Algorithms and Applications for Nonlinear Model Predictive Control with a Long Prediction Horizon

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Supervisor: Prof. Alessandro Beghi

February 26, 2018

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Contents

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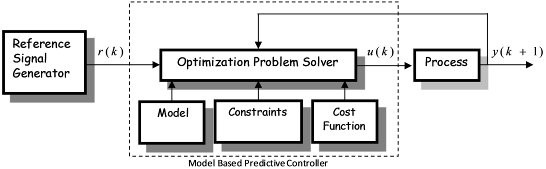
Nonlinear Model Predictive Control



Source of nonlinearity

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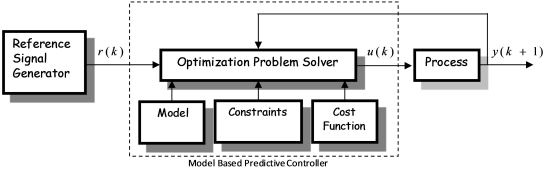

Source of nonlinearity

1. dynamics

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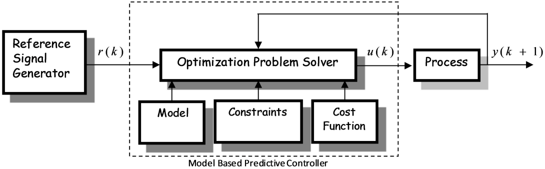

Source of nonlinearity

1. dynamics
2. constraints

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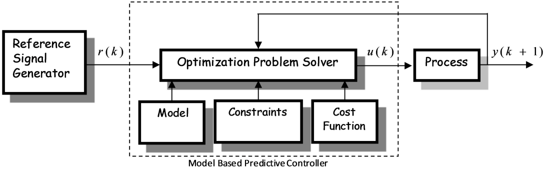

Source of nonlinearity

1. dynamics
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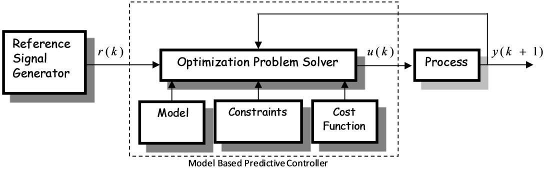

Source of nonlinearity

1. dynamics
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4. **working ranges** (references)

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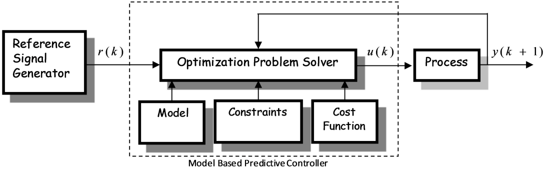
Nonlinear Model Predictive Control



Source of nonlinearity

1. dynamics
2. constraints
3. cost function
4. **working ranges** (references)
5. **effect of feedback**

Motivation

Current state of research

1. The relationship between model and the behavior of the NMPC is not clear

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1. The relationship between model and the behavior of the NMPC is not clear
2. Solving optimization problem on-line in real-time is always a challenge

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Motivation

Current state of research

1. The relationship between model and the behavior of the NMPC is not clear
2. Solving optimization problem on-line in real-time is always a challenge
3. Computational complexity grows with the length of prediction horizon, which is usually desired to be long enough

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An introductory example

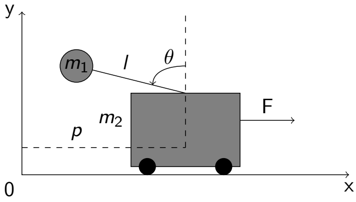

Figure: A schematic illustration of the inverted pendulum control problem.

$$\ddot{p} = \frac{-m_1 l \sin(\theta)\dot{\theta}^2 + m_1 g \cos(\theta)\sin(\theta) + F}{m_2 + m_1 - m_1(\cos(\theta))^2},$$

$$\ddot{\theta} = \frac{F\cos(\theta) - m_1 l \cos(\theta)\sin(\theta)\dot{\theta}^2 + (m_2 + m_1) g \sin(\theta)}{l(m_2 + m_1 - m_1(\cos(\theta))^2)},$$

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An introductory example

The same system, two control tasks, two different results

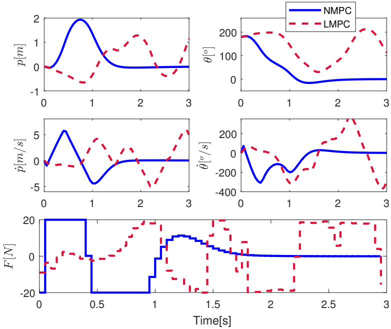

Figure: Left: invert the pendulum;

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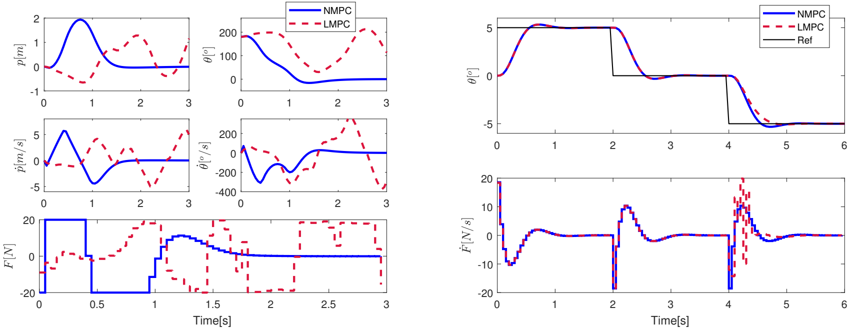

Figure: Left: invert the pendulum; Right: shake the pendulum.

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Research Summary

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3. tailored algorithms have been developed for NMPC with a long prediction horizon
4. an open source NMPC tool is developed for real-time applications

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Background

To understand our model better, we need a metric called

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To understand our model better, we need a metric called

Measure of Nonlinearity

- distance between a nonlinear and a linear system
- gap metric between two linearized systems
- local **curvature measure**

Curvature like measure of nonlinearity

Consider a nonlinear, at least C^2 continuous and differentiable. The Taylor expansion of $\mathcal{Z}$ at a point x_0 with an increment p reads

$$\mathcal{Z}(x_0 + p) = \mathcal{Z}(x_0) + \frac{\partial \mathcal{Z}}{\partial x}(x_0)p + \frac{1}{2!}p^\top \frac{\partial^2 \mathcal{Z}}{\partial x^2} p^\top + \mathcal{O}(\|p^3\|),$$

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Classical CMoN

$$\bar{\kappa} := \frac{\| \frac{\partial^2 \mathcal{Z}}{\partial x^2} p^2 \|}{\| \frac{\partial \mathcal{Z}}{\partial x}(x_0) p \|^2}$$

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The proposed CMoN

$$\kappa = \frac{\|\mathcal{Z}(x_0 + p) - \mathcal{Z}(x_0) - \frac{\partial \mathcal{Z}}{\partial x}(x_0)p\|}{\|\frac{\partial \mathcal{Z}}{\partial x}(x_0)p\|}$$

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1. higher order terms are considered.
2. only the first order derivative is needed
3. local MoN is measured

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A bridge between NMPC and LMPC

For digital processors, NMPC can be seen as LMPC working on systems with time-varying dynamics

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- When NMPC? when LMPC?

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  $\rightarrow$ Sensitivity (linearization) updating logic

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  $\rightarrow$ Parametric Programming

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Solving Structured QP subproblem

$$
\begin{aligned}
\min_{\Delta \mathbf{s}, \Delta \mathbf{u}} \sum_{k=0}^{N-1} & (\frac{1}{2} \begin{bmatrix} \Delta s_k \\ \Delta u_k \end{bmatrix}^\top H_k^i \begin{bmatrix} \Delta s_k \\ \Delta u_k \end{bmatrix} + g_k^{i\top} \begin{bmatrix} \Delta s_k \\ \Delta u_k \end{bmatrix}) \\
& + \frac{1}{2} \Delta s_N^\top H_N^i \Delta s_N + g_N^{i\top} \Delta s_N \\
s.t. \Delta s_0 & = \hat{x}_0 - s_0, \\
\Delta s_k & = A_{k-1}^i \Delta s_{k-1} + B_{k-1}^i \Delta u_{k-1} + d_{k-1}^i, \; k = 1, \ldots, N \\
C_k^i & \begin{bmatrix} \Delta s_k \\ \Delta u_k \end{bmatrix} \leq -c_k^i, \; k = 0, 1, \ldots, N-1, \\
C_N^i & \Delta s_N \leq -c_N^i
\end{aligned}
$$

where $A_k^i = \frac{\partial \Xi}{\partial s}(s_k^i, u_k^i), B_k^i = \frac{\partial \Xi}{\partial u}(s_k^i, u_k^i)$ are called *sensitivities* (linearizations)

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Real-Time Iteration (RTI, Diehl, 2002)

The solution manifold after one QP is a tangential predictor of the exact solution of the Nonlinear Program (NLP) which must be solved on-line

Embed CMoN into RTI

"LMPC"

$$A_k^i = A, B_k^i = B, \forall k = 0, 1, \ldots, N-1,\ i = 0, 1, \ldots$$

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Embed CMoN into RTI

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$$A_k^i = A, B_k^i = B, \forall k = 0, 1, \dots, N-1,\ i = 0, 1, \dots$$

How to mix NMPC and LMPC?

$$A_k^i = A_k^{i-1}, B_k^i = B_k^{i-1}, \text{for some } k$$

where i stands for sampling instants.

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How to mix NMPC and LMPC?

Fixed-Time Updating Logic $\mathcal{Q}_1$: FTB-RTI

Update N_f out of N sensitivities with largest CMoN

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Advantages:

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- Most nonlinear part of the predicted trajectory is linearized

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Disadvantage:

- How to choose N_f ?

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How to mix NMPC and LMPC

Use a threshold η_{pri} to access CMoN to be "linear " or "nonlinear"

Fixed Threshold Updating Logic $\mathcal{Q}_2$: CMoN-RTI

$$A_k^i, B_k^i = \begin{cases} A_k^{i-1}, B_k^{i-1}, \ \kappa_k^i < \eta_{pri}, \text{ (locally linear)}, \\ \frac{\partial \Xi}{\partial s}(s_k^i, u_k^i), \frac{\partial \Xi}{\partial u}(s_k^i, u_k^i), \ \kappa_k^i \geq \eta_{pri}, \text{ (locally nonlinear)}. \end{cases}$$

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Advantages:

- Logic $\mathcal{Q}_2$ can adapt to system operating conditions
- Simple to implement

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Disadvantage

- How to choose η_{pri} ?

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Simulation Results on Inverted Pendulum

Reference changes every 3 seconds

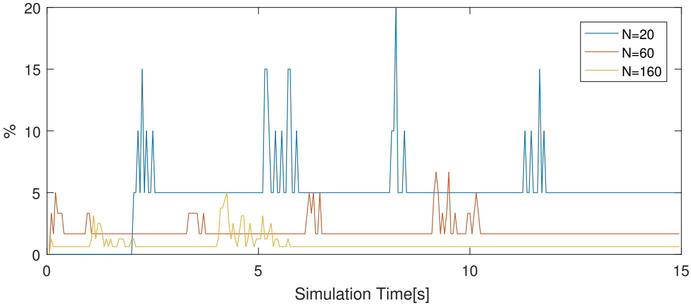

Figure: The percentage of exactly updated sensitivity blocks at each sampling instant during NMPC simulation using CMoN-RTI, with prediction horizon lengths $N = 20, 60, 160$.

Simulation Results

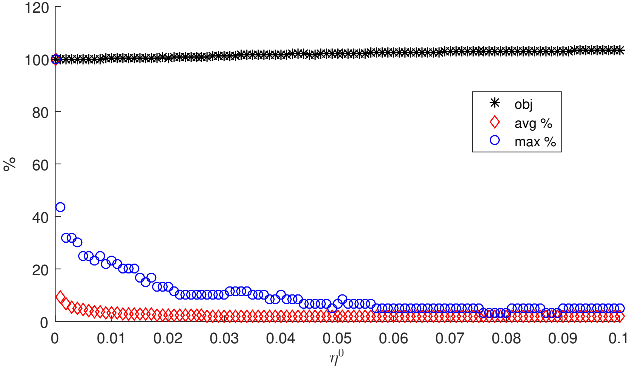

Figure: Effect of the threshold η_{pri} on the value of objective function, the averagely and maximally updated sensitivities when $N = 60$.

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Automatically mix NMPC and LMPC

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Dual CMoN

$$\tilde{\kappa}_k^i := \frac{\|(\nabla \Xi^\top(w_k^i) - \nabla \Xi^\top(w_k^{i-1}))\Delta\lambda_{k+1}^{i-1}\|}{\|\nabla \Xi^\top(w_k^{i-1}))\Delta\lambda_{k+1}^{i-1}\|}.$$

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$$A_k^i, B_k^i = \begin{cases} A_k^{i-1}, B_k^{i-1}, \ \kappa_k^i < \eta_{pri}^i \ \& \ \tilde{\kappa}_k^i < \eta_{dual}^i, \\ \frac{\partial \Xi}{\partial s}(s_k^i, u_k^i), \frac{\partial \Xi}{\partial u}(s_k^i, u_k^i), \ \kappa_k^i \geq \eta_{pri}^i \text{ or } \tilde{\kappa}_k^i \geq \eta_{dual}^i. \end{cases}$$

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Advantages:

- The thresholds depend on sampling instant i, hence capturing latest operating condition

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Question

- How to choose $\eta_{pri}^i, \eta_{dual}^i$?

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Accuracy of the QP Solution

Parametric QP

$$\min_{\Delta \mathbf{w}} \frac{1}{2}\Delta \mathbf{w}^\top H \Delta \mathbf{w} + g^\top \Delta \mathbf{w}$$
$$s.t.\ b(\mathbf{w}) + (B + P)\Delta \mathbf{w} = 0,$$
$$c(\mathbf{w}) + C\Delta \mathbf{w} \le 0,$$

where P is the sensitivity error.

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$$\begin{aligned} \min_{\Delta \mathbf{w}} \frac{1}{2}&\Delta \mathbf{w}^\top H \Delta \mathbf{w} + g^\top \Delta \mathbf{w} \\ s.t.\ b(&\mathbf{w}) + (B + P)\Delta \mathbf{w} = 0, \\ &c(\mathbf{w}) + C\Delta \mathbf{w} \leq 0, \end{aligned}$$

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Theorem

The "distance to optimum" (DtO) in QP solution is a function of the CMoN thresholds η_{pri} *and* η_{dual}

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Theorem

The "distance to optimum" (DtO) in QP solution is a function of the CMoN thresholds η_{pri} *and* η_{dual}

Inversely, if we pre-define a tolerance on DtO, we obtain η_{pri} and η_{dual}

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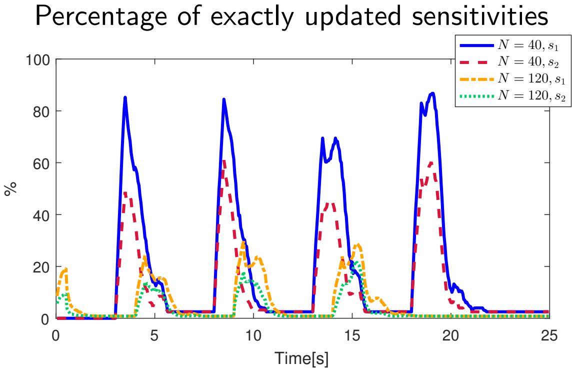
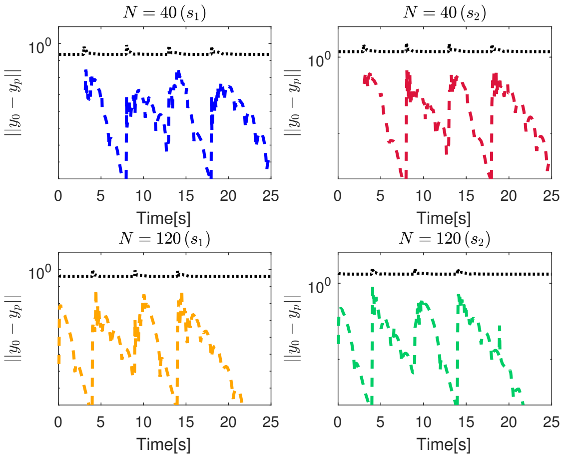
Simulation Results on Inverted Pendulum



Simulation Results on Inverted Pendulum

The accuracy of the QP solution



Level of Sparsity

The QP subproblem in NMPC is structured and is in general sparse

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The QP subproblem in NMPC is structured and is in general sparse

1. Level 0: fully sparse
   $\rightarrow$ all state and control variables are decision variables, Hessian is block diagonal

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The QP subproblem in NMPC is structured and is in general sparse

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2. Level 2: fully dense
   $\rightarrow$ only control variables are decision variables, Hessian is dense and costs $\mathcal{O}(N^2)$

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What in between?

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What in between?

Level 1: partially sparse

$\rightarrow$ part of state and all control variables are decision variables, Hessian is block diagonal

Partial Condensing

In prediction horizon, N points are divided into N_b blocks, each comprising $N_c = N/N_b$ points

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Complexity of partial condensing after partial sensitivity update

costs *at most* $\mathcal{O}(N_b N_c^2)$ Flops, and is usually less in practice

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Numerical Example on Inverted Pendulum

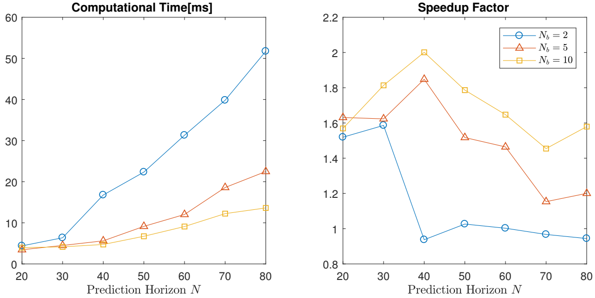
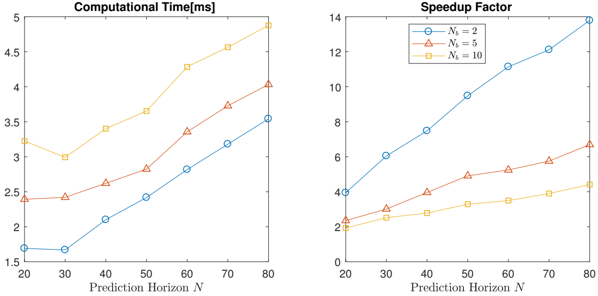


Figure: The average CPU time[ms] of partial condensing with partially updated sensitivities for each sampling instant (left), and the speedup factor w.r.t. the standard partial condensing algorithms(right).

Partial Condensing

A CMoN-free variant

costs $\mathcal{O}(N)$ Flops $\rightarrow$ linear in prediction length



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Solve sparse QP problems

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partial sensitivity update

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Solve sparse QP problems

partial sensitivity update

+

ADMM

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Solve sparse QP problems

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ADMM

For computing the primal step in ADMM

Solve sparse QP problems

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Flop Comparison

$$\frac{\text{proposed}}{\text{state-of-art}} < \frac{N_f}{N}$$

where $N_f \ll N$ is the number of updated sensitivities.

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MATMPC

MATMPC, a MATLAB-based NMPC Package

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Open source available on Github!

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Inverted Pendulum

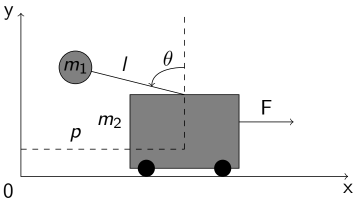
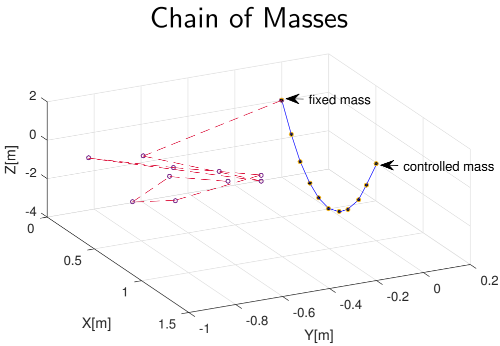


Figure: A schematic illustration of the inverted pendulum control problem.

Toy Examples in MATMPC



Real world applications in MATMPC

Nine DOF Dynamic Driving Simulator

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Real world applications in MATMPC

Active Seat for Simulator

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Real world applications in MATMPC

Hexacopter and Quadcopter

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Thank you!

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