

Università degli Studi di Padova



Population models for complex non-linear phenomena in biology: from mitochondrial dynamics to brain networks

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What if something goes wrong?



... and many others ...

we still don't know how to treat them

- we still don't know how to treat them
- we still don't know how to predict them

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- they are related to energy-impairment



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Our Aim

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Our Aim

To make use of Systems Theory to analyze and characterize some aspects on the complex relationship

BRAIN \longleftrightarrow ENERGY

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Our Aim

To make use of Systems Theory to analyze and characterize some aspects on the complex relationship

BRAIN \longleftrightarrow ENERGY oscillations

MITOCHONDRIA



Mitochondrial Dysfunction in Neurodegenerative Disorders

2016

Second Edition

Amy K. Reeve Eve M. Simcox Michael R. Duchen Doug M. Turnbull *Editors*











OSCILLATIONS

OSCILLATIONS

FULL & CLUSTERS SYNC

SCIENTIFIC **REPORTS**

OPEN Association of specific frequency bands of functional MRI signal oscillations with motor symptoms and depression in Parkinson's disease

2015

Xiaopeng Song¹, Xiao Hu², Shuqin Zhou¹, Yuanyuan Xu², Yi Zhang¹, Yonggui Yuan⁴, Yijun Liu¹, Huaiqiu Zhu¹, Weiguo Liu² & Jia-Hong Gao^{1,6} OSCILLATIONS

FULL & CLUSTERS SYNC

RESEARCH ARTICLE

The Journal of Clinical Investigation



Neuronal firing patterns outweigh circuitry oscillations in parkinsonian motor control

Ming-Kai Pan,^{12,3} Sheng-Han Kuo,⁴ Chun-Hwei Tai,² Ivun-You Liou,⁵ Iu-Chun Pei,⁶ Chia-Yuan Chang,⁶ Yi-Mei Wang,³ Wen-Chuan Liu,² Tien-Rei Wang,² Wen-Sung Lai,⁶³ and Chung-Chin Kuo^{2,28}

SCIENTIFIC REPORTS

OPEN Association of specific frequency oscillations with motor symptoms Accepted: 13 October 2015 and depression in Parkinson's

OSCILLATIONS

FULL & **CLUSTERS** SYNC

en-Sung Lail⁶⁷ and Chung-Chin Kuo^{2,7,8} Contents lists available at ScienceDirect Clinical Neurophysiology journal homepage: www.elsevier.com/locate/clinph 2017 Alzheimer's disease disrupts alpha and beta-band resting-state CrossMark oscillatory network connectivity Loes Koelewijn 3-*, Aline Bompas ^a, Andrea Tales ^b, Matthew J. Brookes ^c, Suresh D. Muthukumaraswamy ^d, Antony Bayer*, Krish D. Singh* SCIENTIFIC **Reports OSCILLATIONS OPEN** Association of specific frequency FULL & **CLUSTERS** oscillations with motor symptoms SYNC Accepted: 13 October 2015 and depression in Parkinson's





Mitochondria & Mitochondrial Dynamics



Mitochondria & Mitochondrial Dynamics

- double membrane-bound organelles found in all eukaryotic organisms
- possess their own genome (mitochondrial DNA)





Mitochondrion

- involved in several tasks
- generate most of the cells supply of ATP (= energy)

Mitochondria & Mitochondrial Dynamics





- constant changes in shape, size, number and location
- affected by mitochondrial morphology
- controlled mainly by the processes of fission and fusion
- M. T. Figge et al. *PLoS Comput Biol*, 2012
- A. K. and T. B. L. Kirkwood. PNAS, 2011
- P. K. Mouli et al. Biophysical Journal, 2009
- P. K. Patel et al. PLoS Comput Biol, 2013
- V. M. Sukhorukov et al. PLoS Comput Biol, 2012
- Z. Y. Tam et al. *PLoS ONE*, 2013
- Z. Y. Tam et al. PLoS Comput Biol, 2013



G. Dalmasso et al. PLoS ONE, 2017







Z. Y. Tam et al. PLoS Comput Biol, 2013



G. Dalmasso et al. PLoS ONE, 2017







G. Dalmasso et al. PLoS ONE, 2017













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α :	biogenesis
δ :	damage
μ :	mitophagy
arphi:	fusion
ψ_1 , ψ_2 :	fission
β , η , ε :	atp production
u :	energy stress

Differential equations



Differential equations



9

Differential equations



9

ATP feedback advantage



Equilibrium point

$$\begin{split} \bar{x}_{d} &= \frac{\psi_{1} + \psi_{2}}{\psi_{2}\varphi} \left(\frac{\alpha_{1} - k\delta - \delta \bar{x}_{ATP}}{k + \bar{x}_{ATP}} \right) \quad \bar{x}_{h} = \frac{(\psi_{1} + \psi_{2})\mu}{\delta(\psi_{1} + \psi_{2}) - \psi_{1}\varphi \bar{x}_{d}} \cdot \bar{x}_{d} \\ \bar{x}_{f} &= \frac{\varphi}{\psi_{1} + \psi_{2}} \cdot \bar{x}_{h} \bar{x}_{d} \qquad \quad \bar{x}_{ATP} = \text{solution of 3rd-deg poly} \end{split}$$

Equilibrium point

$$\begin{split} \bar{x}_{d} &= \frac{\psi_{1} + \psi_{2}}{\psi_{2}\varphi} \left(\frac{\alpha_{1} - k\delta - \delta \bar{x}_{ATP}}{k + \bar{x}_{ATP}} \right) \quad \bar{x}_{h} = \frac{(\psi_{1} + \psi_{2})\mu}{\delta(\psi_{1} + \psi_{2}) - \psi_{1}\varphi \bar{x}_{d}} \cdot \bar{x}_{d} \\ \bar{x}_{f} &= \frac{\varphi}{\psi_{1} + \psi_{2}} \cdot \bar{x}_{h} \bar{x}_{d} \qquad \quad \bar{x}_{ATP} = \text{solution of 3rd-deg poly} \end{split}$$





Synchronization: the Kuramoto Model







Kuramoto Model

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^n a_{ij} \sin(\theta_j - \theta_i)$$



 ω_i : natural frequency

Kuramoto Model

$$\dot{ heta}_i = \omega_i + \sum_{j=1}^n a_{ij} \sin(heta_j - heta_i)$$



$$\begin{split} \mathcal{G} &= (\mathcal{V}, \mathcal{E}): \text{ graph} \\ \mathcal{V} &= \{1, \dots, n\}: \text{ set of nodes} \\ \mathcal{E} &\subseteq \mathcal{V} \times \mathcal{V}: \text{ set of edges} \\ \mathbf{A} &= [a_{ij}]: \text{ weighted adjacency matrix} \end{split}$$

Kuramoto Model

$$\hat{\theta}_i = \omega_i + \sum_{j=1}^n a_{ij} \sin(\theta_j - \theta_i)$$



- ω_i : isolated dynamics
- [*a_{ij}*]: adjacency matrix

 $\sin(\theta_j - \theta_i)$: coupling function

Synchronization: the Kuramoto Model













Phase locking	$ heta_j(t)- heta_i(t) \equiv\gamma$	$\forall i, j, t$
Phase synchronization	$ heta_j(t)- heta_i(t) \equiv 0$	$\forall i, j, t$
Frequency synchronization	$\left \dot{ heta}_{j}(t)-\dot{ heta}_{i}(t) ight \equiv0$	$\forall i, j, t$




	Full network synchronization	Clustered synchronization
Phase cohesiveness		
Phase locking		
Phase synchronization		
Frequency synchronization		

	Full network synchronization	Clustered synchronization
Phase cohesiveness	\checkmark	\checkmark
Phase locking		\checkmark
Phase synchronization		\checkmark
Frequency synchronization		\checkmark

	Full network synchronization	Clustered synchronization
Phase cohesiveness	\checkmark	\checkmark
Phase locking		\checkmark
Phase synchronization		
Frequency synchronization		\checkmark







$$\mathcal{P} = \{\mathcal{P}_1, \ldots, \mathcal{P}_4\}: \qquad \bigcup_k \mathcal{P}_k = \mathcal{V} \quad \mathcal{P}_i \bigcap \mathcal{P}_j = \emptyset$$



Phase & Frequency synchronizable \mathcal{P}

 $\exists \theta(0)$:



Phase & Frequency synchronizable ${\cal P}$

$$egin{cases} heta_i(t) = heta_j(t) \ \dot{ heta}_i(t) = \dot{ heta}_j(t) \ \dot{ heta}_i(t) = \dot{ heta}_j(t) \end{cases}$$

$$\forall i, j \in \mathcal{P}_k \text{ and } \forall k$$



Analysis	Necessary and sufficient conditions for clusters invariance

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 $k \in \mathcal{P}_{\ell}$

Generalized Equitable Partition (GEP) $\mathcal{P}_1 = \{1, 2, 3\} \rightarrow \omega_1$ $\mathcal{P}_2 = \{4, 5, 6\} \rightarrow \omega_2$





 $k \in \mathcal{P}_2$

Generalized Equitable Partition (GEP) $\begin{array}{l} \mathcal{P}_1 = \{1,2,3\} \rightarrow \omega_1 \\ \mathcal{P}_2 = \{4,5,6\} \rightarrow \omega_2 \end{array}$





Generalized Equitable Partition (GEP) $\mathcal{P}_1 = \{1, 2, 3\} \rightarrow \omega_1$ $\mathcal{P}_2 = \{4, 5, 6\} \rightarrow \omega_2$

Analysis	Necessary and sufficient conditions for clusters invariance
Control	How to modify the adjacency matrix to impose clusters invariance with structural constraints



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Control	How to modify the adjacency matrix to impose clusters invariance with structural constraints







Task:Modify A to make \mathcal{P} invariant without modyfying
the dotted edges



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the dotted edges

$$\begin{array}{l} \min_{\mathbf{\Delta}} \|\mathbf{\Delta}\|_{F}^{2} \\ \text{s.t.} (\mathbf{A} + \mathbf{\Delta}) \text{ respects } \mathbf{GEP} \\ \mathbf{\Delta} \in \mathcal{H} \quad \text{structural constraints} \end{array}$$



Task:Modify A to make \mathcal{P} invariant without modyfying
the dotted edges

 $\begin{array}{ll} \underset{\Delta}{\min} \|\Delta\|_{F}^{2} & \mbox{Frobenius norm} \\ \mbox{s.t.} (\mathbf{A} + \mathbf{\Delta}) \mbox{ respects GEP} \\ \mathbf{\Delta} \in \mathcal{H} & \mbox{structural constraints} \end{array}$



























anatomical connections



Structural Connectivity







Effective Connectivity







Functional Connectivity












analysis

- clusters dynamics
- fMRI vs MEG
- REST vs TASK



analysis

- clusters dynamics
- fMRI vs MEG
- REST vs TASK



analysis

- clusters dynamics
- fMRI vs MEG
- REST vs TASK

opposite behavior



analysis

- clusters dynamics
- fMRI vs MEG
- REST vs TASK

• opposite behavior

coherent behavior







Structural connectivity

Structural connectivity

fMRI signals

Structural connectivity





























- population model
- mito dynamics + energy
- feedback regulation
- stability & sensitivity analyses
- non-linear feedback control
- damaged vs healthy model
- more classes

results

open research



OSCILLATIONS FULL & CLUSTERS SYNC

results	 full phase cohesiveness clusters phase cohesiveness phase locking sync invariance 	
open research	 clusters sync attractivity brain fitting	OSCILLATIONS FULL & CLUSTERS SYNC





- results
- data analysis
- modeling
- rest vs task
- fMRI vs MEG

- stroke modeling
- rest vs task
- fMRI vs EEG
- FC dynamics

open research

Thanks to...



Thanks to...

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