis of an efficient method [Baggio, IEEE TAC, 2018 (to appear)]

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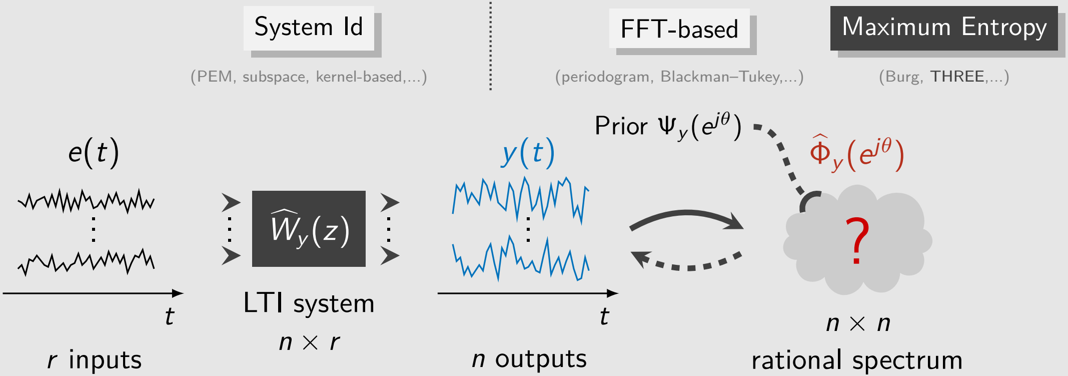

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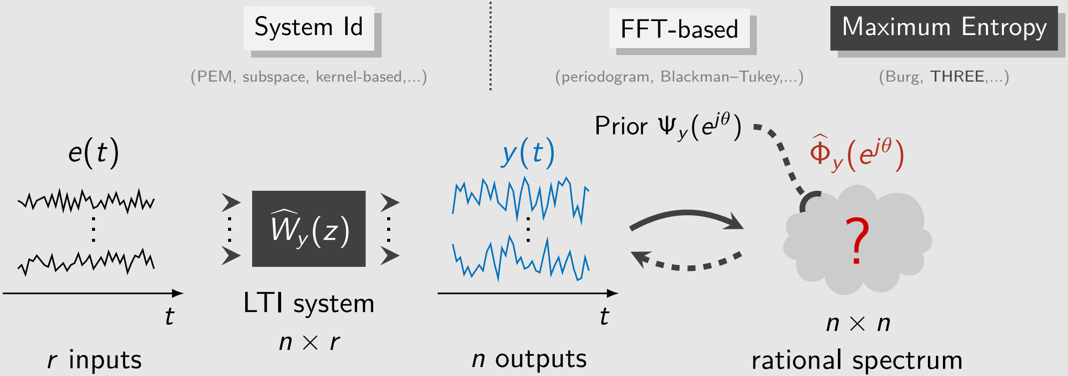

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more in a few slides...

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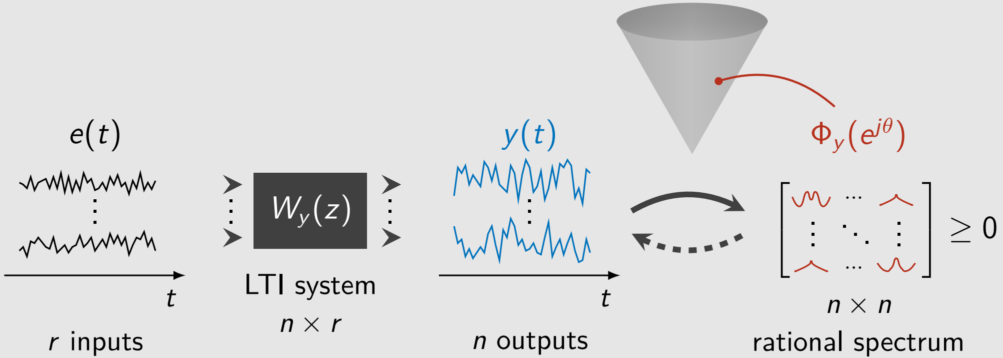

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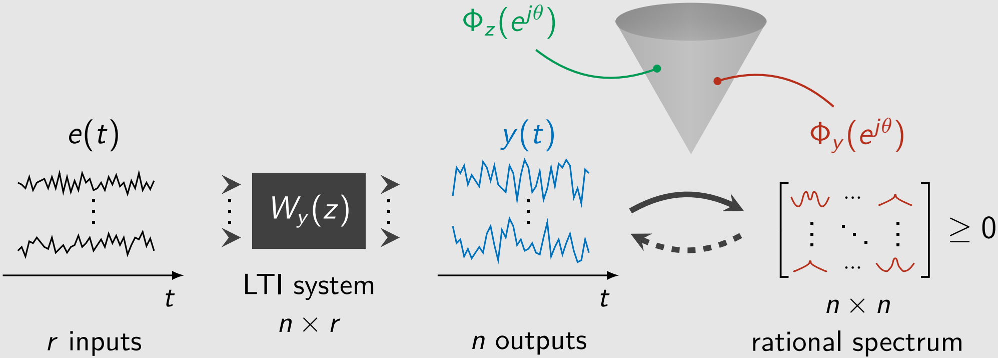

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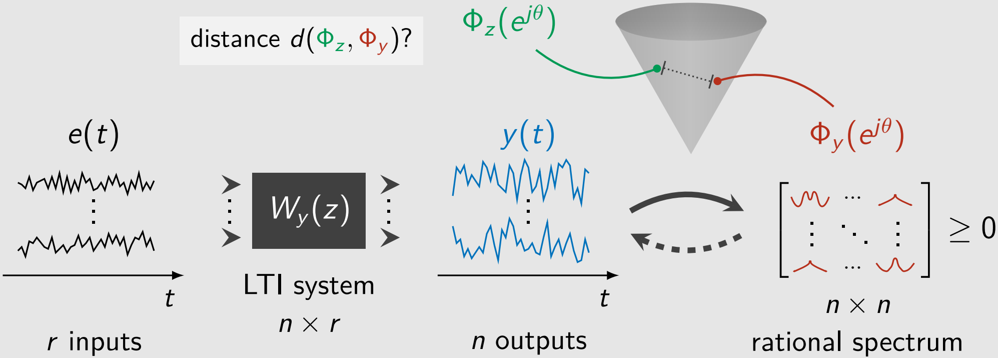

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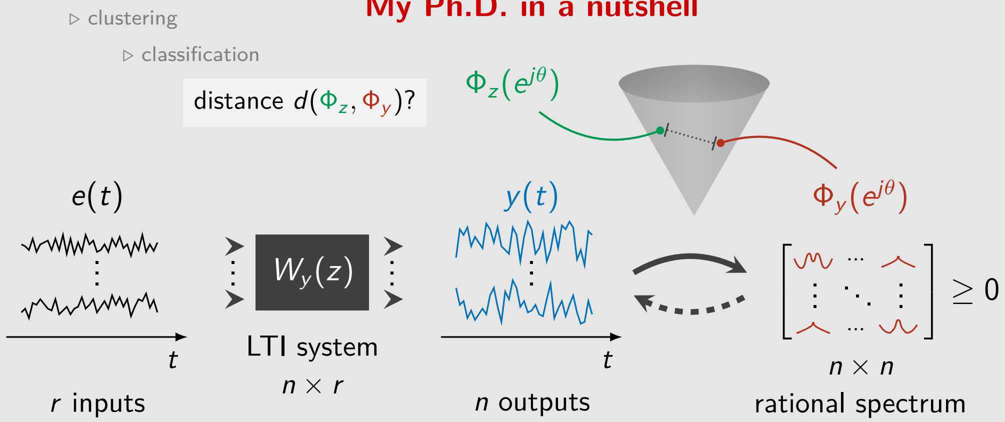
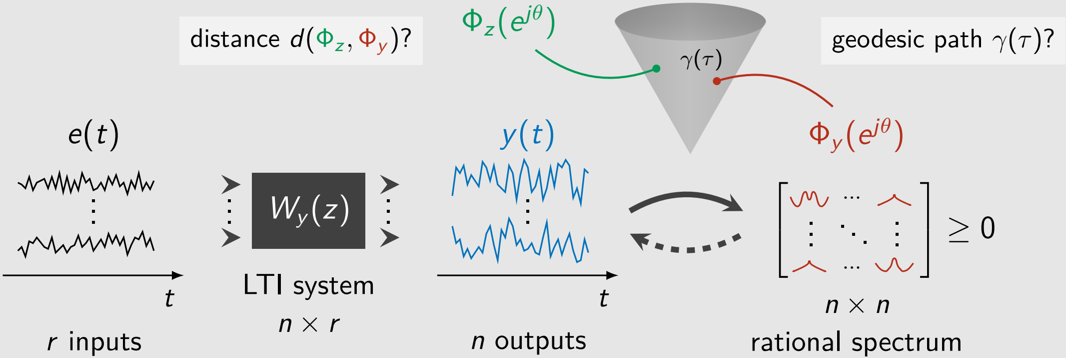


My Ph.D. in a nutshell

- ▷ clustering
  - ▷ classification



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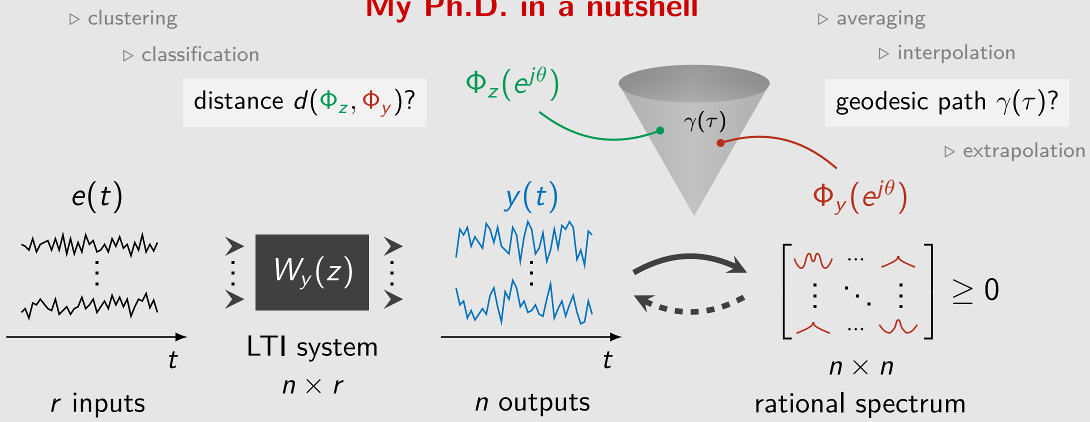

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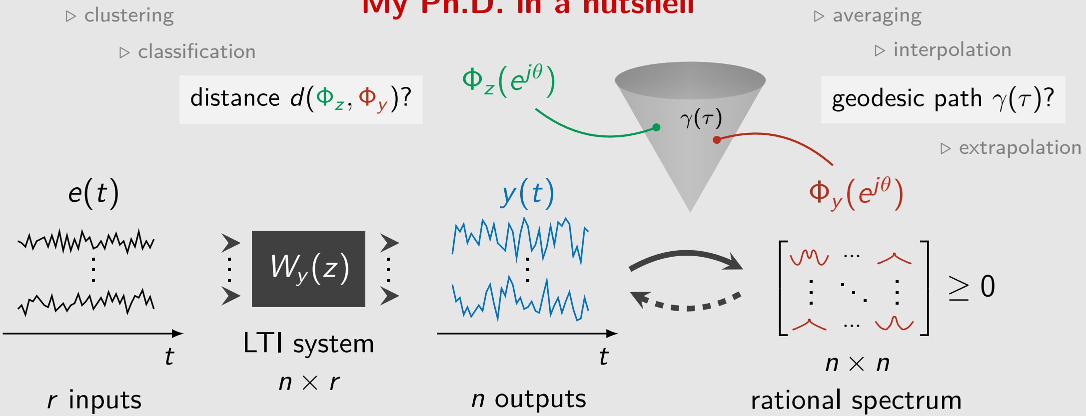

✓ Conal metrics for rational spectra [Baggio, Ferrante & Sepulchre, Under Review, 2018]

Spectral estimation

Setup

1. Let $y = \{y(t)\}_{t\in\mathbb{Z}}$ be a zero mean, real-valued, second-order stationary and purely nondeterministic process
2. Let $\{y(t)\}_{t=1}^{N}$ be a finite observation record of y

Spectral estimation

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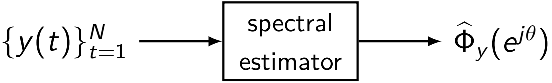
Task

Estimate the spectral density $\Phi_y(e^{j\theta})$ of y from $\{y(t)\}_{i=1}^{N}$



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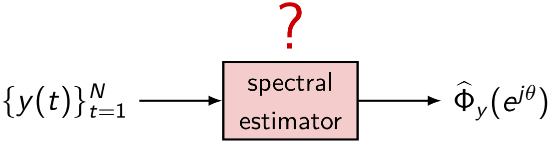

THREE-like spectral estimators

[Byrnes, Georgiou & Lindquist, IEEE TSP, 2000]

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 48, NO. 11, NOVEMBER 2000 3189

A New Approach to Spectral Estimation: A Tunable High-Resolution Spectral Estimator

Christopher I. Byrnes, *Fellow, IEEE*, Tryphon T. Georgiou, *Fellow, IEEE*, and Anders Lindquist, *Fellow, IEEE*

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A New Approach to Spectral Estimation: A Tunable
...ral Estimator

..., IEEE, and Anders Lindquist, *Fellow, IEEE*

2910 IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 49, NO. 11, NOVEMBER 2003

Kullback–Leibler Approximation of Spectral Density
Functions

Tryphon T. Georgiou, *Fellow, IEEE*, and Anders Lindquist, *Fellow, IEEE*

954 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 53, NO. 4, MAY 2008

Hellinger Versus Kullback–Leibler Multivariable
Spectrum Approximation

Augusto Ferrante, Michele Pavon, and Federico Ramponi

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 57, NO. 10, OCTOBER 2012 2561

Time and Spectral Domain Relative Entropy: A New
Approach to Multivariate Spectral Estimat...

Augusto Ferrante, Chiara Masiero, and Michele Pavon

892 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 59, NO. 4, APRIL 2014

A New Family of High-Resolution
Multivariate Spectral Estimators

Mattia Zorzi

4580 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 62, NO. 9, SEPTEMBER 2017

Likelihood Analysis of Power Spectra and
Generalized Moment Problems

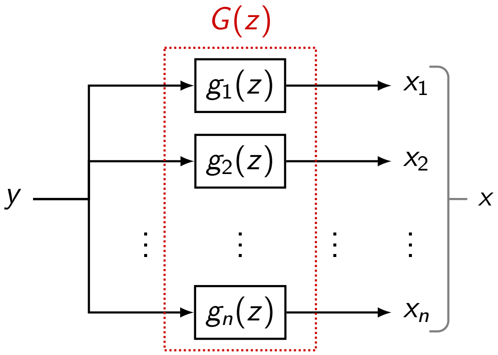
Tryphon T. Georgiou, *Fellow, IEEE*, and Anders Lindquist, *Life Fellow, IEEE*

...and many more

THREE-like spectral estimators

Ingredients

1. A bank of linear time-invariant filters $G(z) = (zI - A)^{-1}B$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, with A strictly (Schur) stable and (A, B) reachable



steady-state covariance

$$\Sigma = \mathbb{E}[xx^\top] > 0$$

$$\Sigma = \int_{-\pi}^{\pi} G(e^{j\theta})\Phi_y(e^{j\theta})G^*(e^{j\theta})\,\frac{d\theta}{2\pi}$$

moment constraint!

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$$\Downarrow$$

$$\Sigma > 0 \text{ s.t. } \Sigma = \int G\Phi_y G^*$$

2. A prior spectral density estimate $\Psi_y(e^{j\theta}) > 0$, $\theta \in [-\pi, \pi]$

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3. A distance measure between spectral densities $d(\Psi_y, \Phi_y)$

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Task

$$\text{Find } \widehat{\Phi}_y = \arg \min_{\Phi_y \in \mathcal{S}(\mathbb{T})} d(\Psi_y, \Phi_y) \;\; \text{s.t.} \;\; \Sigma = \int G\Phi_y G^*$$

Kullback–Leibler framework

[Georgiou & Lindquist, IEEE TIT, 2003]

Kullback–Leibler divergence: $\mathbb{S}(\Psi_y \| \Phi_y) := \int \Psi_y \log \frac{\Psi_y}{\Phi_y}$

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No prior information: $\mathbb{S}(I \| \Phi_y) = -\int \log \Phi_y$

entropy gain!

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Problem KL

Find $\widehat{\Phi}_y = \arg \min_{\Phi \in \mathcal{S}(\mathbb{T})} \mathbb{S}(\Psi_y \| \Phi_y)$ s.t. $\Sigma = \int G \Phi_y G^*$

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Existence of solutions?

$$\Sigma \in \text{Range}\,\Gamma, \quad \Gamma \colon X \mapsto \int G X G^*,$$

X complex-valued continuous function on the unit circle

(many other equivalent conditions...)

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Solutions?

Solution to Problem KL is unique and has the form

$$\widehat{\Phi}_y = \frac{\Psi_y}{G^* \Lambda G}, \quad \Lambda \in \mathbb{C}^{n \times n}, \; \Lambda = \Lambda^*,$$

1 $G^* \Lambda G > 0$ for all θ

2 $\int G \frac{\Psi_y}{G^* \Lambda G} G^* = \Sigma$

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$$\hat{\Phi}_y = \frac{\Psi_y}{G^* \Lambda G}, \quad \Lambda \in \mathbb{C}^{n \times n}, \ \Lambda = \Lambda^*, \quad \int \Psi_y = 1$$

1 $G^* \Lambda G > 0$ for all θ

2 $\int G \frac{\Psi_y}{G^* \Lambda G} G^* = I$

* after "normalization" of Ψ_y and Σ

Numerical solution

How to solve Problem KL ***numerically***?

gradient-based approach

[Georgiou & Lindquist, IEEE TIT, '03]

fixed-point algorithm

[Pavon & Ferrante, IEEE TAC, '06]

Numerical solution

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✗ numerical issues due to re-parametrization of admissible solutions

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Pavon–Ferrante algorithm

[Pavon & Ferrante, IEEE TAC, 2006]

$$\Lambda_{k+1} = \Lambda_k^{1/2} \int G \frac{\Psi_y}{G^* \Lambda_k G} G^* \Lambda_k^{1/2} \qquad \Lambda_0 \in \mathbb{C}^{n\times n},\ \Lambda_0 > 0,$$

(preserves unit trace and positivity)

$$\mathrm{tr}(\Lambda_0) = 1$$

Pavon–Ferrante algorithm

[Pavon & Ferrante, IEEE TAC, 2006]

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If the iteration converges to a *positive definite* fixed point $\bar{\Lambda} > 0$

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✓ 1 $G^* \bar{\Lambda} G > 0$ for all θ ✓ 2 $\int G \frac{\Psi_y}{G^* \bar{\Lambda} G} G^* = I$

✓ $\widehat{\Phi}_y = \dfrac{\Psi_y}{G^* \bar{\Lambda} G}$ solution to Problem KL

What about convergence?

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✓ Local convergence to (closure of set of) positive definite fixed points

[Ferrante, Ramponi & Ticozzi, IEEE TAC, 2011]

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positive definite?

Not guaranteed

What about convergence?

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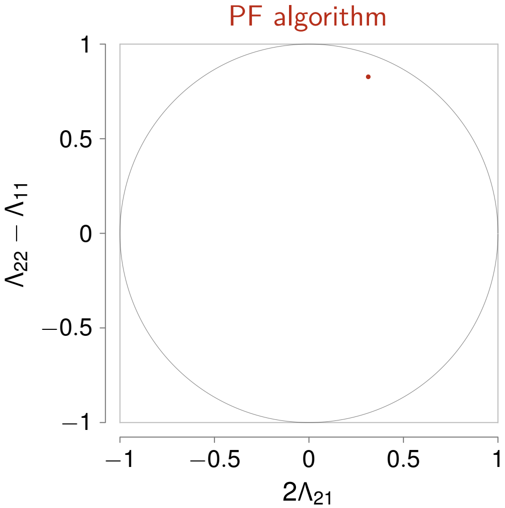
✓ Global convergence to a fixed point

(proof based on a Lyapunov argument)

positive definite?

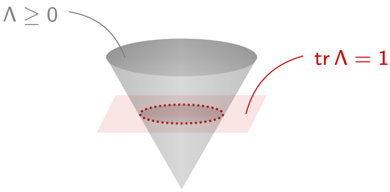
Not guaranteed, but the algorithm can be ***modified*** in order to ensure global convergence to a positive definite fixed point

Modified algorithm: example



$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Psi_y(z) = \frac{1.25}{(z+1.5)(z^{-1}+1.5)}$$



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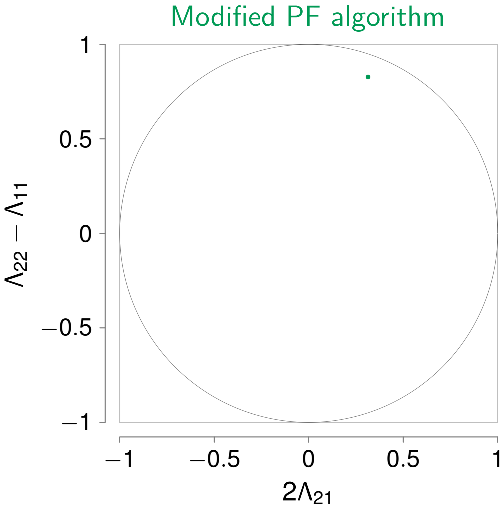
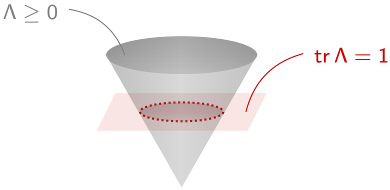
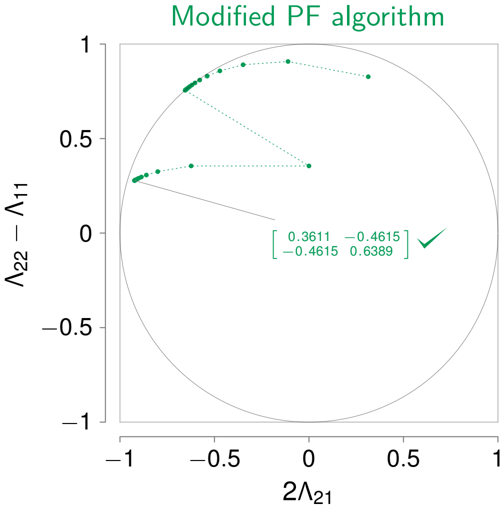


$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Psi_y(z) = \frac{1.25}{(z + 1.5)(z^{-1} + 1.5)}$$



Modified algorithm: example



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key idea

Add a suitable "correction" term whenever the trajectory approaches the boundary

Multivariate parametric extension

[Ferrante, Pavon & Zorzi, book chap., 2010]

scalar

$$\widehat{\Phi}_y = \frac{\Psi_y}{G^* \Lambda G}$$

multivariate

$$\widehat{\Phi}_{y,\Lambda} = W_{y,\Lambda}^{-1} \Psi_y W_{y,\Lambda}^{-*}, \qquad G^* \Lambda G = W_{y,\Lambda}^* W_{y,\Lambda}$$

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$m \times m$ spectral densities

outer spectral factor

$$\Lambda \in \mathcal{L} := \{\Lambda = \Lambda^* \ : \ G^* \Lambda G > 0\}$$

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the parameter

\+ moment constraint: $\int G \widehat{\Phi}_{y,\Lambda} G^* = \Sigma$

Existence of a solution?

Q: Given $\Sigma > 0$, $\Sigma \in \operatorname{Range}\Gamma$, does there exist $\bar\Lambda \in \mathcal{L}$ such that

$$\int G\widehat{\Phi}_{y,\bar\Lambda}G^* = \Sigma \quad ?$$

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partial answer

Yes, if $\Psi_y = M\psi_y$, $M = M^* > 0$ constant, ψ_y scalar spectral density

[Ferrante, Pavon & Zorzi, book chap., 2010]

Existence of a solution?

Q: Given $\Sigma > 0$, $\Sigma \in \operatorname{Range}\Gamma$, does there exist $\bar{\Lambda} \in \mathcal{L}$ such that

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partial answer

Yes, if $\Psi_y = M\psi_y$, $M = M^* > 0$ constant, ψ_y scalar spectral density

[Ferrante, Pavon & Zorzi, book chap., 2010]

complete answer

Yes, for *any* $m \times m$ prior spectral density Ψ_y!

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(proof based on a homotopy argument)

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(proof based on a homotopy argument)

?

uniqueness?
computation of solutions?

To sum up...

Maximum entropy estimation methods offer an attractive and effective alternative to standard spectral estimation techniques. The **THREE** paradigm can be thought of as a (considerable) generalization of these methods.

In the THREE setting, we investigated the **convergence** of an efficient algorithm for the Kullback–Leibler estimation of spectral densities and the **feasibility** of a parametric multivariate extension of the latter problem.

Thank you for your attention!

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