Department of Information Engineering

PhD final exam for the 32nd cycle

Distributed Optimization Strategies for Mobile Multi-Agent Systems

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Supervisor: Prof. Angelo Cenedese

Mar 16th, 2020

Università degli Studi di Padova

Multi-agent systems (MASs)

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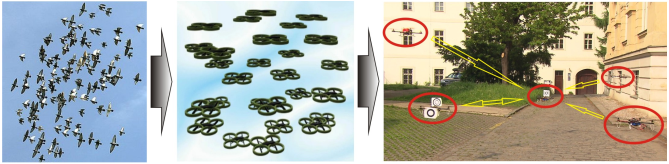

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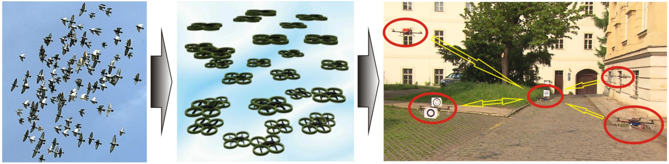
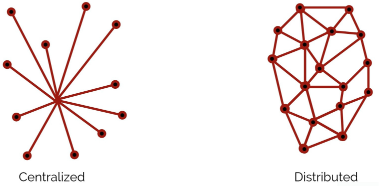


Questions and motivations:

- How to solve tasks that are arduous for the individual?
- How do network components interact within a network?
- How does the network architecture influence the global behavior of the system?



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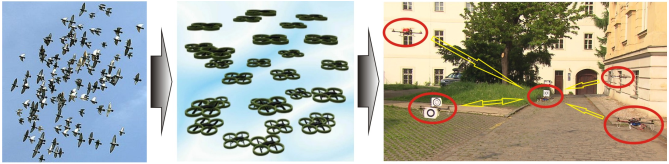

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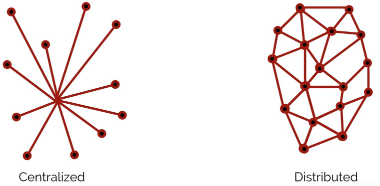


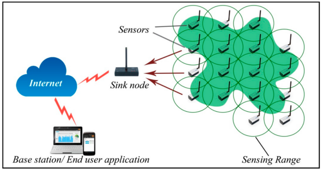
Distinctive features: autonomy, scalability, security, robustness to failure.

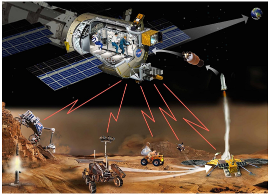
Outline

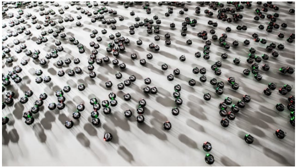
Overview on my reasearch activity

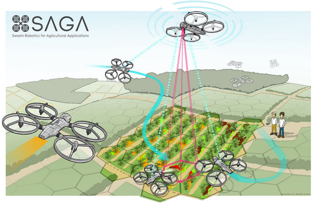
Research thrusts (RTs) & applications

- Distributed strategies for coverage and focus on event with limited sensing capabilities



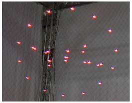


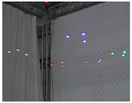


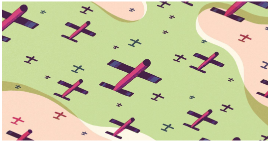


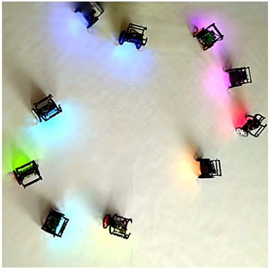
Research thrusts (RTs) & applications

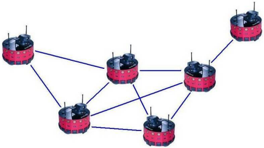
ii Optimal time-invariant formation control

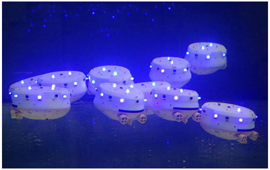












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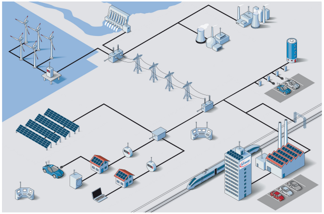
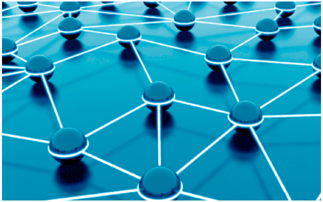
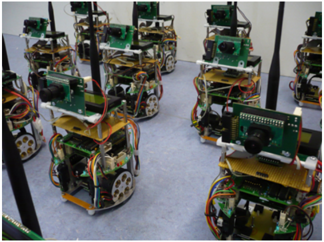
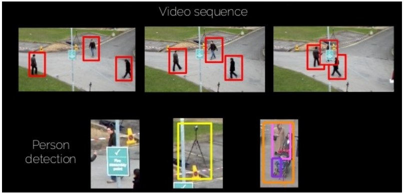


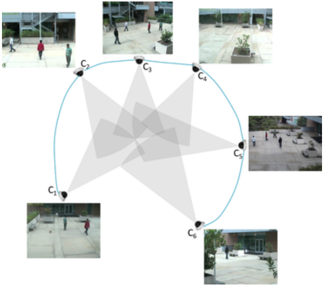




iii Distributed estimation from relative measurements

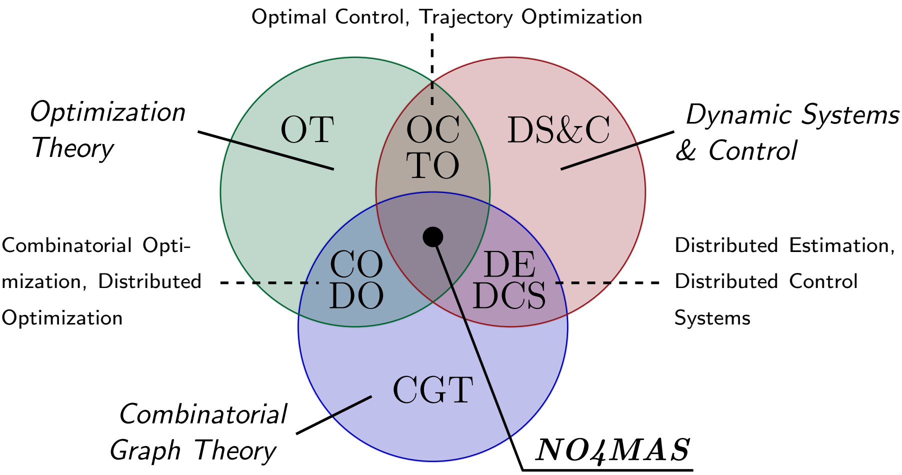
Research thrusts (RTs) & applications





iii Algebraic characterization of certain circulant networks

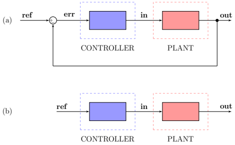
Networked optimization for MASs: common thread



multi-agent leads to multidisciplinary framework

Methods and methodologies

- **Analysis and synthesis of feedback systems**: design of feedback control laws, sensitivity analysis to parameter variations, fulfillment of optimality principles.



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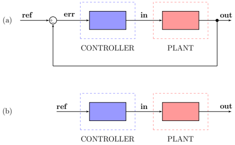

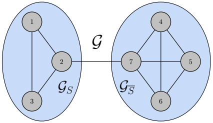

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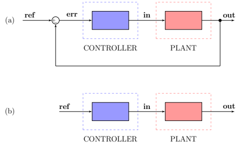

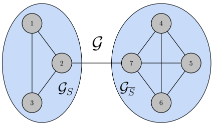

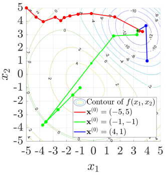

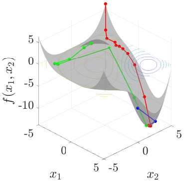

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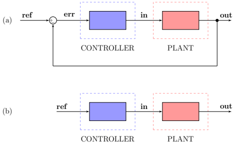
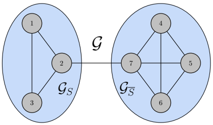
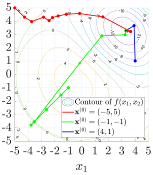
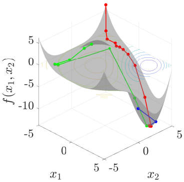
- **Graph-based motion planning and clustering**: greedy algorithms for navigation, edge expansion techniques for partitioning.
- **Iterative methods for optimization**: descent algorithms, approaches for convex optimization.
- **Swarm-robotic-oriented strategies**: geometrical policies for mobile robotics, employ of topological tools.









Overall contribution of the thesis

- formalization of problems having practical consequences in the advancement in the field of MASs
- development of novel analysis and design tools and enrichment of existing mathematical methods
- application of optimization-based strategies to achieve required specifications, drawing inspiration from current literature
- proofs of theoretical statements settled in this framework
- virtual implementation and numerical simulation of the devised techniques to assess case studies

Distributed strategies for coverage and focus on event with limited sensing capabilities

Contributions

- Design and test of a **distributed multi-agent algorithm**;
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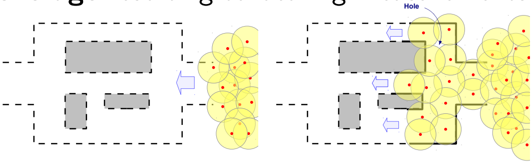

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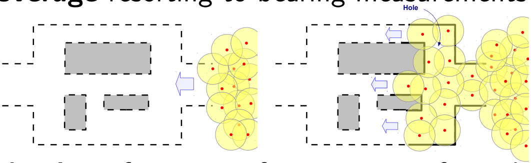
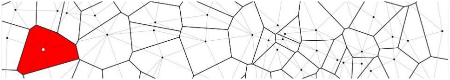


2. **Cluster selection** of a group of agents to perform the detection of an event



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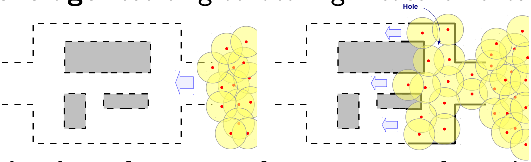

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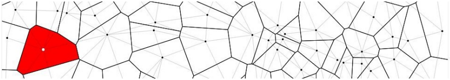
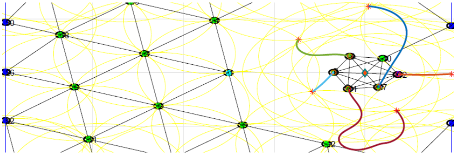


3. Agents' **dispatch** towards the detected event



Assumptions & models 1/2

Models are partly inspired and borrowed by the those used in

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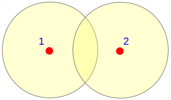
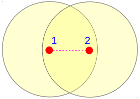
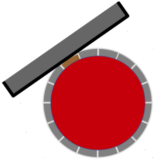
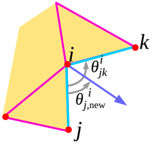
  - sensors to detect **events**
  - navigation by means of **bearing**-based controllers









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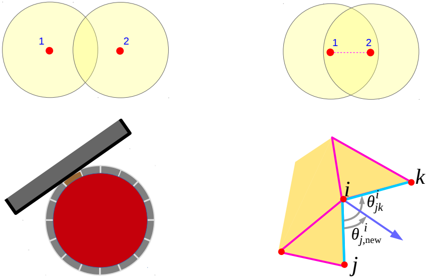
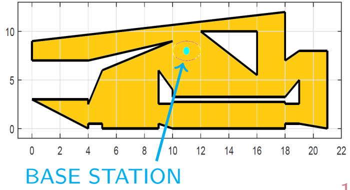
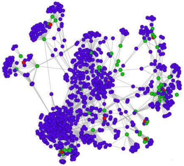


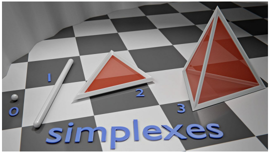
- Virtual environment
  - **synthetic scenario** based on simple geometric features
  - spawn location for agents represented by a **base station**



Assumptions & models 2/2

- Topological tools
  - undirected **graphs** $\longleftrightarrow$ agent interactions
  - **simplexes** and simplicial complex $\longleftrightarrow$ coverage structure





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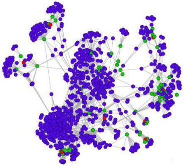
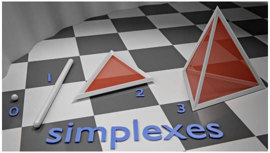
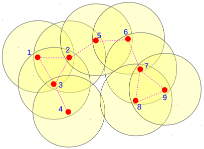
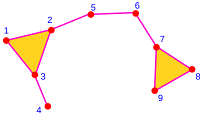
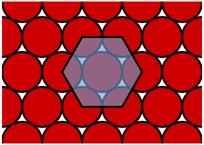




- Deployment policies
  - vertex set structure + agent visibility graph = **Vietoris-Rips complex** to be preserved while deploying
  - **hexagonal packing** = optimal packing to accomplish in order to maximize the covered surface and minimize the number of deployed agents







Algorithm design: overview

Algorithm 1 Outline of the main procedure

$\mathcal{G} \leftarrow$ COVERAGE();
for each agent a_i, s.t. $i = 1, ..., n$ **do**
    $|v_i| \leftarrow \hat{f}_{EV}(\mathbf{p}_i)$;
end for
for all $e_{ij} \in \mathcal{E}$ **do**
    $|e_{ij}| \leftarrow (|v_i| + |v_j|)/2$;
end for
$v^\star \leftarrow$ MAX-CONSENSUS($\mathcal{G}$,BS);
$\mathcal{G}_{CL} \leftarrow \{v^\star\}$
CLUSTERING($v^\star$,1);
for all nodes $v_i \in \mathcal{G}_{CL}$ **do**
    $[c_{di}, \mathrm{f}_{di}] \leftarrow [0, \textbf{false}]$;
end for
while $c_d^\star < MaxIter$ **and** $\mathrm{f}_d^\star =$ **false do**
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Algorithm design: coverage stage

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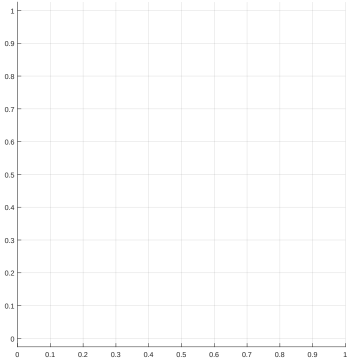

deployment

Algorithm design: cluster selection stage

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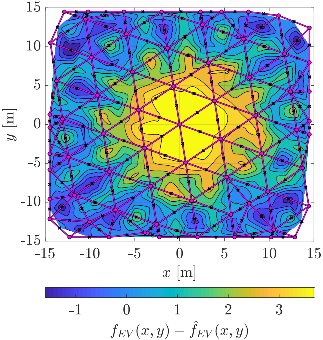

event detection

Algorithm design: dispatch stage

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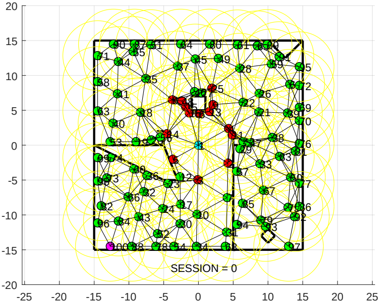
  $|e_{ij}| \leftarrow (|v_i| + |v_j|)/2$;
end for
$v^\star \leftarrow$ MAX-CONSENSUS($\mathcal{G}$,BS);
$\mathcal{G}_{CL} \leftarrow \{v^\star\}$
CLUSTERING($v^\star$,1);
for all nodes $v_i \in \mathcal{G}_{CL}$ **do**
  $[c_{di}, \mathrm{f}_{di}] \leftarrow [0, \textbf{false}]$;
end for
while $c_d^\star < MaxIter$ **and** $\mathrm{f}_d^\star =$ **false do**
  $v^\star \leftarrow$ MAX-CONSENSUS($\mathcal{G}_{CL}, v^\star$);
  $[c_d^\star, \mathrm{f}_d^\star] \leftarrow$ DISPATCH($v^\star$,$c_d^\star + 1$,**true**);
end while



focus on event

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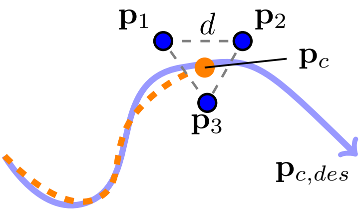
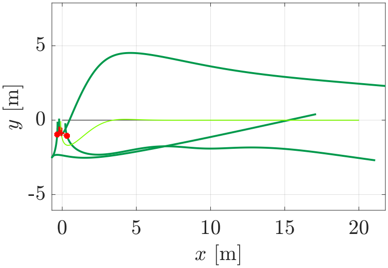
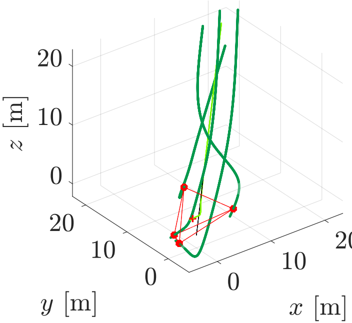
Optimal time-invariant formation control

Contributions

Analysis and design of a **distributed** minimal-energy potential-based control law for a formation tracking problem, involving a second-order linear multi-agent system.







Problem setup: agents' dynamics

Assumptions

- $n > 1$ agents in an M-dimensional space, where $N := Mn$ is set

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Assumptions

- $n > 1$ agents in an M-dimensional space, where $N := Mn$ is set
- each agent i is aware of its absolute position $\mathbf{p}_i \in \mathbb{R}^M$ and velocity $\dot{\mathbf{p}}_i$
- each agent i is controlled in acceleration $\ddot{\mathbf{p}}_i$
- the whole state and the input are given by $\mathbf{x} \in \mathbb{R}^{2N}$, $\mathbf{u} \in \mathbb{R}^N$ respectively s.t.

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} \mathbf{p}_1^\top & \cdots & \mathbf{p}_n^\top & \dot{\mathbf{p}}_1^\top & \cdots & \dot{\mathbf{p}}_n^\top \end{bmatrix}^\top = \begin{bmatrix} \mathbf{p}^\top & \dot{\mathbf{p}}^\top \end{bmatrix}^\top \\ \mathbf{u} &= \begin{bmatrix} \ddot{\mathbf{p}}_1^\top & \cdots & \ddot{\mathbf{p}}_n^\top \end{bmatrix}^\top = \ddot{\mathbf{p}} \end{aligned}$$

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- Info on centroid $\mathbf{x}_c = \begin{bmatrix}\mathbf{p}_c^\top & \dot{\mathbf{p}}_c^\top\end{bmatrix}^\top$ is available, s.t. $\mathbf{p}_c := n^{-1} \sum_{i=1}^n \mathbf{p}_i$

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Assumptions

- $n > 1$ agents in an M-dimensional space, where $N := Mn$ is set
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- The dynamics can be represented by means of the linear system

$$\begin{cases}\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{x}_c = \mathbf{C}\mathbf{x}\end{cases}$$

with $(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \left(\begin{bmatrix}\mathbf{Z}_N & \mathbf{I}_N \\ \mathbf{Z}_N & \mathbf{Z}_N\end{bmatrix}, \begin{bmatrix}\mathbf{Z}_N \\ \mathbf{I}_N\end{bmatrix}, \frac{1}{n}\begin{bmatrix}\mathbf{I}_M & \dots & \mathbf{I}_M & \mathbf{Z}_M & \dots & \mathbf{Z}_M \\ \mathbf{Z}_M & \dots & \mathbf{Z}_M & \mathbf{I}_M & \dots & \mathbf{I}_M\end{bmatrix}\right)$

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- Desire path tracked by the system centroid: $\mathbf{x}_{c,des} = \begin{bmatrix} \mathbf{p}_{c,des}^\top & \dot{\mathbf{p}}_{c,des}^\top \end{bmatrix}^\top$

Problem setup: cost functional minimization 1/2

Let $\mathcal{T}$ be the trajectory manifold of $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$. We aim at solving

$$\min_{\xi \in \mathcal{T}} h\left(\xi\right), \quad \mathbf{x} := \begin{bmatrix} \mathbf{p}^\top & \dot{\mathbf{p}}^\top \end{bmatrix}^\top, \quad \mathbf{u} := \ddot{\mathbf{p}}$$

s.t. $$h(\mathbf{x}(\cdot), \mathbf{u}(\cdot)) := \int_0^T l\left(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau\right) d\tau + m\left(\mathbf{x}(T)\right).$$

Problem setup: cost functional minimization 1/2

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$$l(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau) := l^{tr}(\mathbf{x}_c(\tau)) + l^{in}(\mathbf{u}(\tau)) + l_d^{fo}(\mathbf{p}(\tau)) + l^{al}(\dot{\mathbf{p}}(\tau)) \geq 0$$

Problem setup: cost functional minimization 1/2

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s.t. $$h(\mathbf{x}(\cdot), \mathbf{u}(\cdot)) := \int_0^T l(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau)\, d\tau + \boxed{m(\mathbf{x}(T))}.$$

$$l(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau) := l^{tr}(\mathbf{x}_c(\tau)) + l^{in}(\mathbf{u}(\tau)) + l_d^{fo}(\mathbf{p}(\tau)) + l^{al}(\dot{\mathbf{p}}(\tau)) \geq 0$$

$$\boxed{m(\mathbf{x}(T)) := l^{tr}(\mathbf{x}_c(T)) + l_d^{fo}(\mathbf{p}(T)) + l^{al}(\dot{\mathbf{p}}(T)) \geq 0}$$

Problem setup: cost functional minimization 1/2

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s.t. $$h(\mathbf{x}(\cdot), \mathbf{u}(\cdot)) := \int_0^T l\left(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau\right) d\tau + m\left(\mathbf{x}(T)\right).$$

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OIFT: Optimal time-Invariant Formation Tracking
(for a second-order MAS)

Problem setup: cost functional minimization 2/2

Each term involved in the instantaneous cost l and in the final cost m characterizes one specific task. Let us examine the instantaneous cost

$$l(\mathbf{x}(\tau), \mathbf{u}(\tau), \tau) = l^{tr}(\mathbf{x}_c(\tau)) + l^{in}(\mathbf{u}(\tau)) + l_d^{fo}(\mathbf{p}(\tau)) + l^{al}(\dot{\mathbf{p}}(\tau)).$$

Problem setup: cost functional minimization 2/2

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$$l^{tr}(\mathbf{x}_c(\tau)) := \frac{1}{2} \sum_{i=1}^{n} \|\mathbf{x}_c(\tau) - \mathbf{x}_{c,des}(\tau)\|^2_{\mathbf{Q}_{c,\dot{c},i}}$$

Problem setup: cost functional minimization 2/2

Each term involved in the instantaneous cost l and in the final cost m characterizes one specific task. Let us examine the instantaneous cost

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Problem setup: cost functional minimization 2/2

Each term involved in the instantaneous cost l and in the final cost m characterizes one specific task. Let us examine the instantaneous cost

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$$l^{in}(\mathbf{u}(\tau)) := \frac{1}{2} \sum_{i=1}^{n} \|\mathbf{u}_i(\tau)\|^2_{\mathbf{R}_i}$$

$$l_d^{fo}(\mathbf{p}(\tau)) := \frac{k_F}{4} \sum_{i=1}^{n} \sum_{\forall j \neq i} \sigma_{d_{ij}}(r_{ij}^2(\tau)), \quad r_{ij} = \|\mathbf{p}_i - \mathbf{p}_j\|$$

Problem setup: cost functional minimization 2/2

Each term involved in the instantaneous cost l and in the final cost m characterizes one specific task. Let us examine the instantaneous cost

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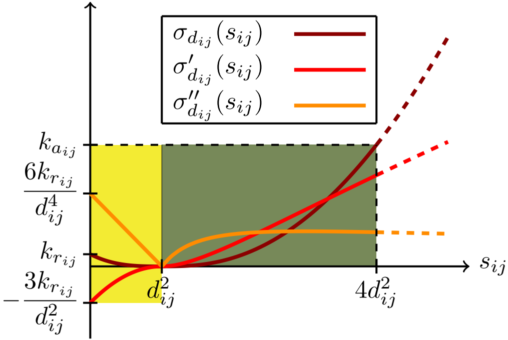
$$l^{al}(\dot{\mathbf{p}}(\tau)) := \frac{k_A}{4}\sum_{i=1}^{n}\sum_{\forall j\neq i} \|\dot{\mathbf{p}}_i - \dot{\mathbf{p}}_j\|^2_{q_{A_{ij}}}$$

Problem setup: potential-based formations

Formations are achieved through a distance-based control law. Setting $s_{ij} := r_{ij}^2$, the structure of term $l_d^{fo}(\mathbf{p})$ depends on the potential function

$$\sigma_{d_{ij}}(s_{ij}) := \begin{cases} k_{r_{ij}}(1 - s_{ij}/d_{ij}^2)^3 & \text{for } 0 \le s_{ij} \le d_{ij}^2 \\ k_{a_{ij}}(\sqrt{s_{ij}}/d_{ij} - 1)^3 & \text{for } s_{ij} \ge d_{ij}^2 \end{cases} \quad \in \mathscr{C}^2(\mathbb{R})$$



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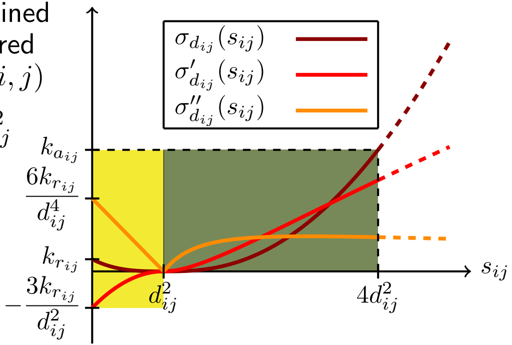
- The minimum for $\sigma_{d_{ij}}$ is attained at $r_{ij} = d_{ij} \Rightarrow d_{ij}$ is the desired formation distance between (i, j)
- $\sigma'_{d_{ij}}(s_{ij}) \le 0$ for $0 \le s_{ij} \le d_{ij}^2$
- $\sigma'_{d_{ij}}(s_{ij}) \ge 0$ for $s_{ij} \ge d_{ij}^2$
- $\sigma''_{d_{ij}}(s_{ij}) \ge 0$ for all s_{ij}



Control law design: variational approach

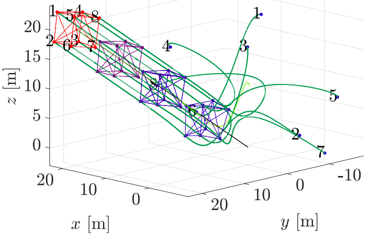
Let us define $\bar{Q}_c := \sum_{j=1}^{n} \bar{Q}_{c,j}/n$ and $\bar{Q}_{\dot{c}} := \sum_{j=1}^{n} \bar{Q}_{\dot{c},j}/n$, with $\bar{Q}_{\dot{c}}$ non singular. Assuming to adopt a *distributed* PD controller $\mathbf{u} = \begin{bmatrix} \mathbf{u}_1^\top & \cdots & \mathbf{u}_n^\top \end{bmatrix}^\top$ govern the dynamics of the MAS, it is possible to prove that functional h is stationary under the distributed control law

$$
\begin{aligned}
\mathbf{u}_i := & -\mathbf{R}_i^{-1}\left[k_{P,i}^{tr}\overline{\mathbf{Q}}_c(\mathbf{p}_c - \mathbf{p}_{c,des}) + k_{D,i}^{tr}\overline{\mathbf{Q}}_{\dot{c}}(\dot{\mathbf{p}}_c - \dot{\mathbf{p}}_{c,des})\right] \\
& -\mathbf{R}_i^{-1}\left[k_{P,i}^{fo}k_F \sum_{j\in\mathcal{N}_i} \sigma'_{d_{ij}}(r_{ij}^2)\mathbf{e}_{ij} + k_{D,i}^{al}k_A \sum_{j\in\mathcal{N}_i} q_{Aij}\dot{\mathbf{e}}_{ij}\right] \\
& -\mathbf{R}_i^{-1}k_D^{fo}k_F \sum_{j\in\mathcal{N}_i}\left[2\sigma''_{d_{ij}}(r_{ij}^2)\mathbf{e}_{ij}\mathbf{e}_{ij}^\top + \chi_{>0}(\sigma'_{d_{ij}}(r_{ij}^2))\sigma'_{d_{ij}}(r_{ij}^2)\mathbf{I}_M\right]\dot{\mathbf{e}}_{ij}
\end{aligned}
$$

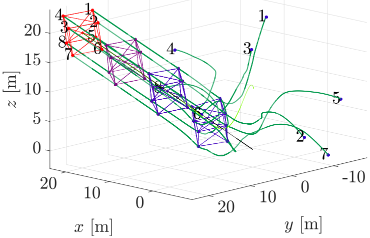
where $\mathbf{e}_{ij} := \mathbf{p}_i - \mathbf{p}_j$, $(k_{P,i}^{tr}, k_{D,i}^{tr}, k_{P,i}^{fo}, k_{D,i}^{al}, k_D^{fo})$ are feeback gains, $\mathcal{N}_i$ is the neighborhood of agent i and $\chi_{>0}$ is the characteristic function for positive numbers.

Centralized vs Distributed comparison 1/2

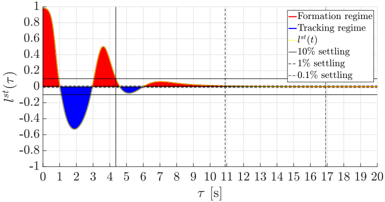
The numerical tool PRONTO has been used to provide an optimality reference for the OIFT in the centralized case.



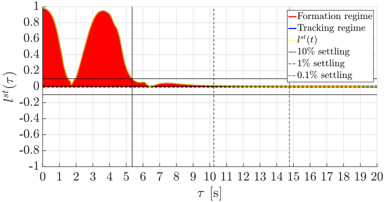
(a) PRONTO: Position trajectories



(b) Distributed: Position trajectories



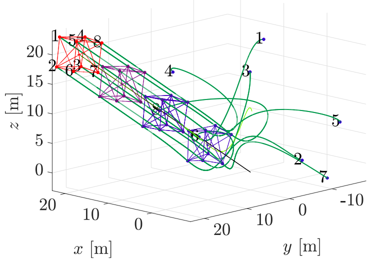
(e) PRONTO: Settling time



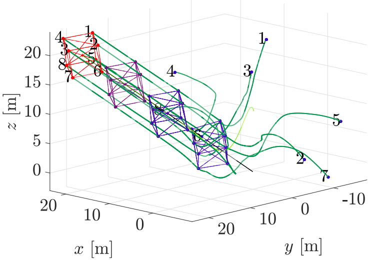
(f) Distributed: Settling time

Centralized vs Distributed comparison 2/2

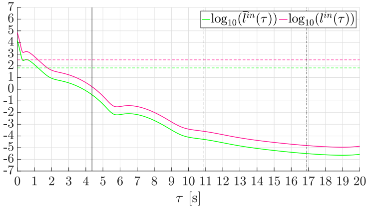
The numerical tool PRONTO has been used to provide an optimality reference for the OIFT in the centralized case.



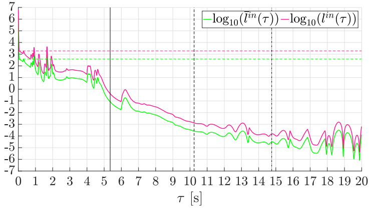
(a) PRONTO: Position trajectories



(b) Distributed: Position trajectories



(c) PRONTO: Input energy consumption



(d) Distributed: Input energy consumption

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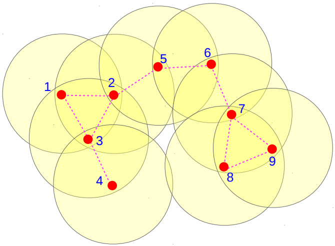
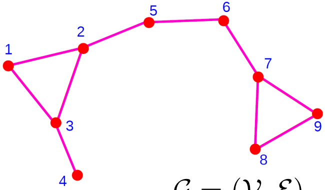
Distributed estimation from relative measurements

Contributions

Formalization and comparison of three iterative linear algorithms for the distributed state estimation from relative measurements (RMs) in a MAS.

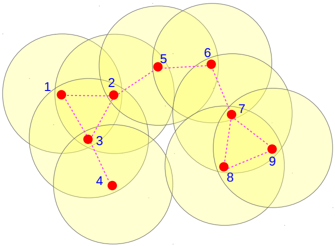


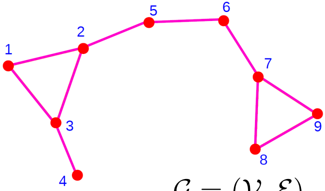


$$\mathcal{G} = (\mathcal{V}, \mathcal{E})$$

Contributions

Formalization and comparison of three iterative linear algorithms for the distributed state estimation from relative measurements (RMs) in a MAS.





Problem statement. Minimize the diffusive squared error:

$$\underset{\{\mathbf{x}_1,\ldots,\mathbf{x}_n\}}{\arg\min}\ \frac{1}{2}\sum_{v_i\in\mathcal{V}}\sum_{v_j\in\mathcal{N}_i}(\mathbf{x}_i-\mathbf{x}_j+\tilde{\mathbf{x}}_{ij})^\top(\mathbf{x}_i-\mathbf{x}_j+\tilde{\mathbf{x}}_{ij})$$

where $\mathbf{x}_i$ is the state of node $v_i \in \mathcal{V}$, $\mathcal{N}_i$ is the neighborhood of v_i and $\tilde{\mathbf{x}}_{ij} = \tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_j$ is the noisy RM.

Distributed solutions 1/2

Let us consider the problem in only 1 dimension, w.l.o.g. and let

$$\tilde{\mathbf{x}} := \left[\sum_{v_j \in \mathcal{V}_1} (\tilde{x}_{j1} - \tilde{x}_{1j}) \quad \dots \quad \sum_{v_j \in \mathcal{V}_n} (\tilde{x}_{jn} - \tilde{x}_{nj}) \right]^\top .$$

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General **distributed solution**: linear state-space system driven by an exogenous input $\mathbf{u}_\vartheta = \mathbf{u}_\vartheta(\tilde{\mathbf{x}})$ dependent on the RMs and a state update provided by $\mathbf{F}_\vartheta$ dependent on the network topology.

$$\Sigma_\vartheta : \quad \mathbf{x}(t+1) = \mathbf{F}_\vartheta \mathbf{x}(t) + \mathbf{u}_\vartheta, \qquad \vartheta \in \{0, \eta, \rho, \epsilon\}$$

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Scheme	Parameter	State matrix	Input vector
Σ_0		$\mathbf{F}_0 = \mathbf{D}^{-1}\mathbf{A}$	$\mathbf{u}_0 = \frac{1}{2}\mathbf{D}^{-1}\tilde{\mathbf{x}}$
Σ_η	$\eta \in [0, 1)$	$\mathbf{F}_\eta = (\eta \mathbf{I}_n + (1-\eta)\mathbf{F}_0)$	$\mathbf{u}_\eta = (1-\eta)\mathbf{u}_0$
Σ_ρ	$\rho \geq 0$	$\mathbf{F}_\rho = \left(\mathbf{D} + \frac{\rho}{2}\mathbf{I}_n\right)^{-1}\left(\mathbf{A} + \frac{\rho}{2}\mathbf{I}_n\right)$	$\mathbf{u}_\rho = \left(\mathbf{D} + \frac{\rho}{2}\mathbf{I}_n\right)^{-1}\mathbf{D}\mathbf{u}_0$
Σ_ϵ	$\epsilon \in \left(0, \frac{2}{\lambda^{\mathbf{L}}_{n-1}}\right)$	$\mathbf{F}_\epsilon = \mathbf{I}_n - \epsilon \mathbf{L}$	$\mathbf{u}_\epsilon = \epsilon\, \mathbf{D}\mathbf{u}_0$

Distributed solutions 2/2

For $\vartheta \in \{\eta, \rho, \epsilon\}$ the solution of Σ_ϑ converges to the **centralized solution**

$$\mathbf{x}^\star = \frac{1}{2}\mathbf{L}^\dagger \tilde{\mathbf{x}}$$

where $\mathbf{L}^\dagger$ is the pseudo-inverse of the Laplacian matrix associated to $\mathcal{G}$.

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Scheme	Best convergence rate	Optimal parameter selection
Σ_0	$\mathrm{r}_0 = \begin{cases} \lambda_{n-1}^{\mathcal{L}} - 1, & \text{if } \varsigma_{\mathcal{L}} > 1 \\ 1 - \lambda_1^{\mathcal{L}}, & \text{if } \varsigma_{\mathcal{L}} \leq 1 \end{cases}$	no parameter available
Σ_η	$\mathrm{r}_{\eta^\star} = \begin{cases} 1 - \lambda_1^{\mathcal{L}}/\varsigma_{\mathcal{L}}, & \text{if } \varsigma_{\mathcal{L}} > 1 \\ 1 - \lambda_1^{\mathcal{L}}, & \text{if } \varsigma_{\mathcal{L}} \leq 1 \end{cases}$	$\eta^\star = \begin{cases} 1 - 1/\varsigma_{\mathcal{L}}, & \text{if } \varsigma_{\mathcal{L}} > 1 \\ 0, & \text{if } \varsigma_{\mathcal{L}} \leq 1 \end{cases}$
Σ_ρ	$\mathrm{r}_{\rho^\star} = \begin{cases} \mathrm{r}_{\rho^+}, & \text{if } \varsigma_{\mathcal{L}} > 1 \\ 1 - \lambda_1^{\mathcal{L}}, & \text{if } \varsigma_{\mathcal{L}} \leq 1 \end{cases}$	$\rho^\star = \begin{cases} \rho^+, & \text{if } \varsigma_{\mathcal{L}} > 1 \\ 0, & \text{if } \varsigma_{\mathcal{L}} \leq 1 \end{cases}$
Σ_ϵ	$\mathrm{r}_{\epsilon^\star} = 1 - \lambda_1^{\mathrm{L}}/\varsigma_{\mathrm{L}}$	$\epsilon^\star = 1/\varsigma_{\mathrm{L}}$

where $\mathcal{L} = \mathbf{D}^{1/2}\mathbf{L}\mathbf{D}^{1/2}$ and $\varsigma_{\mathrm{L}} = (\lambda_1^{\mathrm{L}} + \lambda_{n-1}^{\mathrm{L}})/2$.

Sensitivity analysis 1/2

Consider a discrete linear state-space system $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})_\vartheta$ with transfer function $\mathbf{W}(z, \vartheta) = \mathbf{C}(\mathbf{I}z - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$ depending on parameter ϑ.

Sensitivity: $S_\vartheta(z) = \dfrac{\partial \ln(\det[\mathbf{W}(z, \vartheta)])}{\partial \ln(\vartheta)}$. Relative sensitivity: $\bar{S}_\vartheta(z) = \dfrac{S_\vartheta(z)}{\vartheta}$.

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Meaning of the relative sensitivity for 1-dimensional W:

$$\begin{aligned} W(z, \vartheta + \Delta\vartheta) &\simeq W(z, \vartheta) + \frac{\partial W(z, \vartheta)}{\partial \vartheta} \Delta\vartheta \\ &= W(z, \vartheta)(1 + \bar{S}_\vartheta(z)\Delta\vartheta) \end{aligned}$$

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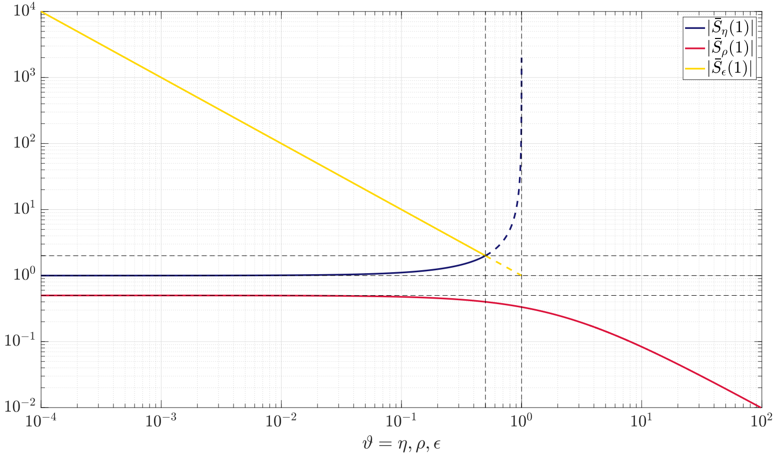
&= W(z, \vartheta)(1 + \bar{S}_\vartheta(z)\Delta\vartheta)
\end{aligned}
$$

Simplification for the relative sensitivity formula:

$$
\bar{S}_\vartheta(z) = \operatorname{tr}\left[\mathbf{W}(z, \vartheta)^{-\top} \frac{\partial \mathbf{W}(z, \vartheta)}{\partial \vartheta}\right]
$$

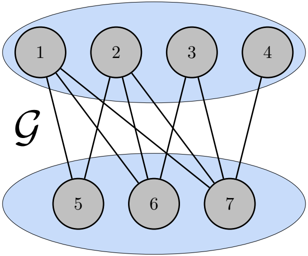
Sensitivity analysis 2/2

$$|\bar{S}_\eta(1)| = \frac{1}{\eta - 1}, \qquad |\bar{S}_\rho(1)| = \frac{1}{2\mathrm{vol}(\mathcal{G}) + \rho}, \qquad |\bar{S}_\epsilon(1)| = \frac{1}{\epsilon}$$



Case study: bipartite network 1/2

- Graph $\mathcal{G}$ has $n = 7$ nodes and it is bipartite



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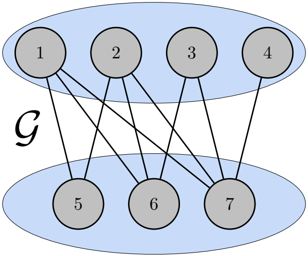

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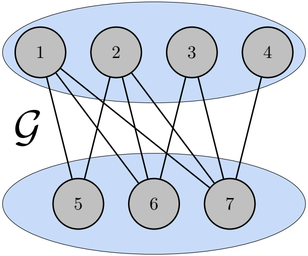

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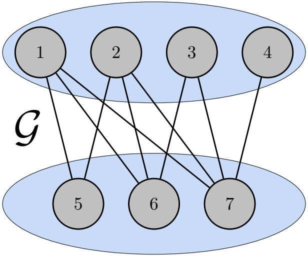
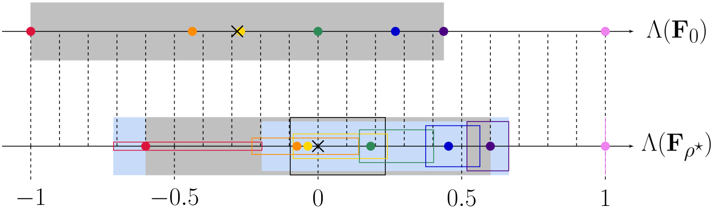
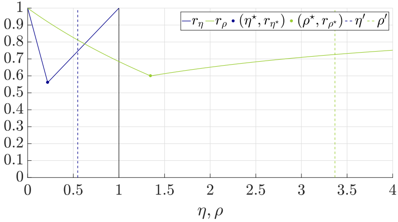
- Due to bipartiteness, Σ_0 is not expected to converge towards $\mathbf{x}^\star$
- Simulations on Σ_ϵ are not considered due to high sensitivity
- Bounds for the eigenvalues of F_ρ can be provided, given ρ: helps to figure out the rate of convergence

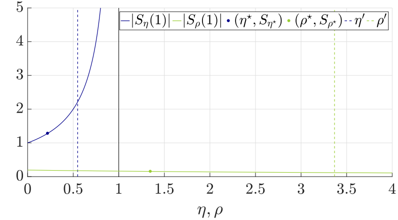




Case study: bipartite network 2/2

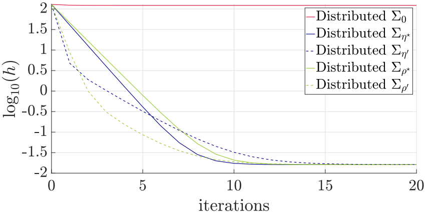
Tuning of parameters

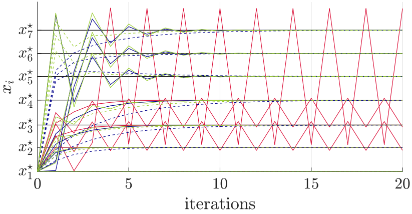




Sensitivity comparison

Performances





Estimation dynamics

References for RT (iii)

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- Landau H., Odlyzko A., 1981 "Bounds for Eigenvalues of Certain Stochastic Matrices"

Algebraic characterization of certain circulant networks

Contributions

- General aim: investigate stability, performances of graph-based protocols and the communication exchange over networks.

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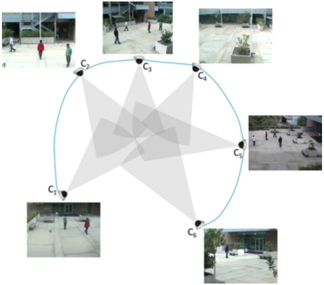
- In particular, circulant networks are widely employed in the design of distributed consensus-like algorithms. E.g., camera networks whose nodes share a common field of view:



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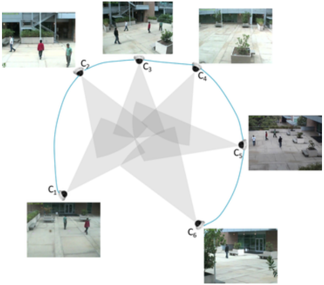

- A spectral characterization of the Laplacian matrix related to a class of circulant graphs is provided through the Dirichlet kernel.

Preliminaries: circulant graphs

Circulant matrix

$$\mathbf{F} = \mathrm{circ}(\boldsymbol{\varpi}) := \begin{bmatrix} \varpi_0 & \varpi_1 & \cdots & \varpi_{n-2} & \varpi_{n-1} \\ \varpi_{n-1} & \varpi_0 & \cdots & \varpi_{n-3} & \varpi_{n-2} \\ \vdots & \ddots & \cdots & \ddots & \vdots \\ \varpi_2 & \varpi_3 & \cdots & \varpi_0 & \varpi_1 \\ \varpi_1 & \varpi_2 & \cdots & \varpi_{n-1} & \varpi_0 \end{bmatrix} \in \mathbb{R}^{n\times n}$$

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Circulant matrix spectrum

$$\lambda^{\mathbf{F}}(j) = \sum_{k=0}^{n-1} \left[\varpi_k \exp\left(-\frac{2k\pi\mathbf{i}}{n} j \right) \right] \quad \text{for } j = 0, \ldots, n-1$$

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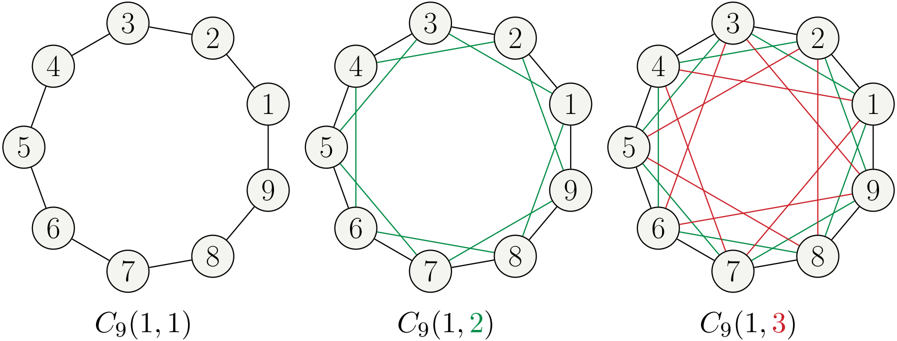
Spectral equivalence between normalize Laplacian and Randić matrices

$$\lambda^{\mathbf{F}}(j) = \lambda^{\mathscr{R}}(j) = 1 - \lambda^{\boldsymbol{\mathcal{L}}}(j) \quad \text{for } j = 0, \dots, n-1$$

Preliminaries: κ-ring graphs

κ-ring graphs $C_n(1, \kappa)$ are a class of circulant graphs constructed by multiple circulant edge layers



#Vertices	#Edges	Diameter	Radius	Girth	Regularity
$\|\mathcal{V}\| = n \geq 4$	$\|\mathcal{E}\| = n\kappa$	$\phi = \lceil n/2^{\kappa} \rceil$	$\mathrm{r} = \phi$	$\mathrm{g} = \begin{cases} n, & \text{if } \kappa = 1 \\ 3, & \text{otherwise} \end{cases}$	$\mathrm{d} = 2\kappa$

Main results: spectral characterization

Definition (Dirichlet kernel)

$\mathcal{D}_\kappa : \mathbb{R} \to \mathbb{R}$ of order $\kappa \in \mathbb{N}$ such that

$$\mathcal{D}_\kappa(x) := \begin{cases} \dfrac{\sin((\kappa + 1/2)x)}{2\sin(x/2)}, & \text{if } x \neq 2\pi l, \ \forall l \in \mathbb{Z}; \\ \kappa + 1/2, & \text{otherwise.} \end{cases}$$

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Theorem (Spectral characterization of κ-ring graphs)

$\mathbf{L}$ graph Laplacian of κ-ring graph $C_n(1,\kappa)$, $\theta := \pi/n$. Eigenvalues $\lambda^{\mathbf{L}}(j) \in \Lambda(\mathbf{L})$ can be expressed in function of the Dirichlet kernel as

$$\lambda^{\mathbf{L}}(j) = 1 + 2\left(\kappa - \mathcal{D}_\kappa(2\theta j)\right), \qquad \text{for } j = 0, \ldots, \lfloor n/2 \rfloor;$$
$$\lambda^{\mathbf{L}}(n-j) = \lambda^{\mathbf{L}}(j), \qquad \text{for } j = 1, \ldots, \lfloor n/2 \rfloor.$$

$\lambda^{\mathbf{L}}(j) \in [0, 4\kappa]$, $\forall j = 0, \ldots, n-1$, $\lambda_0^{\mathbf{L}} := \lambda^{\mathbf{L}}(0) = 0$ is simple and, if $\exists j^\star \in \mathbb{N}$ s.t. $\lambda^{\mathbf{L}}(j^\star) = 4\kappa$, $j^\star \in (0, n)$, then $\lambda^{\mathbf{L}}(j^\star)$ is simple.

Main results: Spectral characterization

Proof. Exploiting the spectrum of the circulant matrices and setting

$$[\boldsymbol{\varpi}]_i := \begin{cases} \mathrm{d}^{-1}, & \text{if } e_{i1} \in \mathcal{E}; \\ 0, & \text{otherwise}; \end{cases}$$

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eigenvalues of the Randić matrix $\mathscr{R}$ can be rewritten as

$$\begin{aligned} \lambda^{\mathscr{R}}(j) &= \frac{1}{\mathrm{d}} \sum_{k=1}^{\mathrm{d}/2} [\exp(-\mathbf{i}2k\theta j)] + \frac{1}{\mathrm{d}} \sum_{k=n-\mathrm{d}/2}^{n-1} [\exp(-\mathbf{i}2k\theta j)] \\ &= \frac{1}{\mathrm{d}} \sum_{k=1}^{\mathrm{d}/2} [\exp(-\mathbf{i}2k\theta j)] + \frac{1}{\mathrm{d}} \sum_{k=1}^{\mathrm{d}/2} [\exp(\mathbf{i}2k\theta j)] \\ &= \frac{2}{\mathrm{d}} \left(\frac{1}{2} \sum_{|k| \le \mathrm{d}/2} [\exp(\mathbf{i}2k\theta j)] - \frac{1}{2} \right) \\ &= \kappa^{-1} (\mathcal{D}_\kappa(2\theta j) - 1/2) \end{aligned}$$

protocol performances improve as κ increases!

Main results: Spectral characterization

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$$[\boldsymbol{\varpi}]_i := \begin{cases} \mathrm{d}^{-1}, & \text{if } e_{i1} \in \mathcal{E}; \\ 0, & \text{otherwise;} \end{cases}$$

eigenvalues of the Randić matrix $\mathscr{R}$ can be rewritten as

$$\begin{aligned} \lambda^{\mathscr{R}}(j) &= \frac{1}{\mathrm{d}} \sum_{k=1}^{\mathrm{d}/2} [\exp(-\mathbf{i}2k\theta j)] + \frac{1}{\mathrm{d}} \sum_{k=n-\mathrm{d}/2}^{n-1} [\exp(-\mathbf{i}2k\theta j)] \\ &= \frac{1}{\mathrm{d}} \sum_{k=1}^{\mathrm{d}/2} [\exp(-\mathbf{i}2k\theta j)] + \frac{1}{\mathrm{d}} \sum_{k=1}^{\mathrm{d}/2} [\exp(\mathbf{i}2k\theta j)] \\ &= \frac{2}{\mathrm{d}} \left(\frac{1}{2} \sum_{|k| \leq \mathrm{d}/2} [\exp(\mathbf{i}2k\theta j)] - \frac{1}{2} \right) \\ &= \kappa^{-1} (\mathcal{D}_{\kappa}(2\theta j) - 1/2) \end{aligned}$$

protocol performances improve as κ increases!

Leveraging the d-regularity, the rest of the statement can be proven resorting to Landau H., Odlyzko A., 1981 *"Bounds for Eigenvalues of Certain Stochastic Matrices"*. □

Main results: Fiedler value

The previous theorem offers a deep insight on the connection between the Dirichlet kernel and the eigenvalues of $\mathbf{L}$.

The analysis continues focusing on the extremal eigenvalues of the restricted spectrum $\Lambda_0(\mathbf{L}) := \Lambda(\mathbf{L}) \setminus \{\lambda_0^{\mathbf{L}}\} \subseteq (0, 4\kappa]$, denoting the eigenvalues of $\Lambda(\mathbf{L})$ with $0 = \lambda_0^{\mathbf{L}} < \lambda_1^{\mathbf{L}} \leq \ldots \leq \lambda_{n-1}^{\mathbf{L}}$.
Only the result on the Fiedler value is reported in what follows.

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Only the result on the Fiedler value is reported in what follows.

Corollary (Fiedler value of κ-ring graphs)

The smallest positive eigenvalue $\lambda_1^{\mathbf{L}}$ of the graph Laplacian $\mathbf{L}$ associated to the κ-ring graph $C_n(1, \kappa)$ is given by

$$\lambda_1^{\mathbf{L}} := \min_{j=1\ldots n-1} \lambda^{\mathbf{L}}(j) = \lambda^{\mathbf{L}}(1) = \lambda^{\mathbf{L}}(n-1) \in (0, 2\kappa).$$

Eigenvalue $\lambda_1^{\mathbf{L}}$ gives us information on the right limit $\lambda_1^{\mathbf{F}}$ of the unit circle allowing to determine protocol performances.

References for RT (iv)

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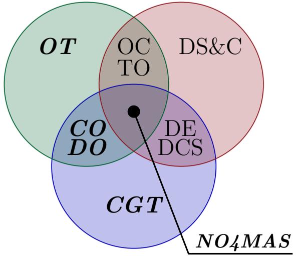
Conclusions

Main research goals

- Distributed strategies for coverage and focus on event with limited sensing capabilities

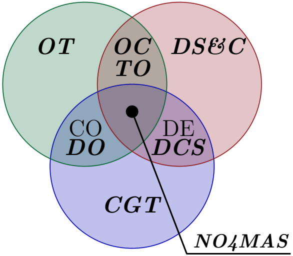


INVESTIGATION OBJECTIVES

- Automatic and dynamic deployment
- Event detection
- Clustering
- Robotic dispatch
- Virtual modeling & simulation

Main research goals

i Distributed strategies for coverage and focus on event with limited sensing capabilities



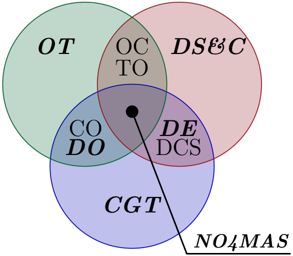
ii Optimal time-invariant formation control

INVESTIGATION OBJECTIVES

- Formation flocking
- Distributed control design
- Trajectory exploration
- Comparison of performances

Main research goals

i Distributed strategies for coverage and focus on event with limited sensing capabilities



ii Optimal time-invariant formation control

INVESTIGATION OBJECTIVES

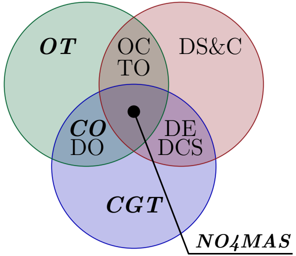
- Networked estimation
- Distributed algorithm design
- Performance analysis & comparison

iii Distributed estimation from relative measurements

Main research goals

i Distributed strategies for coverage and focus on event with limited sensing capabilities

ii Optimal time-invariant formation control



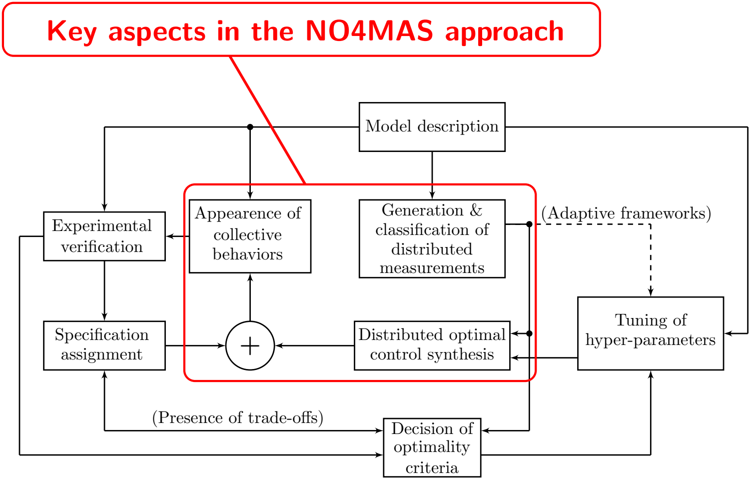
INVESTIGATION OBJECTIVES

- Network analysis
- Detailed spectral characterization of a class of graphs

iv Algebraic characterization of certain circulant networks

iii Distributed estimation from relative measurements

General approach to NO4MAS: design & validation



Arrows express dependencies.

Publications 1/2

- RT (ii) acceptance of conference paper "**Optimal Time-Invariant Formation Tracking for a Second-Order Multi-Agent System**", to ECC 2019.
- RT (iii) acceptance of conference paper "**On the Distributed Estimation from Relative Measurements: a Graph-Based Convergence Analysis**", to ECC 2019.
- Collaboration with Ph.D. student Luca Varotto: acceptance of conference paper "**Distributed Localization of Visual Sensor Networks based on Dual Quaternions**", to ECC 2019.
- RT (i): acceptance of conference paper "**Distributed Strategies for Dynamic Coverage with Limited Sensing Capabilities**", to MED 2019.

Publications 2/2

- RT (iii): acceptance of conference paper "**A Proximal Point Approach for Distributed System State Estimation**", to IFAC 2020.
- RT (iv): writing of journal article "**On the Relation between the Eigenvalues Induced by a Class of Circulant Graphs and the Dirichlet Kernel**", to Linear Algebra and its Applications.
- RT (iii): writing of journal article "**Regularized Graph-based Iterative Approaches for the Distributed Estimation from Relative Measurements**", to Transaction on Control of Network Systems.
- RT (ii): writing as journal article "**Optimal Time-Invariant Distributed Formation Tracking for a Second-Order Multi-Agent System**", to European Journal of Control.

Future directions

1. Currently: working as a **post-doc** under the supervision of Daniel Zelazo at the Technion in Haifa, Israel. Research topic: **cyber-security for multi-agent systems**.

2. From April 2020: submission of the pending articles.

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