Model Reduction for Multibody Systems

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Introduction

- motorbike industry needs for dynamic analysis and performance prediction of prototypes
- in 2006 a first version of MinTime3B was released as an attempt to answer this necessity
- MinTime3B calculates an approximate solution of a minimum time problem: given a dynamic description of the bike and a 2D track, it gives the minimum time velocity profile together with a state trajectory.
- MinTime3B optimizes over tracks of second-to-minute duration range
- The core of the program is a multibody code that implements the equations of the motorbike. As the time scale of interest is quite long, fast transients and vibrations are negligible as they bring only useless complexity and numerical issues.

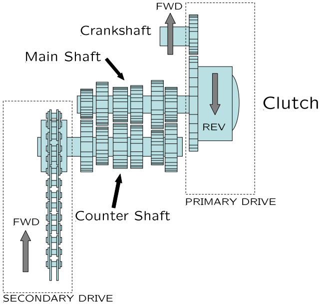
Objectives

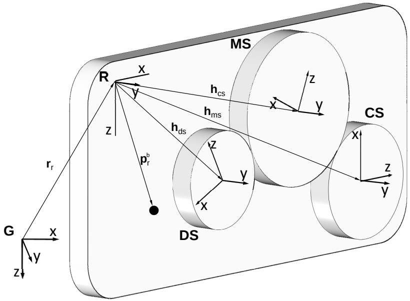
The primary objective of my research has been enriching the vehicle model without increasing the calculation time and avoiding numerical issues. Many modeling aspects have been dealt with during last years: aerodynamics, rolling resistance moments, engine torque profiles, etc. but my thesis focuses on

1. engine and gear-box rotating shafts
2. drive chain

Complete model

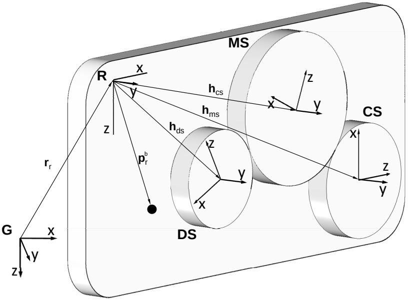
The most immediate model simulates each rotating shaft with a rigid body





Complete model's parameters

m_r, m_{cs} m_{ms}, m_{ds}	Masses of the four bodies
$\mathbb{J}_r$, $\mathbb{J}_{cs}$ $\mathbb{J}_{ms}$, $\mathbb{J}_{ds}$	Inertia tensors of the four bodies
$\mathbf{p}_r^b$	Position of the center of mass of R in body coordinates
$\mathbf{h}_{ds}$, $\mathbf{h}_{ms}$ $\mathbf{h}_{cs}$	Position of the origins of O_k with respect to Σ_r
$\rho_{ds,ms} = \frac{\dot{\theta}_{ds}}{\dot{\theta}_{ms}}$	Gear ratio of the transmission (depends on the selected gear)
$\rho_{ms,cs} = \frac{\dot{\theta}_{ms}}{\dot{\theta}_{cs}}$	Gear ratio of the primary drive

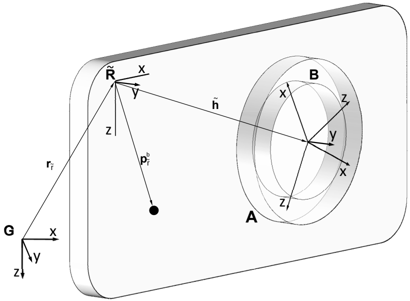


Are all bodies really necessary?

Fast model's parameters

the gyroscopic effects of three rotating bodies with parallel axes can be produced by two coaxial counter-rotating bodies

$m_{\tilde{r}}$ m_b m_a	Masses of the three bodies
$\mathbb{J}_{\tilde{r}}$ $\mathbb{J}_b$ $\mathbb{J}_a$	Inertia tensors of the three bodies
$\mathbf{p}_{\tilde{r}}^b$	Position of the center of mass of $\tilde{R}$ in body coordinates
$\tilde{\mathbf{h}}$	Position of the origins of O_k with respect to $\Sigma_{\tilde{r}}$
$\rho_{a,b} = -1$	Gear ratio



Dynamic systems equivalence 1

Definition

Consider the case of two generic dynamic systems of the form $\dot{\mathbf{x}} = f(\mathbf{x})$ and $\dot{\mathbf{y}} = g(\mathbf{y})$, where $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ are the state vectors and $f, g : \mathbb{R}^n \to \mathbb{R}^n$ smooth vector fields. If f and g are related by a global smooth change of coordinates $\mathbf{y} = \Gamma(\mathbf{x})$, that is

$$f(\mathbf{x}) = D\Gamma^{-1} \circ g \circ \Gamma(\mathbf{x}) \qquad \forall \mathbf{x} \in \mathbb{R}^n ,$$

then f and g are in fact the same dynamic system.

Flow box theorem

The Flow Box Theorem states that in a neighborhood of any regular point all autonomous differential equations are equivalent, up to a change of coordinates. In particular there always exist a particular mapping such that the trajectories of any system can even become straight lines. Nevertheless:

- it is defined only locally
- the change of coordinate is generally very complex
- state variables loose any physical meaning

Requirements

1. the equivalence must hold in all the state space by mean of a single and simple change of coordinates
2. the equivalent system must preserve a mechanical structure, which allows to implement it using a multibody code

Preliminaries

Change of coordinates in mechanical systems

Coordinate transformations between mechanical systems are defined involving only generalized coordinates and then naturally extended to velocities

$$(\tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}}) = \Gamma(\mathbf{q}, \dot{\mathbf{q}}) := (\Phi(\mathbf{q}), \mathbf{D}\Phi(\mathbf{q})\dot{\mathbf{q}}) .$$

Lagrangian approach

A Lagrangian approach has been used to derive the equations of motion in free evolution:

- Hamilton's principle $\delta \int_{t_0}^{t_1} L(\mathbf{q}(t), \dot{\mathbf{q}}(t))dt = 0$
- Euler-Lagrange equations $\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = 0$

Lemma

Be μ the vector of parameters defining the complete model and ν the vector of parameters defining the fast model. For any choice of μ with physical meaning, there exist a vector $\nu(\mu)$ such that the two Lagrangian functions L and $\tilde{L}$ associated to the models are equal up to a change of coordinates:

$$\tilde{L}(\Phi(\mathbf{q}), \mathbf{D}\Phi(\mathbf{q})\dot{\mathbf{q}}) = L(\mathbf{q}, \dot{\mathbf{q}}).$$

- we impose to Φ a linear structure like

$$\tilde{\mathbf{q}} = (\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\varphi}, \tilde{\theta}, \tilde{\psi}, \theta_a)^T = \Phi(\mathbf{q}) = (x, y, z, \varphi, \theta, \psi, \rho_e \theta_{ds})^T$$

(which is trivially a diffeomorphism as long as $\rho_e \neq 0$) with constant Jacobian

$$J_\Phi := \mathbf{D}\Phi = \begin{bmatrix} I_{6\times 6} & 0 \\ 0 & \rho_e \end{bmatrix}.$$

- we impose the equality of potential and kinetic energies:

$$V(\mathbf{q}) = \tilde{V}(\Phi(\mathbf{q}))$$

$$T(\mathbf{q}, \dot{\mathbf{q}}) = \tilde{T}(\Gamma(\mathbf{q}, \dot{\mathbf{q}}))$$

Parameters definitions 1

- we obtain the formulas that define the parameters of the Fast Model to those of the Complete one

$$m_{\tilde{r}} = m_r$$

$$\mathbf{p}_{\tilde{r}}^b = \mathbf{p}_r^b$$

$$m_a = m_b = \frac{1}{2}(m_{ds} + m_{ms} + m_{cs})$$

$$\tilde{\mathbf{h}} = \frac{m_{ds}\mathbf{h}_{ds} + m_{ms}\mathbf{h}_{ms} + m_{cs}\mathbf{h}_{cs}}{m_{ds} + m_{ms} + m_{cs}}$$

$$\rho_e = \sqrt{\frac{\mathbb{J}_{ds}^{yy} + \rho_{ms,ds}^2 \mathbb{J}_{ms}^{yy} + \rho_{cs,ms}^2 \mathbb{J}_{cs}^{yy}}{\mathbb{J}_{ds}^{yy} + \mathbb{J}_{ms}^{yy} + \mathbb{J}_{cs}^{yy}}}$$

$$\mathbb{I}_{\tilde{r}} = \mathbb{I}_r - \mathbb{I}_{a,\tilde{r}}^{pm} - \mathbb{I}_{b,\tilde{r}}^{pm} + \mathbb{I}_{ds,r}^{pm} + \mathbb{I}_{ms,r}^{pm} + \mathbb{I}_{cs,r}^{pm}$$

Parameters definitions 2 (Inertia tensors)

$$
\begin{aligned}
\mathbb{J}_a^{xx} = \mathbb{J}_a^{zz} &= \frac{1}{2}(1+\lambda)(\mathbb{J}_{ds}^{xx} + \mathbb{J}_{ms}^{xx} + \mathbb{J}_{cs}^{xx}) \\
\mathbb{J}_a^{yy} &= \frac{1}{2}(\mathbb{J}_{ds}^{yy} + \mathbb{J}_{ms}^{yy} + \mathbb{J}_{cs}^{yy}) + \frac{1}{2}(\mathbb{J}_{ds}^{yy} + \mathbb{J}_{ms}^{yy} + \mathbb{J}_{cs}^{yy})^{\frac{1}{2}} \\
&\quad (\mathbb{J}_{ds}^{yy} + \rho_{ms,ds}\mathbb{J}_{ms}^{yy} + \rho_{cs,ds}\mathbb{J}_{cs}^{yy}) \\
&\quad (\mathbb{J}_{ds}^{yy} + \rho_{ms,ds}^{2}\mathbb{J}_{ms}^{yy} + \rho_{cs,ds}^{2}\mathbb{J}_{cs}^{yy})^{-\frac{1}{2}} \\
\mathbb{J}_b^{xx} = \mathbb{J}_b^{zz} &= \frac{1}{2}(1-\lambda)(\mathbb{J}_{ds}^{xx} + \mathbb{J}_{ms}^{xx} + \mathbb{J}_{cs}^{xx}) \\
\mathbb{J}_b^{yy} &= \frac{1}{2}(\mathbb{J}_{ds}^{yy} + \mathbb{J}_{ms}^{yy} + \mathbb{J}_{cs}^{yy}) - \frac{1}{2}(\mathbb{J}_{ds}^{yy} + \mathbb{J}_{ms}^{yy} + \mathbb{J}_{cs}^{yy})^{\frac{1}{2}} \\
&\quad (\mathbb{J}_{ds}^{yy} + \rho_{ms,ds}\mathbb{J}_{ms}^{yy} + \rho_{ms,ds}\mathbb{J}_{cs}^{yy}) \\
&\quad (\mathbb{J}_{ds}^{yy} + \rho_{ms,ds}^{2}\mathbb{J}_{ms}^{yy} + \rho_{ms,ds}^{2}\mathbb{J}_{cs}^{yy})^{-\frac{1}{2}}
\end{aligned}
$$

$$
\lambda = \frac{\beta}{2\alpha}\frac{\gamma}{\sqrt{\beta\delta}} = \frac{1}{2}\frac{\mathbb{J}_{ds}^{yy} + \rho_{ms,ds}\mathbb{J}_{ms}^{yy} + \rho_{cs,ds}\mathbb{J}_{cs}^{yy}}{\mathbb{J}_{ds}^{xx} + \mathbb{J}_{ms}^{xx} + \mathbb{J}_{cs}^{xx}}\sqrt{\frac{\mathbb{J}_{ds}^{yy} + \mathbb{J}_{ms}^{yy} + \mathbb{J}_{cs}^{yy}}{\mathbb{J}_{ds}^{yy} + \rho_{ms,ds}^{2}\mathbb{J}_{ms}^{yy} + \rho_{cs,ds}^{2}\mathbb{J}_{cs}^{yy}}}.
$$

These formulas can be generalized to define a Fast model from a Complete model with an arbitrary number of shafts

Forced evolution equivalence 1

A system in free evolution is not sufficient to describe the transmission of a bike, because it does not include the engine torque, the chain tension. We need to extend the equivalence to the case with external inputs.
We need to port the external torques and forces, acting on the complete model, into the fast one.
A couple force-toque acting on a body is called wrench $\mathbf{F}$:

$$\mathbf{F} = \begin{bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{bmatrix} = \begin{bmatrix} \mathrm{f}^x & \mathrm{f}^y & \mathrm{f}^z & \tau^x & \tau^y & \tau^z \end{bmatrix}^T .$$

To each wrench of the complete model corresponds a unique generalized force

$$\Upsilon := (\mathbf{J}^b)^T \mathbf{F} .$$

We use the definition of the change of coordinates derived in the previous lemma, to define the equivalent generalized force of the fast model.

$$\tilde{\Upsilon} := [\mathbf{D}\Phi^*]^{-1}(\mathbf{q})\Upsilon = J_\Phi^{-1}\Upsilon .$$

To maintain the mechanical structure of the fast system we endowed it with inputs, whose generalized forces are equal to $\tilde{\Upsilon}$.

Forced evolution equivalence 2

Applying this method we get the following cases:

- General wrench applied on R:

$$\mathbf{F}_r \rightarrow \mathbf{F}_{\tilde{r}} := \mathbf{F}_r$$

- Wrench without torque along y, applied on a shaft K ($\mathbf{F}^* := \begin{bmatrix} f^x & f^y & f^z & \tau^x & 0 & \tau^z \end{bmatrix}^T$)

$$\mathbf{F}_k^* \rightarrow \mathbf{F}_{\tilde{r}} := Ad^T_{\mathbf{g}_{rk}^{-1}} \mathbf{F}_k^*$$

- Simple torques along y applied on CS or MS can be replaced by a torque on DS plus a corrective torque on R:

$$\boldsymbol{\tau}_k^y \rightarrow (\boldsymbol{\tau}_{ds}, \boldsymbol{\tau}_r) := (\rho_{k,ds} \boldsymbol{\tau}_k, (1 - \rho_{k,ds}) \boldsymbol{\tau}_k)$$

- Simple torque along y applied on DS

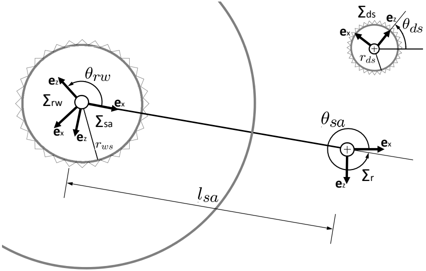
$$\boldsymbol{\tau}_{ds}^y \rightarrow (\boldsymbol{\tau}_{\tilde{r}}^y, \boldsymbol{\tau}_a^y) := \left((1 - \frac{1}{\rho_e}) \boldsymbol{\tau}_{ds}^y, \frac{1}{\rho_e} \boldsymbol{\tau}_{ds}^y \right)$$

Conclusions

- Regardless for the masses, the gear ratios, and the inertia tensors of the original rotating shafts, they can always be replaced by a couple of counter-rotating shafts with appropriate masses and inertia tensors
- It is possible to find a fast model for a complete model with an arbitrary number of shafts
- The fast model is valid in all the state space, it is a mechanical system

Half-bike Model

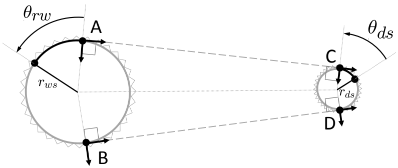
To describe and simulate the drive chain we developed a dynamic model of the rear half of a motorcycle



- The chassis is fixed to the ground, *DS*, *SA* and *RW* can rotate about their joints
- Generalized coordinates: $(\theta_{ds}, \theta_{sa}, \theta_{rw})$
- Input torques:

$$\boldsymbol{\tau}_{ds}(\dot{\theta}_{ds}, t) = \tau_{eng} + \tau_{fri}^{ds}$$
$$\boldsymbol{\tau}_{sa}(\theta_{sa}, \dot{\theta}_{sa}, \dot{\theta}_{rw}, t) = \tau_{spr} + \tau_{dam} + \tau_{bra} + \tau_{fri}^{rw}$$
$$\boldsymbol{\tau}_{rw}(\dot{\theta}_{rw}, t) = \tau_{load} - \tau_{bra} - \tau_{fri}^{rw}$$

Full chain Model



- This chain model is massless and with lumped parameters
- when a segment is tense it is tangent to both the sprockets
- the tension is applied on the tangent points and is expressed with respect to the reference frames A, B, C, D

- the extension of each segment is defined as

$$e_u := l_u - l_u^0 = r_{ws}\theta_{rw} + \overline{AC} - r_{ds}\theta_{ds} - l_u^0$$
$$e_l := l_l - l_l^0 = -r_{ws}\theta_{rw} + \overline{BD} + r_{ds}\theta_{ds} - l_l^0$$

- each segments behaves like a unilateral spring with linear characteristic

$$T_u := \begin{cases} ke_u & e_u \geq 0 \\ 0 & e_u < 0 \end{cases}$$
$$T_l := \begin{cases} ke_l & e_l \geq 0 \\ 0 & e_l < 0 \end{cases}$$

- Only one segment of chain at a time is active

Limits of the Full chain model

High stiffness and discontinuous derivative at the origin cause

- numerical issues when integrating in time
- heavy noise that affects other quantities

Uselessly detailed: high frequency dynamics does not influence the overall performance of the vehicle
the chain extension is negligible for realistic chain stiffness ($10^{-3}m$)

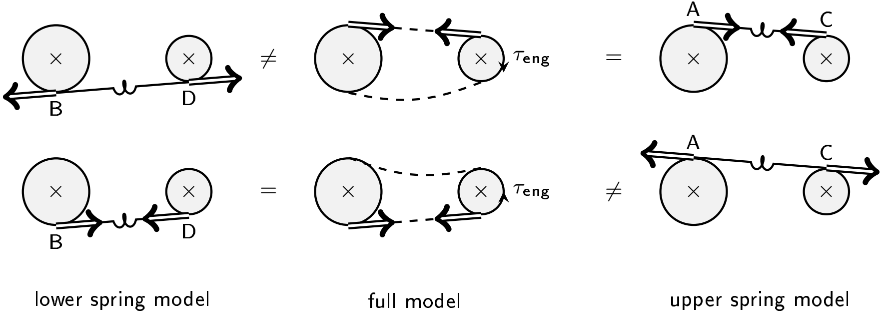
Holonomic constraints

This suggested to set the chain extensions to zero imposing two holonomic constraints:

$$l_u - l_u^0 = 0 \tag{1}$$
$$l_l - l_l^0 = 0\,. \tag{2}$$

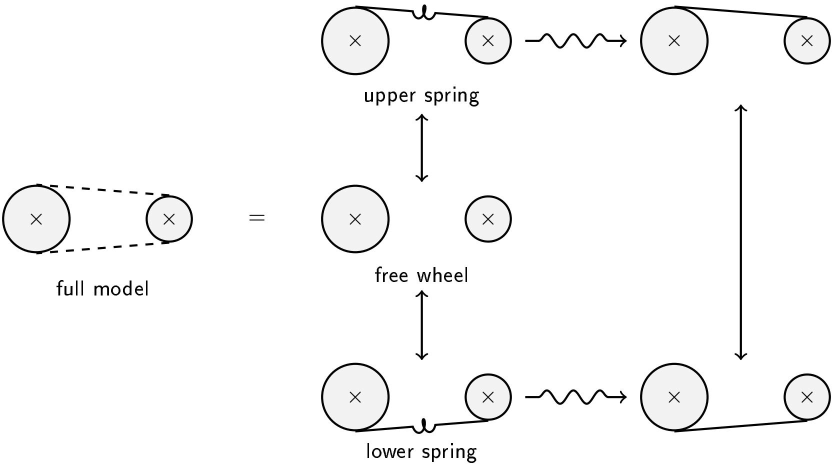
It is not possible to impose both constraints at the same time, as it would reduce two degrees of freedom, and consequently lock the system
At most one constraint at a time can be added

The Full model is thought of as a switching model



Each sub-model has only one fully linear (bilateral) spring and holds when it is tense.

Inextensible chain model



Conclusions and Future Perspective

- The inextensible chain model has been already implemented in MinTime3B and is currently used
- Code optimizations of MinTime3B are being operated on higher levels
- The Fast model of the internal drivetrain will be implemented in the next future