DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

GINGER

UNIVERSITÀ DEGLI STUDI DI PADOVA

Modeling, estimation and control of ring laser gyroscopes for the accurate estimate of the Earth rotation

Davide Cuccato, DEI-INFN. March, 20th 2015.

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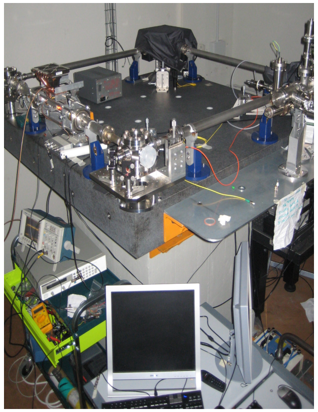
Presentation Outline

- **Introduction**
- **Ring Laser Overview**
- **Ring Laser Dynamics:**
  1. Model
  2. Rotational frequency estimation
  3. Results
- **Conclusions**

Introduction

- Small Size: (5-50 cm) Inertial Guidance

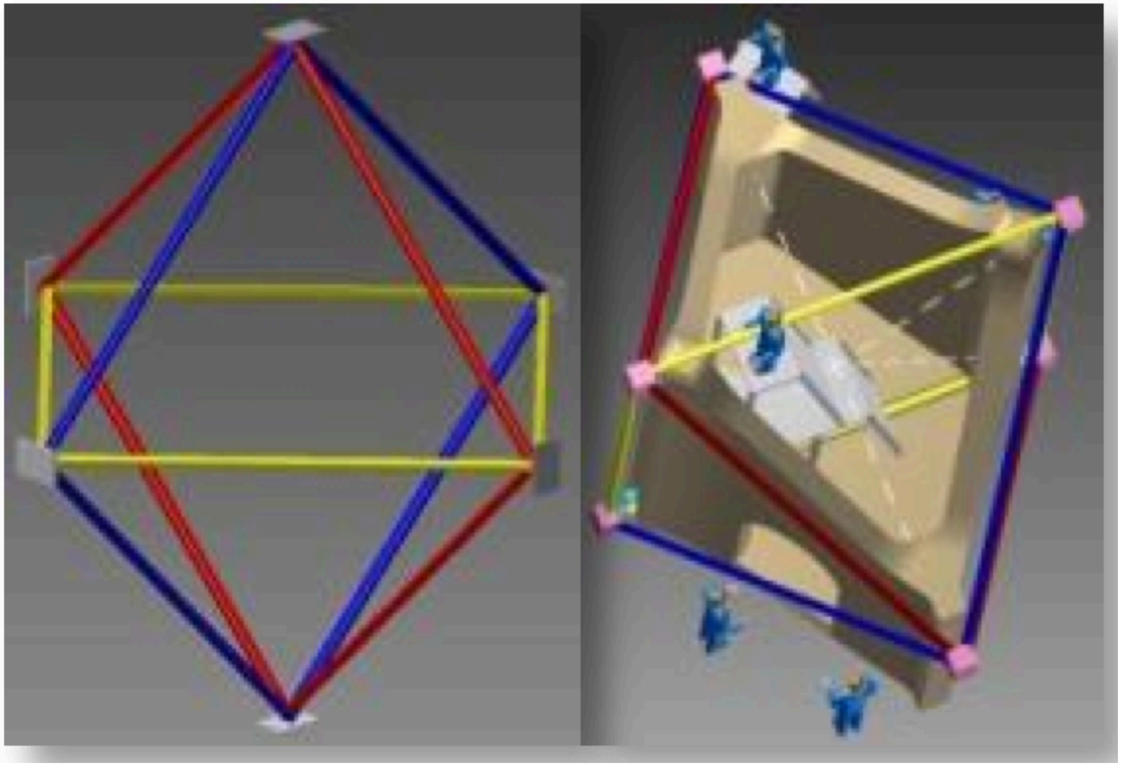
- Medium Size: (1-5 m) Geophysics, Seismology, Metrology



- Large Size: (5-10 m) Geodesy, Geophysics

GINGER: Gyroscope IN GEneral Relativity

- Measure Lense-Thirring at 10-1%.
- Triaxial system, 4-10 m side
- Accurate estimation of 9 digits of the Earth rotation
- Data taking of several days
- Can not be made monolithic



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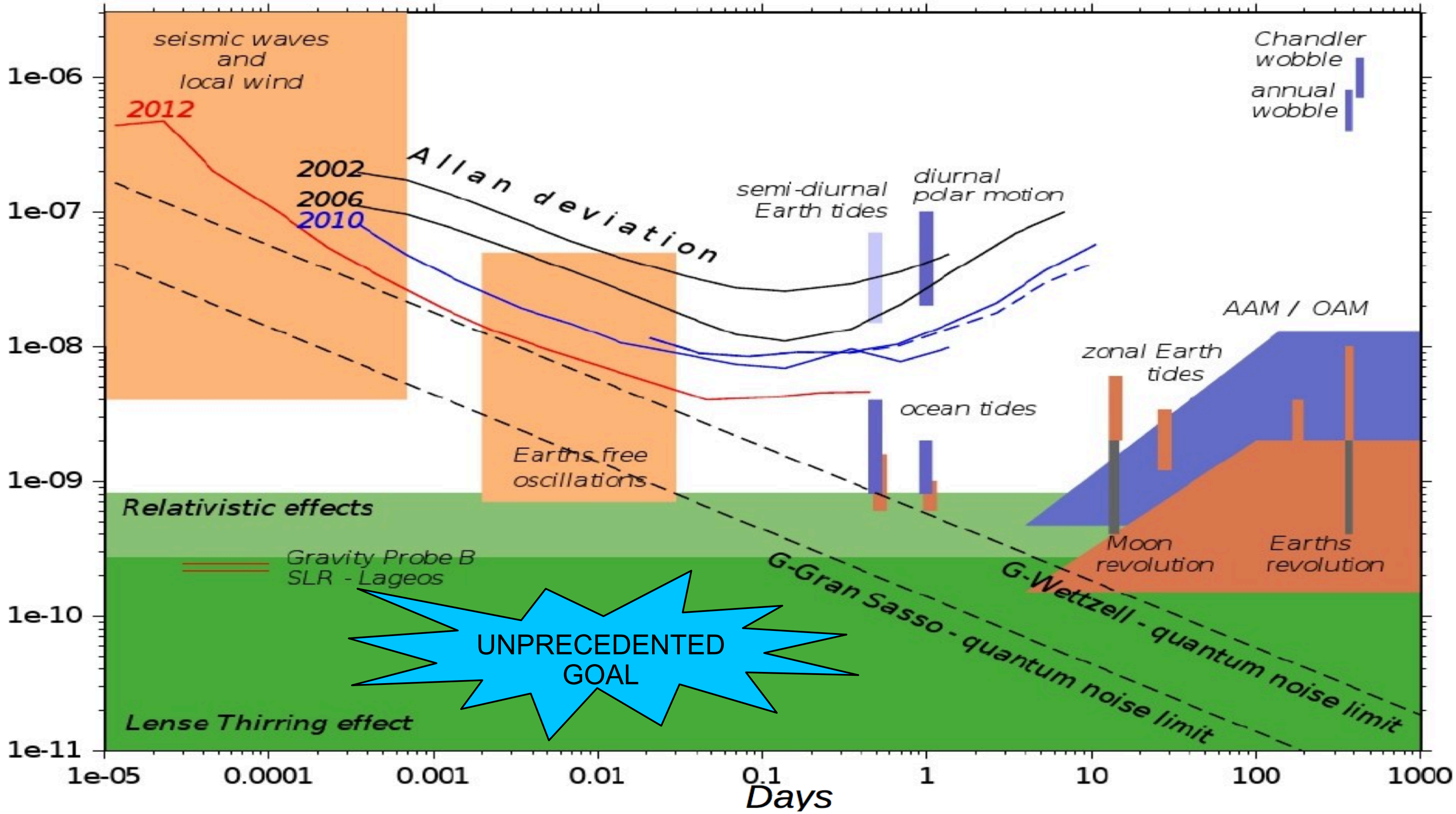
GINGER: Gyroscope IN GEneral Relativity



Introduction: Active Sagnac Interferometry

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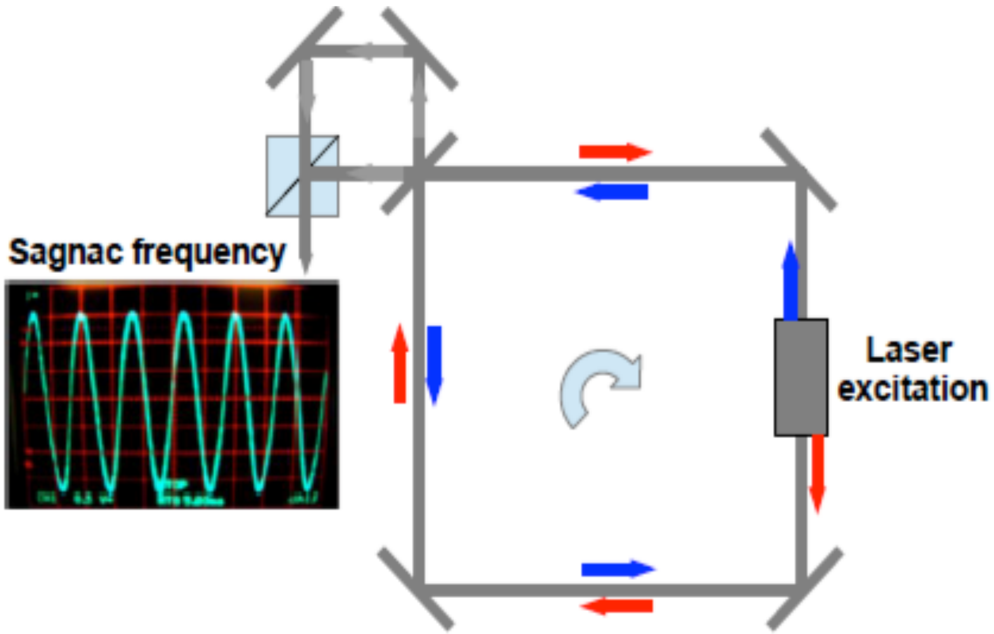
- Light inside a polygonal optical cavity
- Mirrors as vertexes, gas-filled tubes as sides
- Two electromagnetic waves travelling in opposite directions
- The frequency split between opposite travelling waves is mainly due to rotation



Introduction: Active Sagnac Interferometry

Sagnac Frequency

- Scale factor corrections
- Null shift corrections
- Dead band effects
- Accuracy is needed!

$$\nu_s = \left(\frac{4A}{\lambda L} + \Delta\nu_{SF} \right) \mathbf{n} \cdot \mathbf{\Omega} + \Delta\nu_0 + \Delta\nu_{BS} + \eta$$

- Optical cavity geometry
- Laser dynamics

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Ring Laser dynamics: Model

$$\dot{E}_1 = (\alpha_1 + i\omega_s) E_1 + r_2 e^{i\epsilon} E_2 - f_1(I_1, I_2) E_1$$

$$\dot{E}_2 = (\alpha_2 - i\omega_s) E_2 + r_1 e^{i\epsilon} E_1 - f_2(I_1, I_2) E_2$$

Where

$$\begin{cases} I_{1,2} = |E_{1,2}^2| \quad S = |E_1 + E_2|^2 \\ f_{1,2}(I_1, I_2) = \beta I_{1,2} + (\theta + i\tau) I_{2,1} \end{cases}$$

$E_{1,2}(t)$ are linearly coupled through $r_{1,2}$ ϵ

And non-linearly coupled through θ τ

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Ring Laser dynamics: Model

The Ring Laser dynamics in compact from:

$$\dot{\mathbf{E}} = \left[\mathbf{A} - \mathscr{D}(\mathbf{E}).\mathbf{B}.\mathscr{D}(\mathbf{E}^*)\right]\mathbf{E}$$

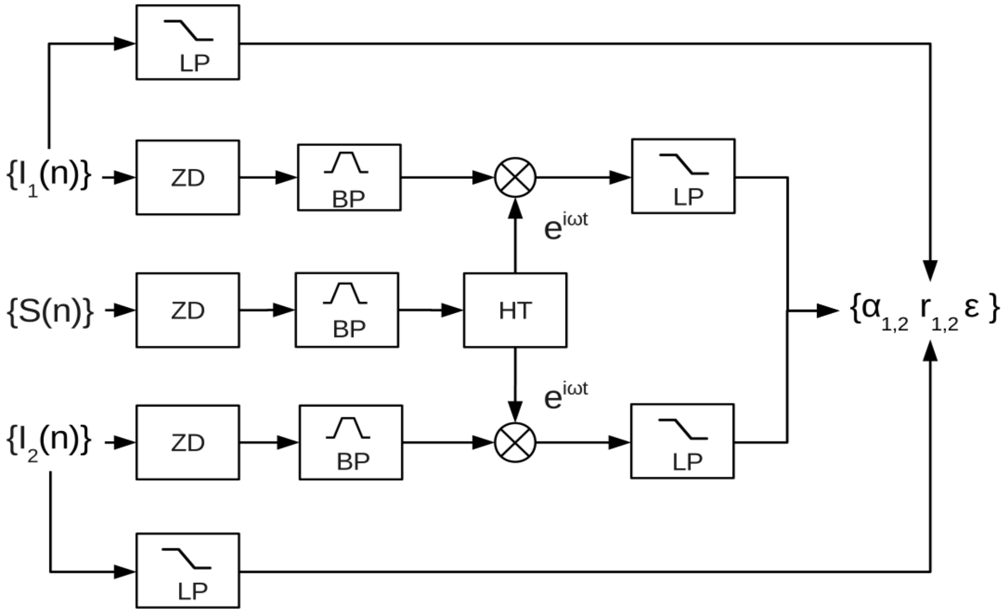
The matrices $\mathbf{A} \equiv \frac{c}{L}\mathbf{P}^{(0)} - \mathbf{M}$ and $\mathbf{B} \equiv \frac{c}{L}\mathbf{P}^{(2)}$ are given by

- **Atomic Polarization:**
  - Active medium parameters
  - Related to He-Ne isotopic mixture
  - Linear and Non Linear Coupling
  - Can be computed using QED
- **Dissipative Effects:**
  - "Passive" parameters
  - Related to cavity mirrors
  - Linear Coupling.
  - Transmission,absorption and scattering, Sagnac effect.

Ring Laser dynamics: Rotational frequency estimation

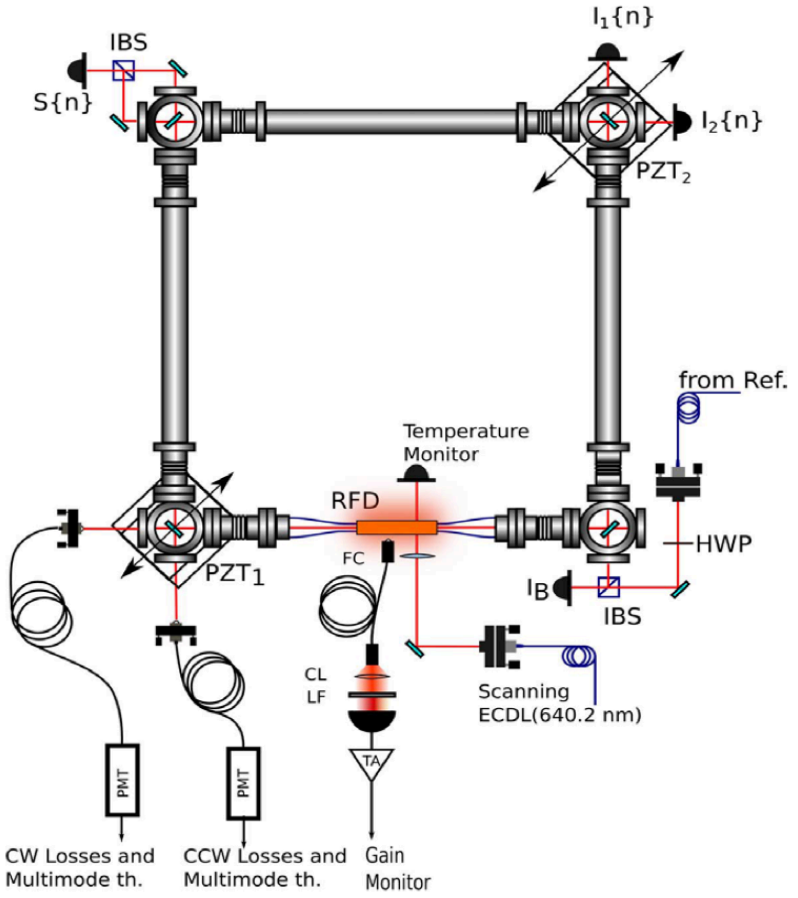
Passive parameters identification

- Pre-filtering for electronic noise rejection
- Perturbative solutions of the RL dynamics



Ring Laser dynamics: Rotational frequency estimation

Active parameters estimation:



- Polarization computation with *Lamb* model
- Gain fluorescence monitor
- ECDL probe of gas mixture
- Ring down times

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Extended Kalman Filter

- State: $E_{1,2}$
- Measures: S, $I_{1,2}$
- Parameters: A and B
- Laser systematic subtraction and rotational frequency estimation

Ring Laser dynamics: Results

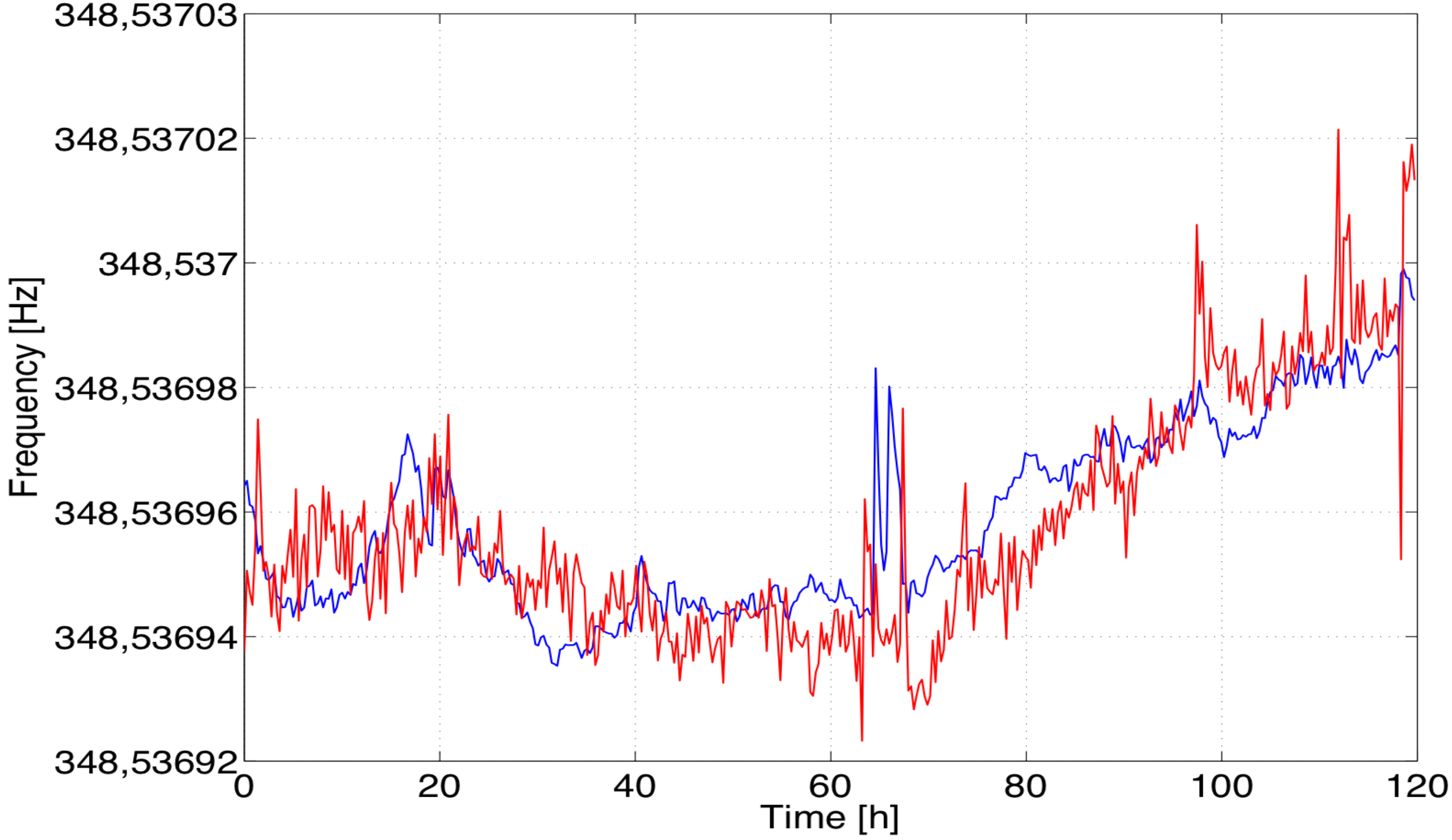
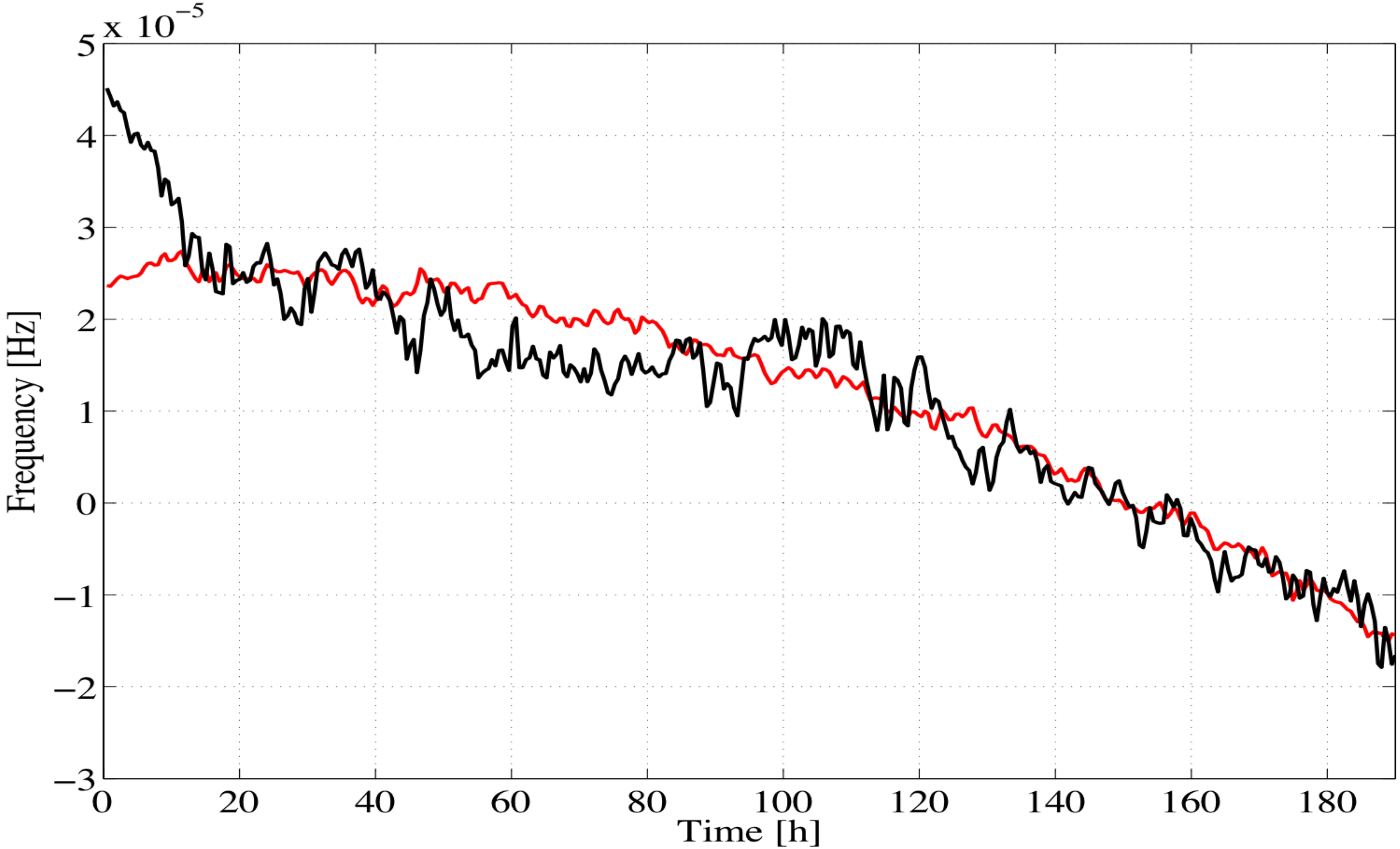
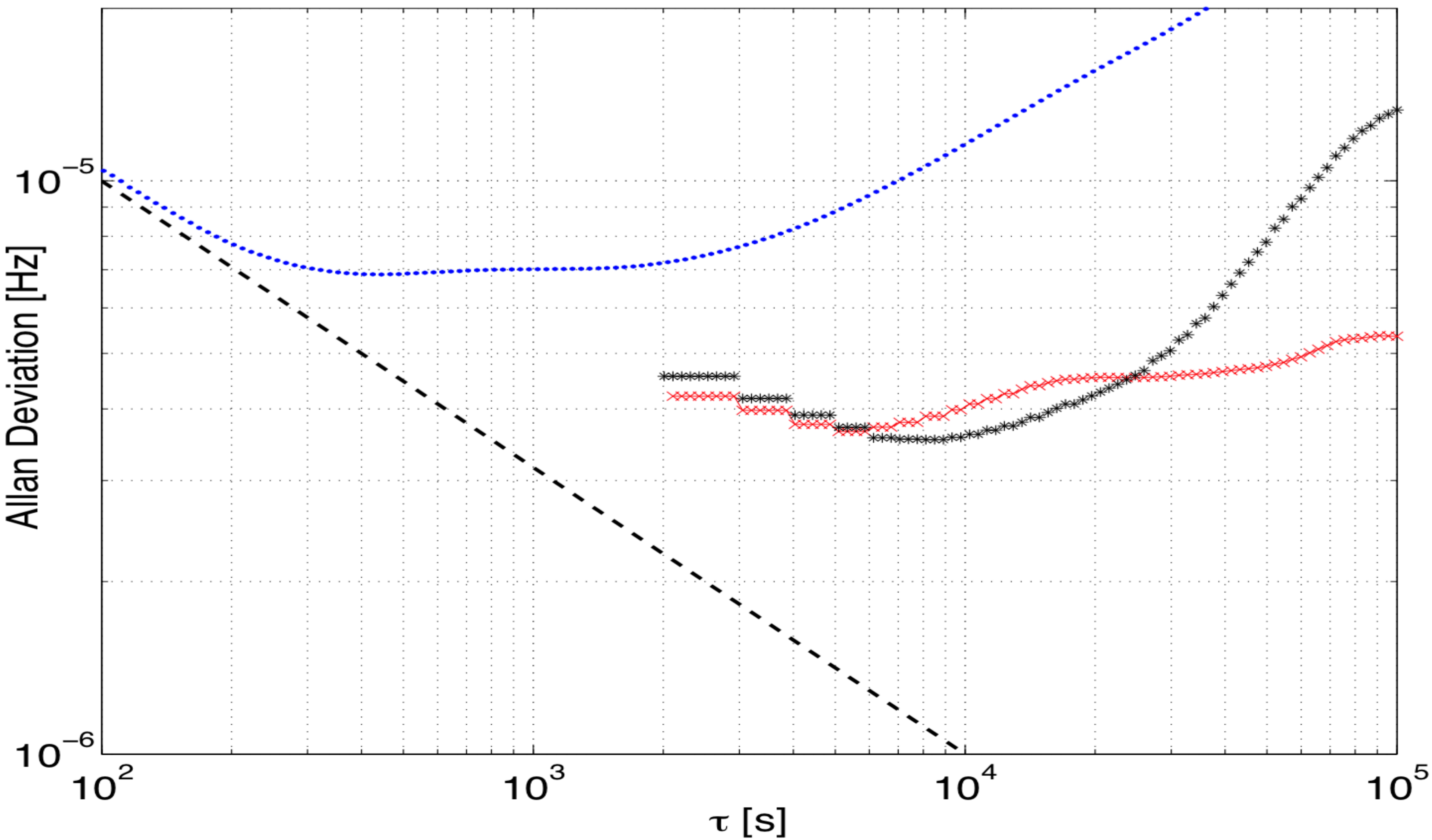


Fig. 5 Comparison of the backscattering (blue) estimated from the intensity channels with the residuals of the Sagnac frequency (red) estimated from the interferogram channel. Note that they correlate on the micro-hertz scale.

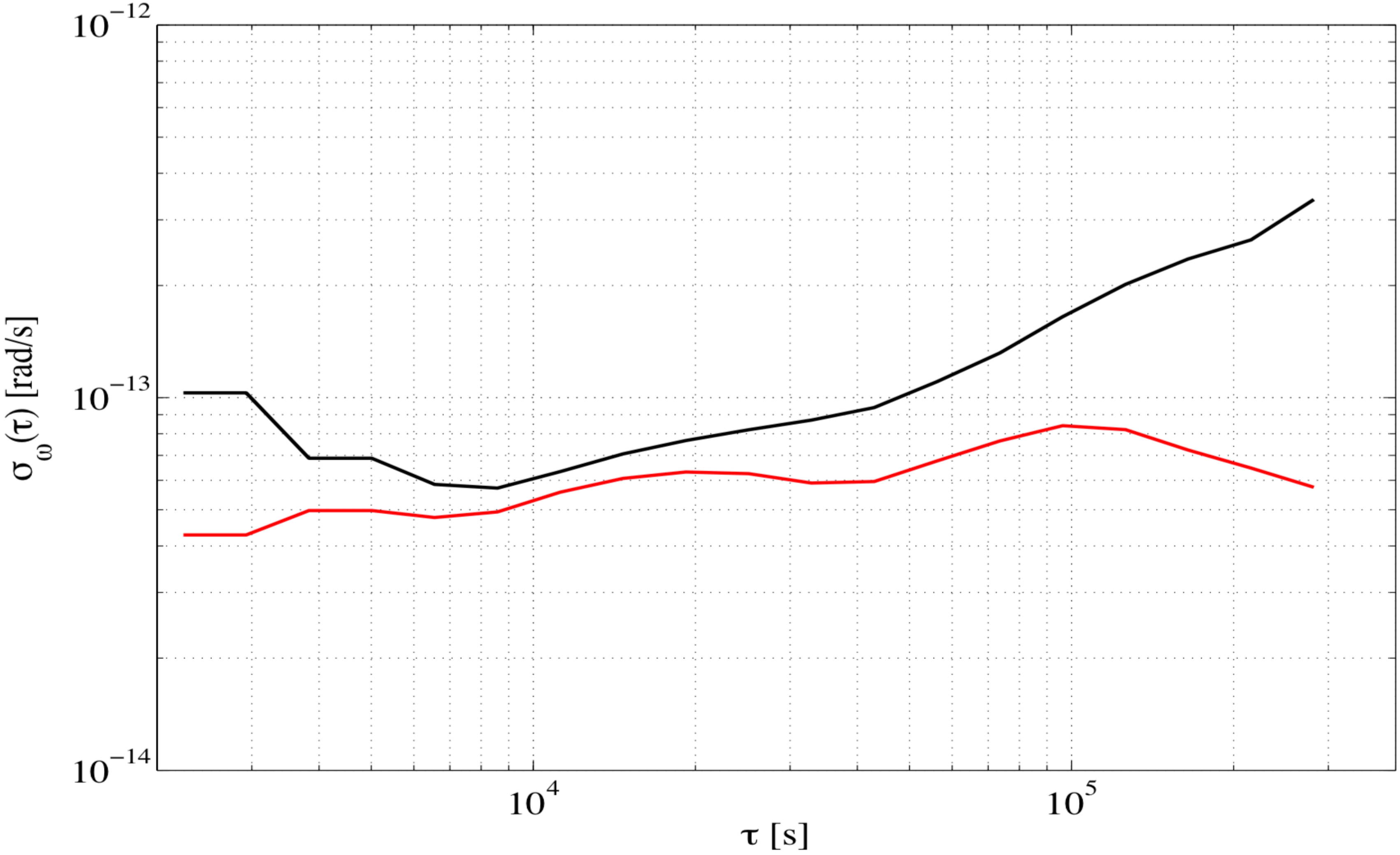
Ring Laser dynamics: Results



Ring Laser dynamics: Results



Ring Laser dynamics: Results



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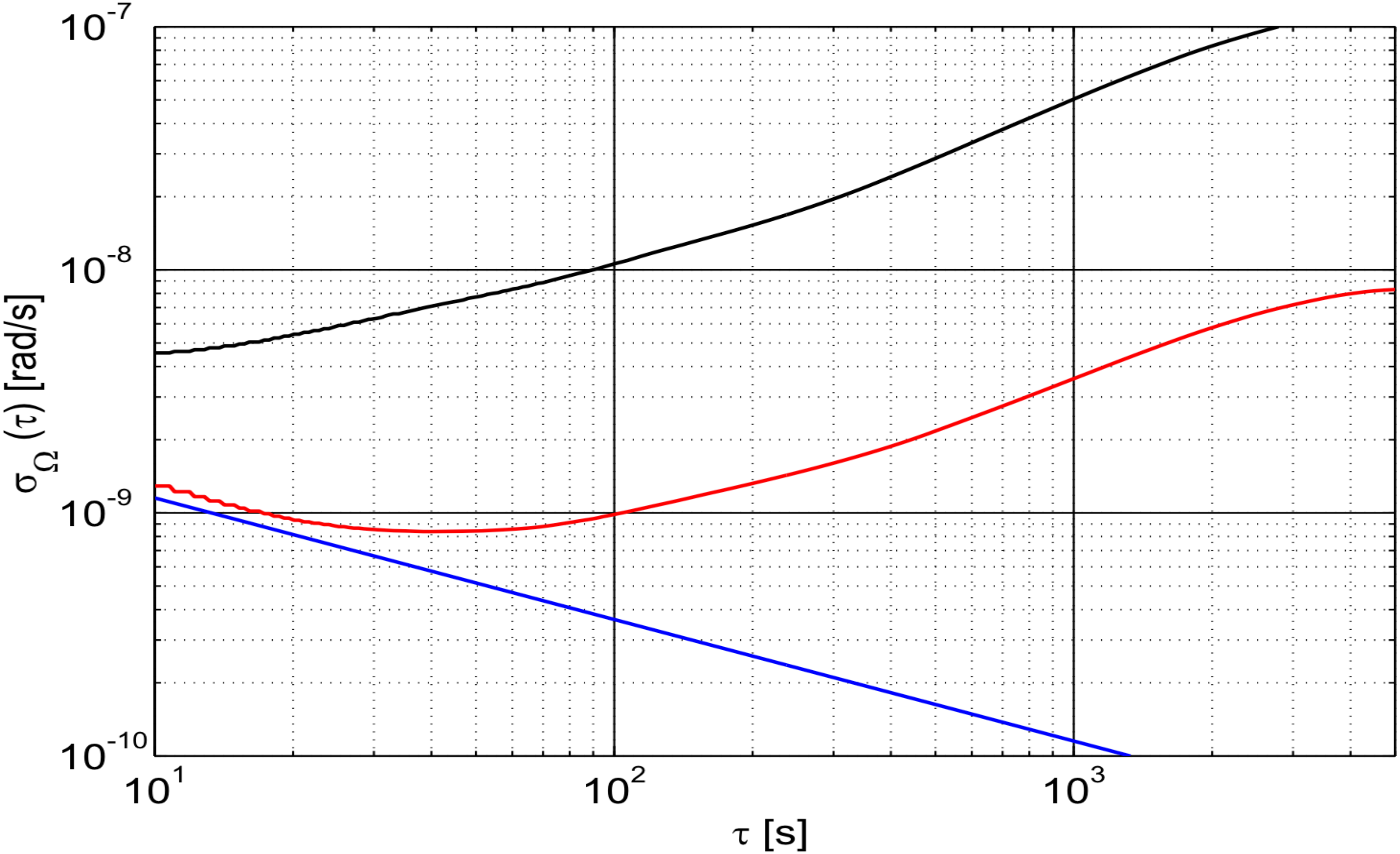
Ring Laser dynamics: Results



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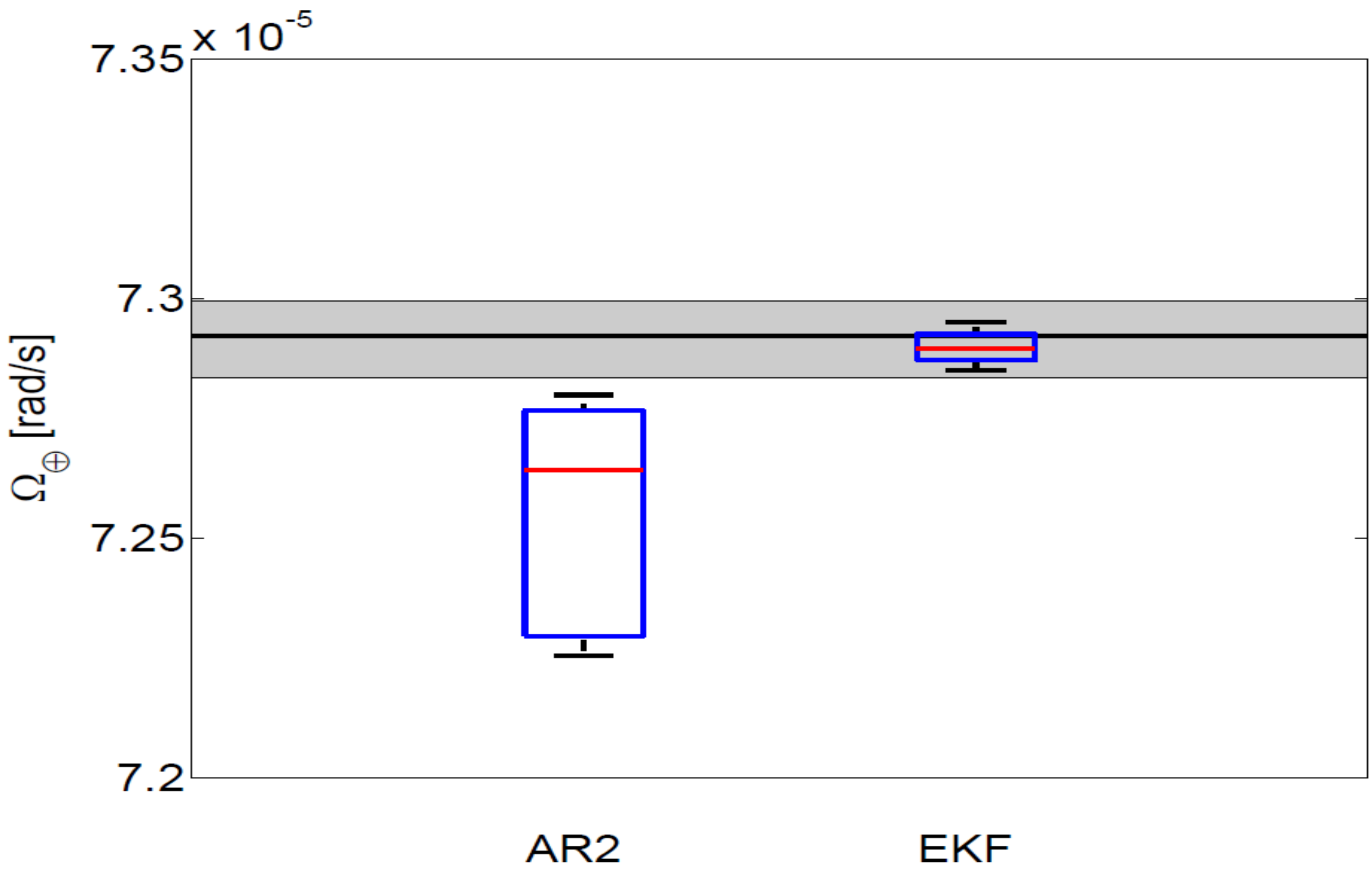
Ring Laser dynamics: Results



Pubblications

Journal papers

- R. Santagata *et. al.* "Optimization of the geometrical stability in square ring laser gyroscopes" Classical and Quantum Gravity, January (2015), to be published.
- J. Belfi *et. al.* "Interferometric length metrology for the dimensional control of ultra-stable ring laser gyroscopes" Classical and Quantum Gravity, 31, 22, (2014).
- D. Cuccato *et. al.* "Controlling the non-linear intracavity dynamics of large He–Ne laser gyroscopes ", Metrologia, 51, 1, (2014).
- A. Di Virgilio *et. al.* "A ring lasers array for fundamental physics" Comptes Rendus Physique, 15, 10, (2014).
- A. Beghi *et. al.* "Compensation of the laser parameter fluctuations in large ring-laser gyros: a Kalman filter approach", Applied Optics, 51, 31, (2012).

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Pubblications

Conference papers

- J. Belfi *et. al.* "Experimental activity toward GINGER (gyroscopes IN general relativity)" Laser Optics, 2014 International Conference, IEEE.
- N. Beverini *et. al.* "Toward the "perfect square" ring laser gyroscope" Photonics Technologies, 2014 Fotonica AEIT Italian Conference on, IEEE.
- J Belfi *et. al.* "Absolute control of the scale factor in GP2 laser gyroscope: Toward a ground based detector of the lense-thirring effect" European Frequency and Time Forum & International Frequency Control Symposium (EFTF/IFC), 2013 Joint, IEEE.
- D. Cuccato *et. al.* "Laser dynamics effects on the systematics of large size laser gyroscopes" European Frequency and Time Forum & International Frequency Control Symposium (EFTF/IFC), 2013 Joint, IEEE.
- J. Belfi *et. al.* "Laser gyroscopes for very high sensitive applications" European Frequency and Time Forum (EFTF), 2012, IEEE.

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Conclusions

- Ring laser dynamics effects on the accuracy rotational frequency estimation **reviewed**
- Offline procedure for the subtraction of laser systematics **designed and demonstrated**
- Geometric Newton algorithm for the computation of the beams position in the optical cavity **designed and demonstrated**
- Pose & Shape decomposition of a square optical cavity **proposed**
- RLG Simulator of all the relevant processes involved in the Ring Laser operation **developed**

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Collaboration

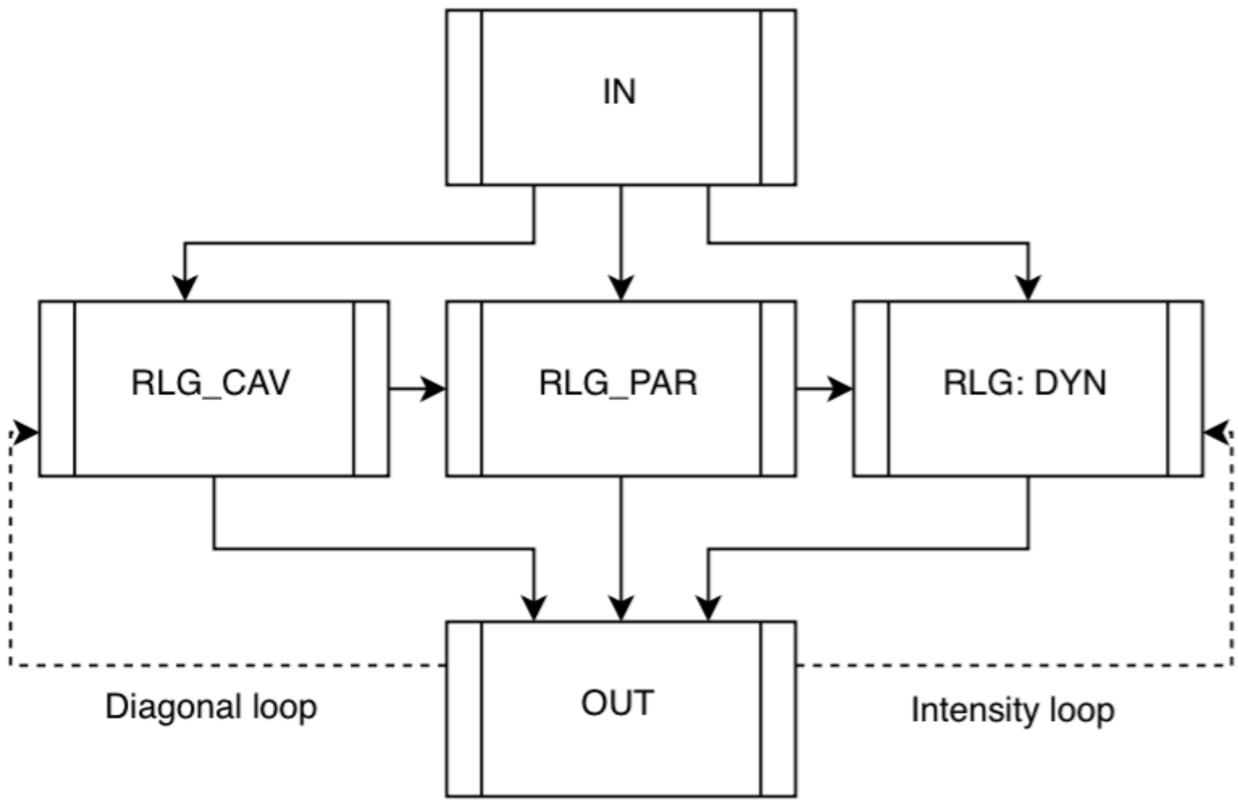
DEI	Alessandro Beghi Davide Cuccato Alberto Donazzan Giampiero Naletto	University of Canterbury New Zealand	Robert Hurst Geoff Stedman Robert Thirkettle Jon-Paul Wells
INFN (PI, LNL)	Jacopo Belfi Angela Di Virgilio Antonello Ortolan	LMU München Germany	Celine Hadziioannou Heiner Igel Maria Nader Joachim Wassermann
University of Pisa	Nicolo Beverini Giorgio Carelli Enrico Maccioni Rosa Santagata	TUM, BKG (Wettzell) Germany	Andre Gebauer Thomas Klügel Ulrich Schreiber Alexander Velikoseltsev
CNR (PD, NA)	Maria G. Pellizzo Alberto Porzio		
Politecnico (Torino)	Matteo L. Ruggero Angelo Tartaglia		

… and many more

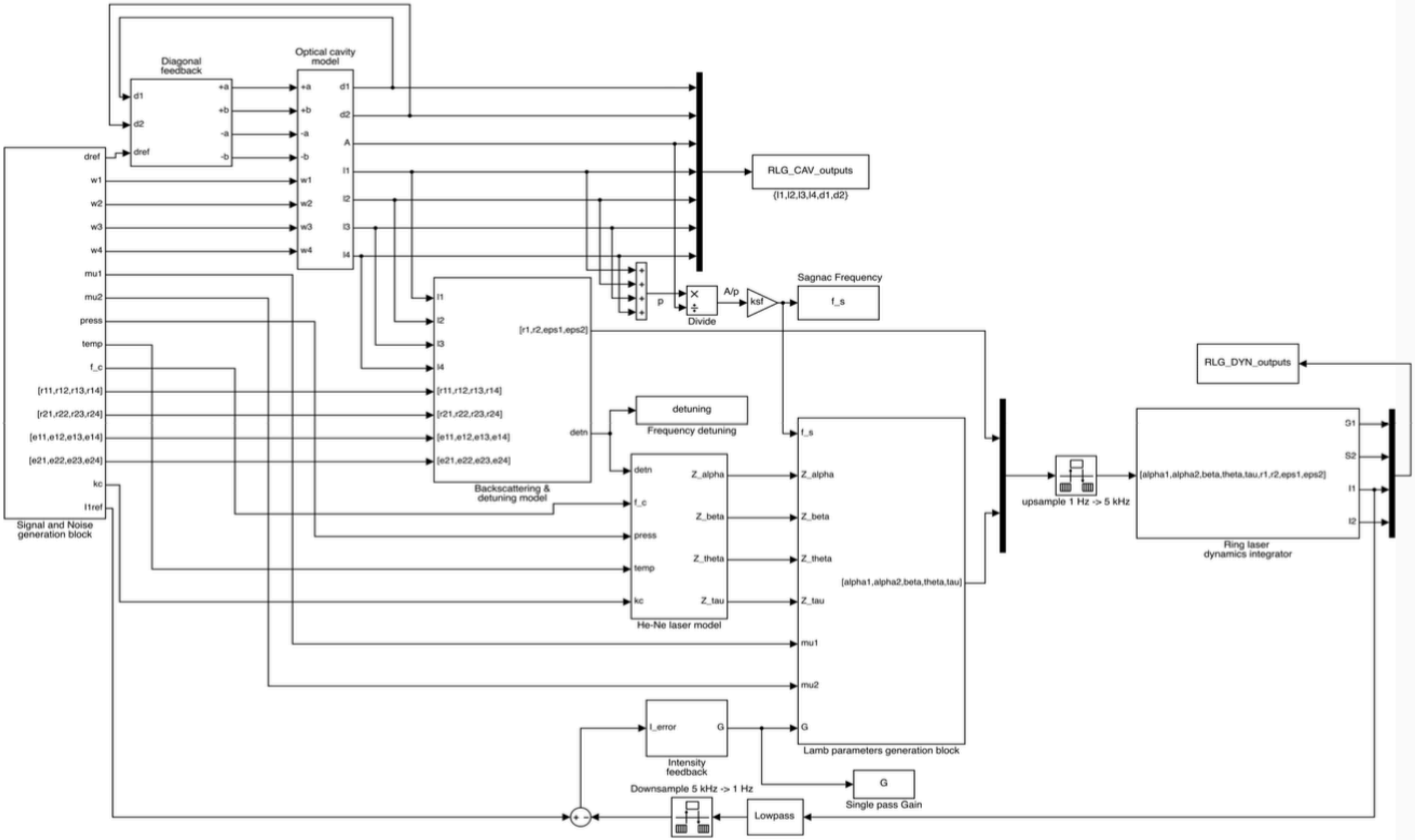
The End

Thanks for the Attention !!!

RLG Simulator: Overview



RLG Simulator: Overview



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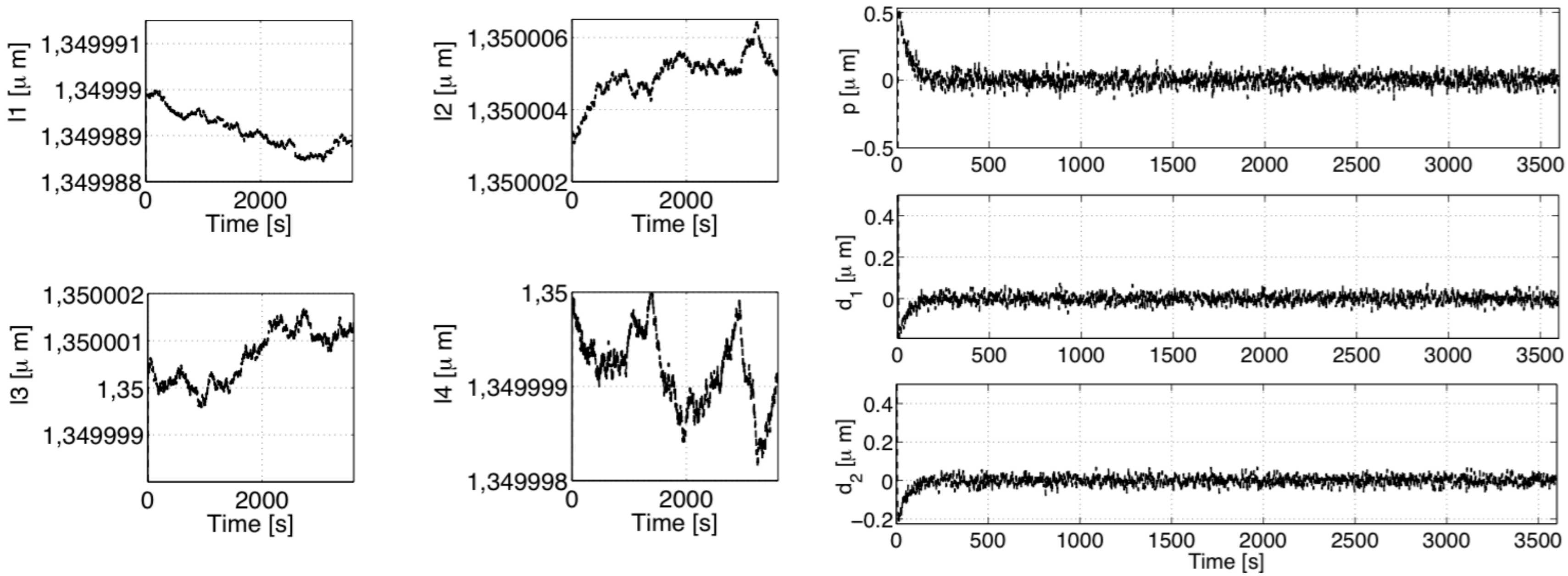
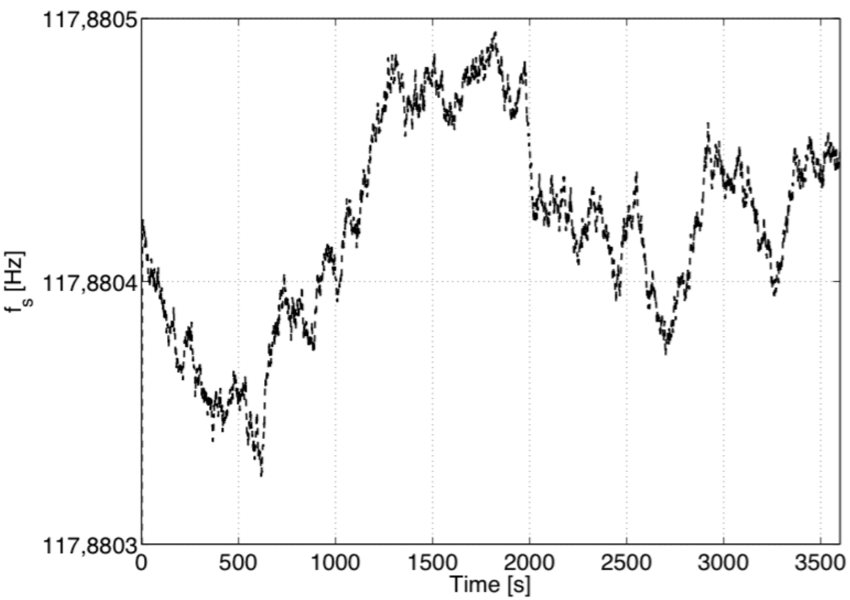
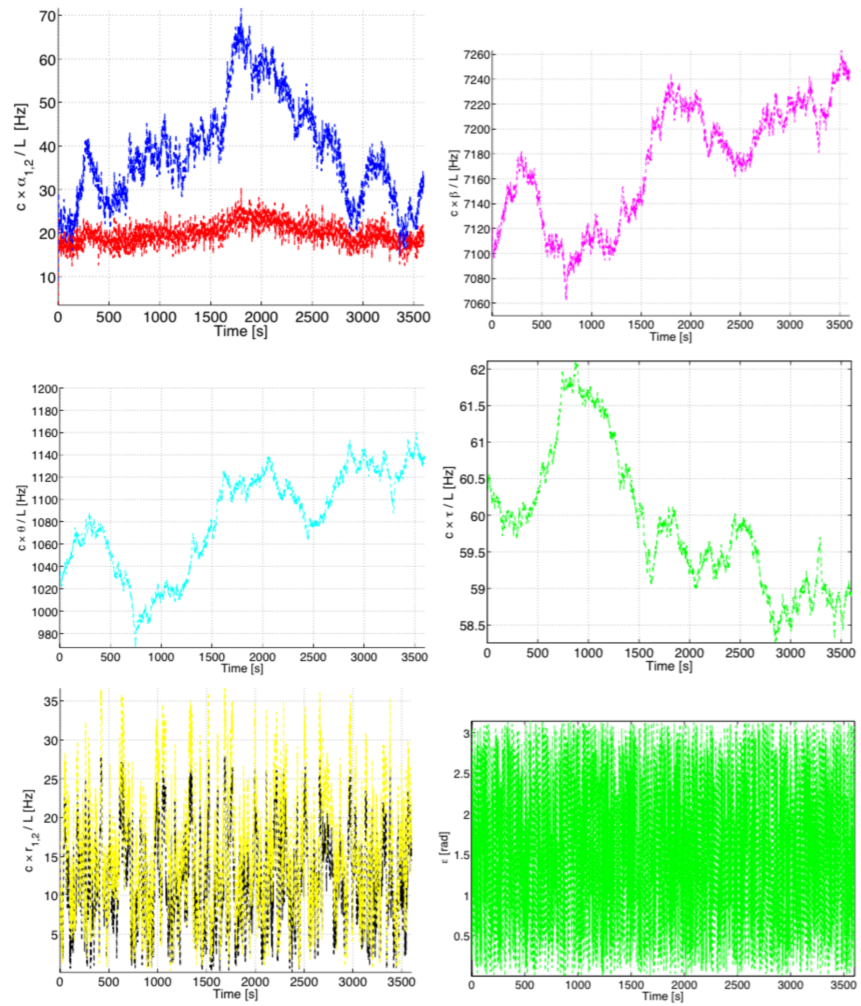
RLG Simulator: GP2 case study

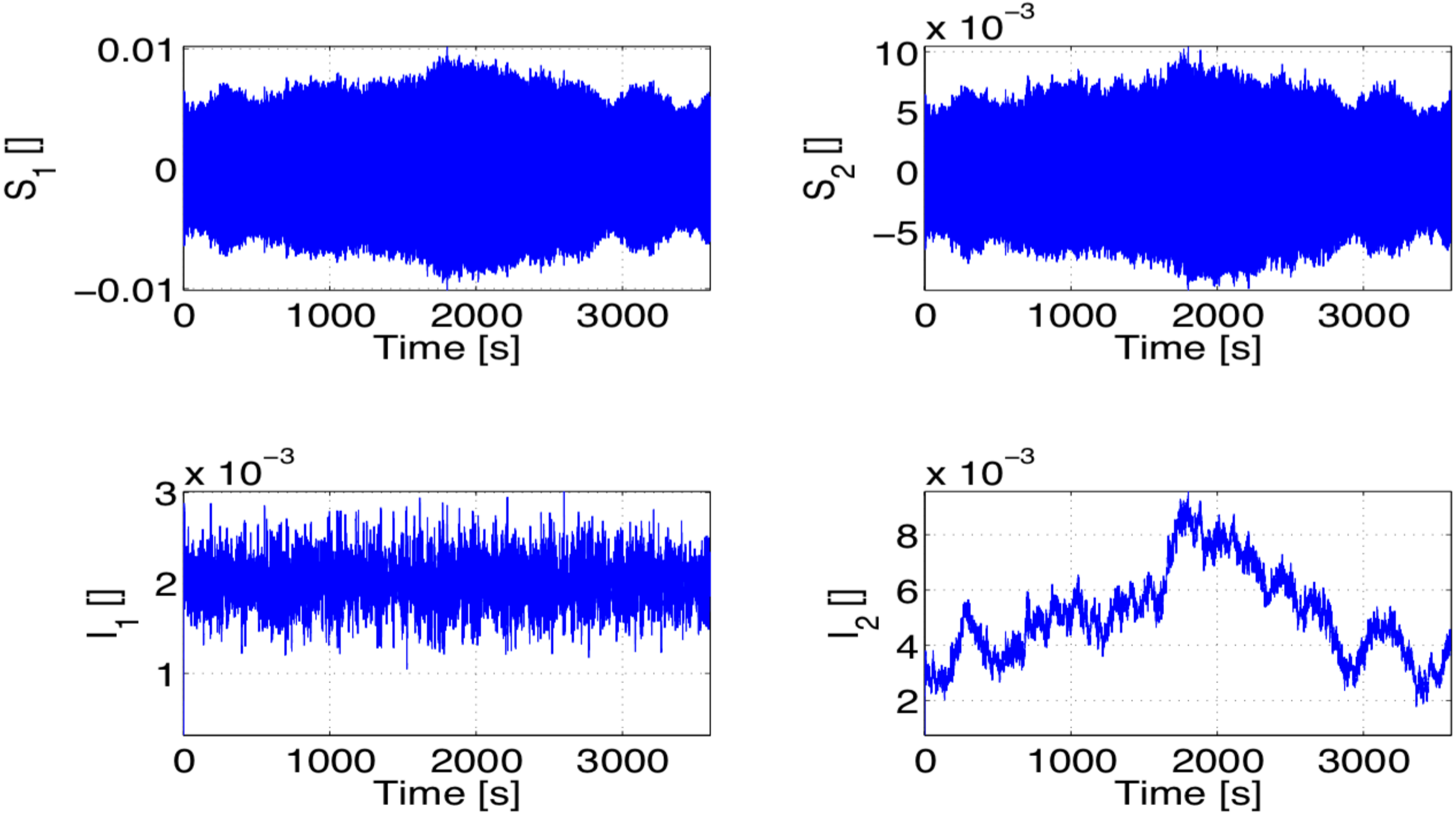


RLG Simulator: GP2 case study





RLG Simulator: GP2 case study

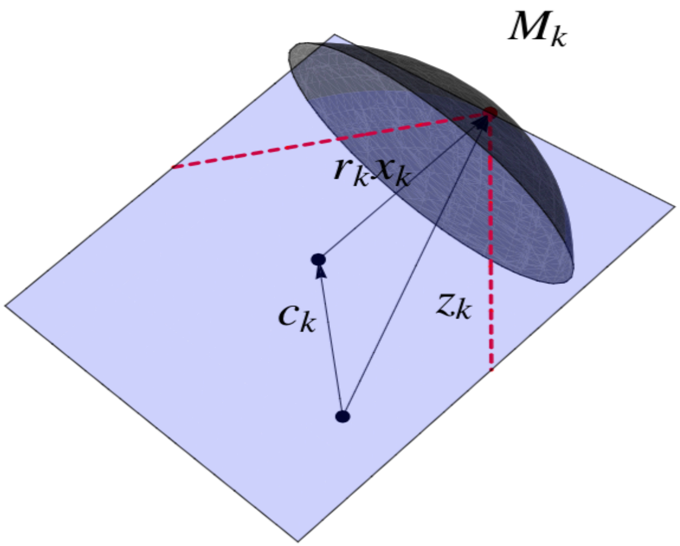


Optical Cavity geometry: Beams position computation

Task: Find the laser beams position for a given cavity configuration

Formalism: Geometric Optics

Problem Data:
- 4 Points in $\mathbb{R}^3$, The spherical mirrors C.O.C. $\mathbf{c}_k$
- 4 positive scalars in $\mathbb{R}$, the spherical mirrors R.O.C. r_k



To find the beams position, the Fermat's Principle (stationarity of the optical path length) is used:

$$\text{grad}\ p(X; C, R) = 0$$

Problem variables:
- 4 Points on the Unit Sphere $\mathbb{S}^2$, i.e. a point of the Oblique Manifold 2x4.

$$X = (\mathbf{x}_1, \ldots, \mathbf{x}_4) \in \mathcal{OB}(2, 4)$$

Laser spot virtual positions: $\mathbf{z}_k = \mathbf{c}_k + r_k \mathbf{x}_k$

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Optical Cavity geometry: beams position computation

Geometric Newton Equation

$$\begin{cases} \text{Hess} f(x)[\eta_x] = & -\text{grad} f(x) \\ \eta_x \in T_x\mathcal{M} \end{cases}$$

Retraction

$$R : T_x\mathcal{M} \to \mathcal{M},$$

$$R(0_{T_x\mathcal{M}}) = x$$

$$DR(0_{T_x\mathcal{M}})[\xi_x] = \xi_x$$

Armijo line search

$$t_k = \alpha\beta^l$$

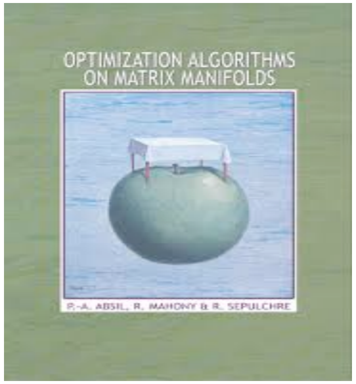
$$h(x) - h\left(R(t_k\eta_x)\right) \geq -\sigma\gamma_k Dh(x)[\eta_x]$$

Algorithm 1 *Geometric Newton with line search.*

Input: $x_0 \in \mathcal{M}$, real valued function f on $\mathcal{M}$

Output: Sequence of iterates $x_1, \dots x_n$

1. *Search Direction:* solve (4.2) in η_{x_k}.
2. *Step Size:* find $t_k = \arg\min_{\lambda} \|\text{grad} f\left(R\left(\lambda\eta_x\right)\right)\|^2$
3. *Update:* Set $x_{k+1} = R_x(t_k\eta_{x_k})$



Riemannian Gradient

$$\forall x \in \mathcal{M}, \ \text{grad } f(x) = P_x\left(\partial f(\bar{x})\right)$$

Riemannian Hessian

$$\forall x \in \mathcal{M}, \ \text{Hess} f(x)[\eta] = P_x\left(D \, \text{grad} \bar{f}(x)[\eta]\right)$$

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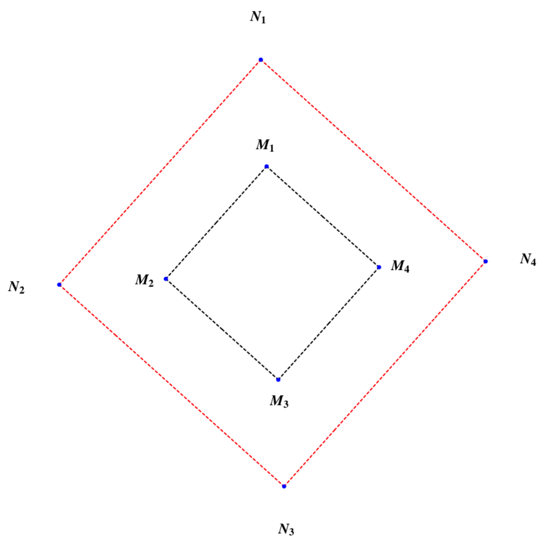
Optical Cavity geometry: Pose & Shape Decomposition

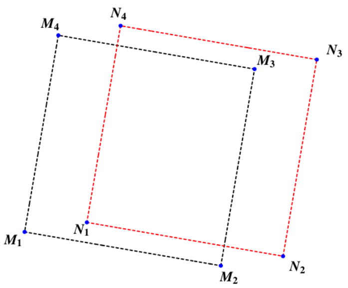
The matrix C accounts for both the pose and the shape of the mirrors

The optical cavity deformations are only induced by shape changes

Shape change



Pose change



Pose & Shape Theorem:

Regularity hypotesis on mirrors centers:

$$\mathcal{P} = \left\{ M \in \mathbb{R}^{3\times 4} | \, M_i \wedge M_{i+1} \neq 0, \, \overline{M} = 0_{3\times 1} \right\}$$

Decomposition:

$$\mathcal{P} = SO(3) \times \mathcal{T} \times \mathcal{V}$$

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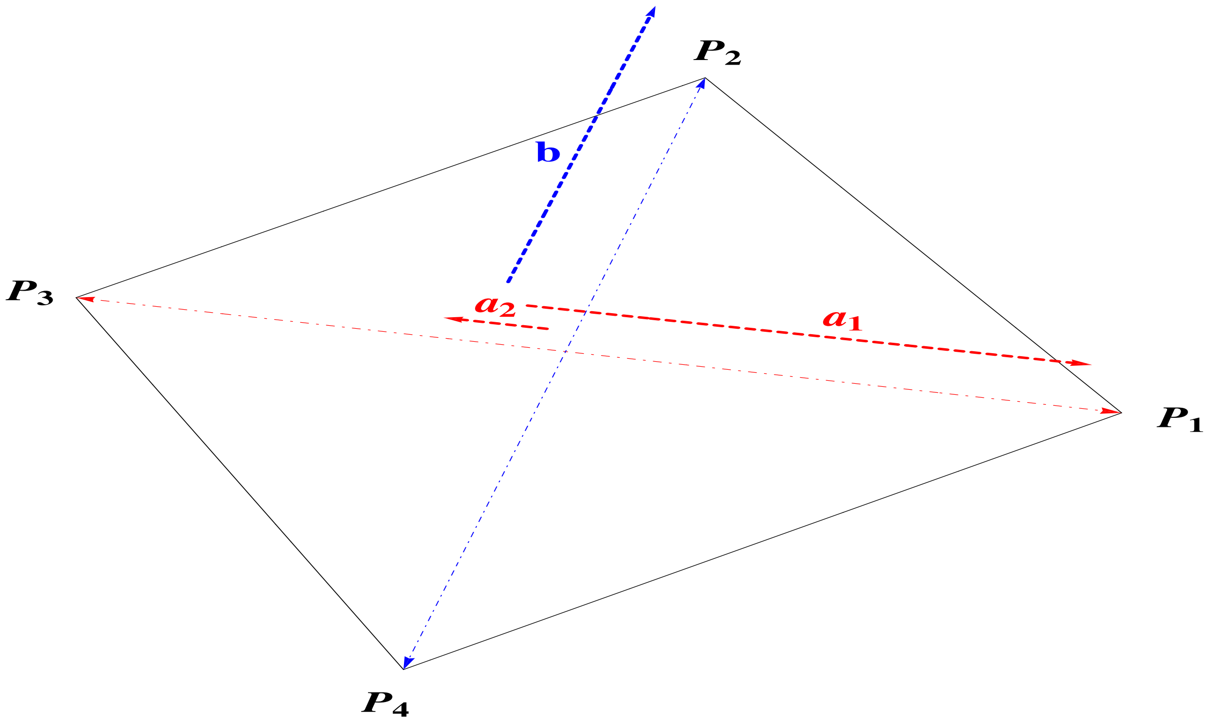
Optical Cavity geometry: Pose & Shape Decomposition

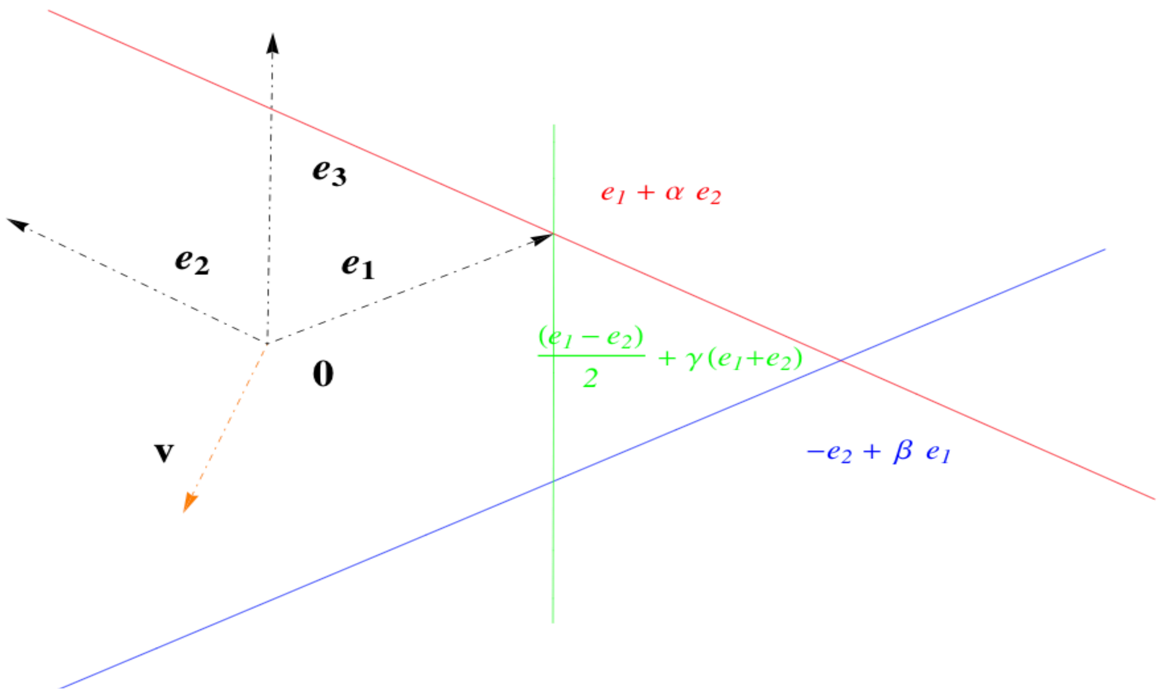
The isosceles trapezoids:



$$\mathcal{T} = \left\{ \begin{bmatrix} a_1 & b & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 3}, a_1, a_2 \in \mathbb{R}^+, b \in \mathbb{R} \right\}$$

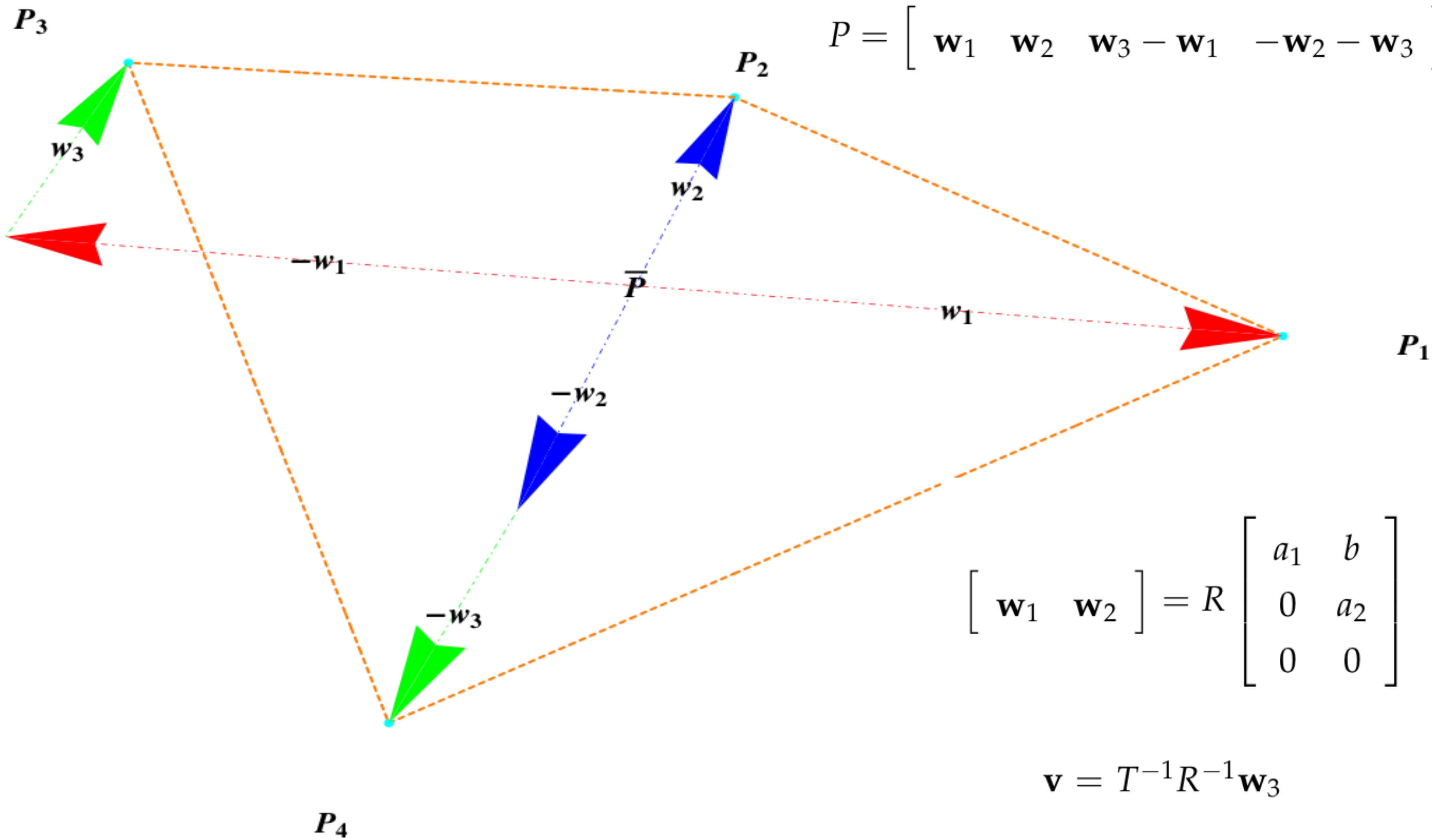
Optical Cavity geometry: Pose & Shape Decomposition

The irregular quadrilaterals:



$$\mathcal{V} = \mathbb{R}^3 \setminus \left\{ \mathrm{e}_1 + \alpha \mathrm{e}_2, -\mathrm{e}_2 + \beta \mathrm{e}_1, \frac{\mathrm{e}_1 - \mathrm{e}_2}{2} + \gamma \left(\mathrm{e}_1 + \mathrm{e}_2 \right), \alpha, \beta, \gamma \in \mathbb{R} \right\},$$

Optical Cavity geometry: Pose & Shape Decomposition

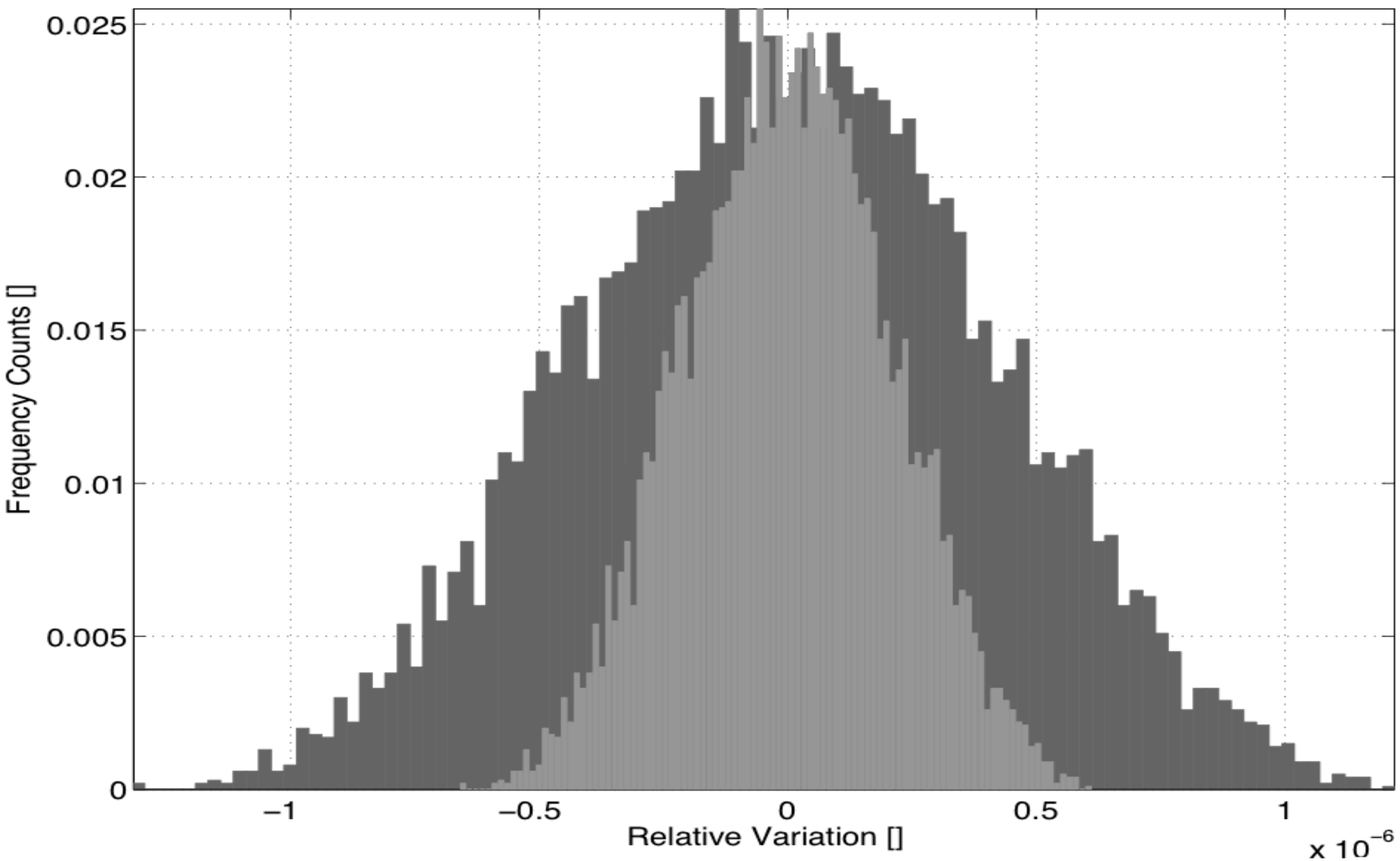


$$P = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 - \mathbf{w}_1 & -\mathbf{w}_2 - \mathbf{w}_3 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 \end{bmatrix} = R \begin{bmatrix} a_1 & b \\ 0 & a_2 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{v} = T^{-1} R^{-1} \mathbf{w}_3$$

Optical Cavity geometry: Results



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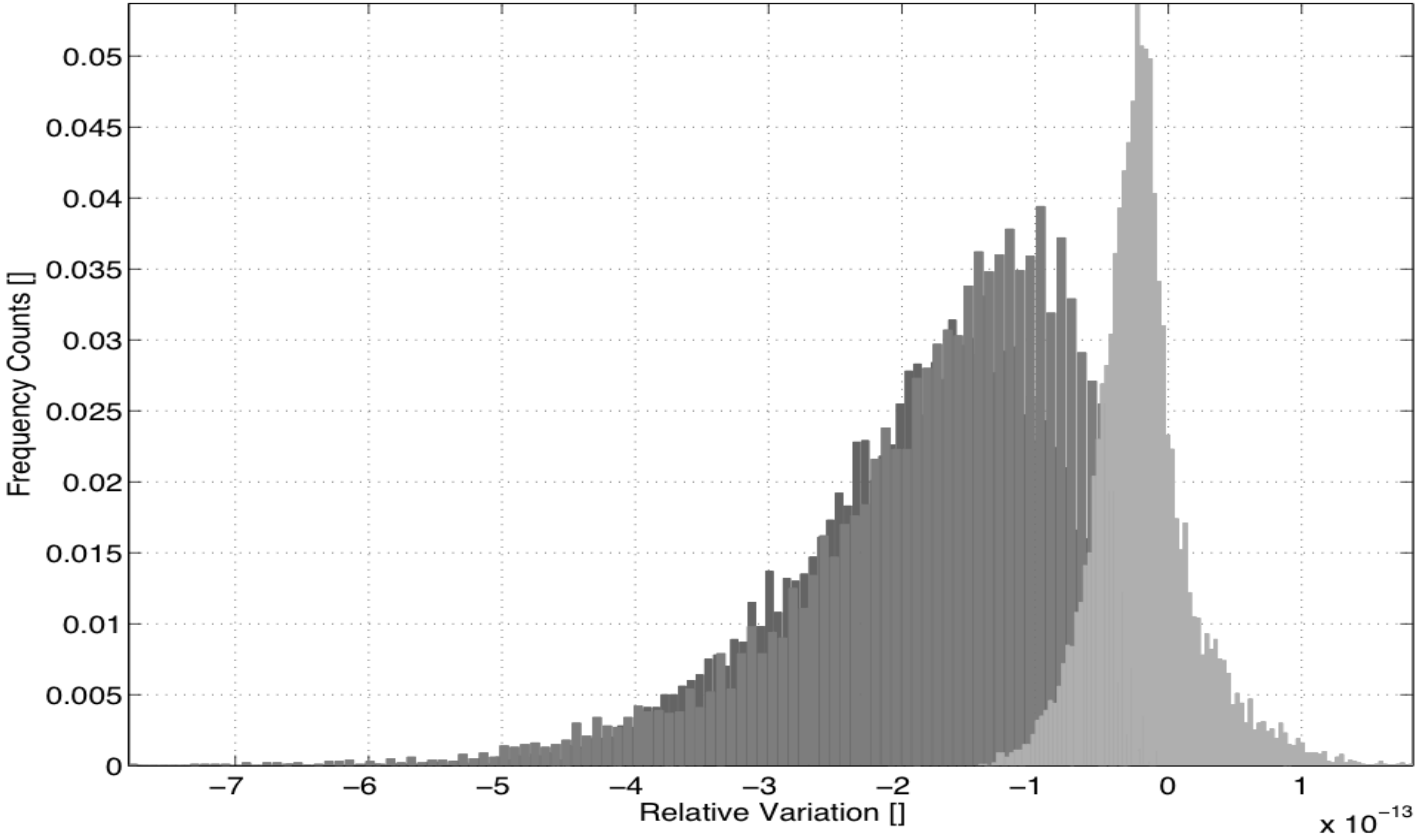
Optical Cavity geometry: Results



Optical Cavity geometry: Results