Mapping and Coverage Control in Robotics Networks

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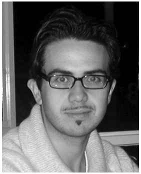

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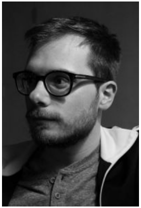

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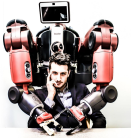
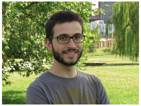
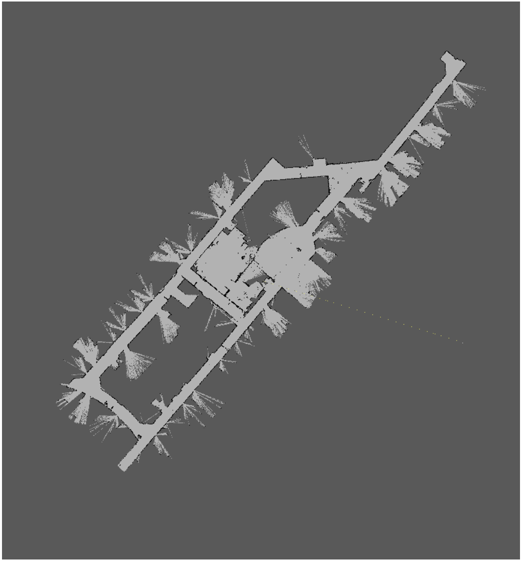


Andrea Antonello



Francesco Branz

Why Localization and Mapping?



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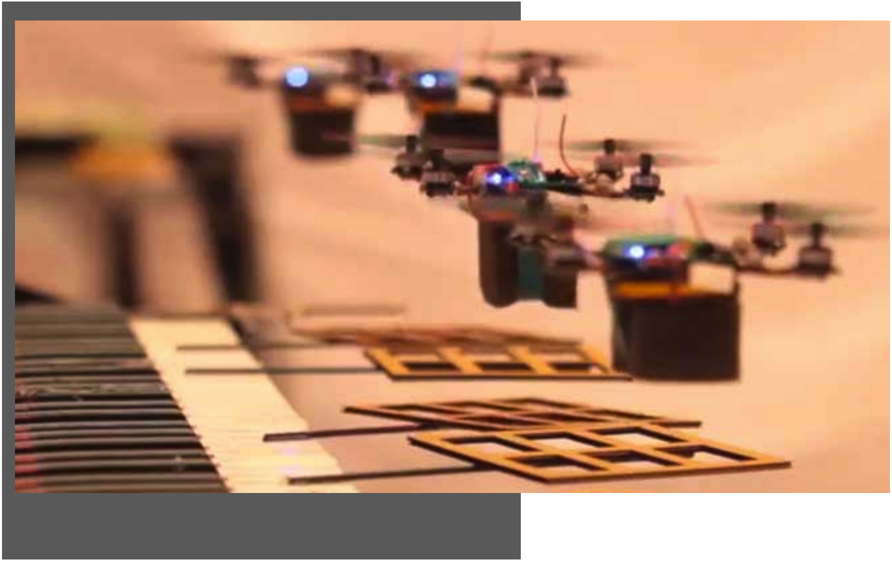

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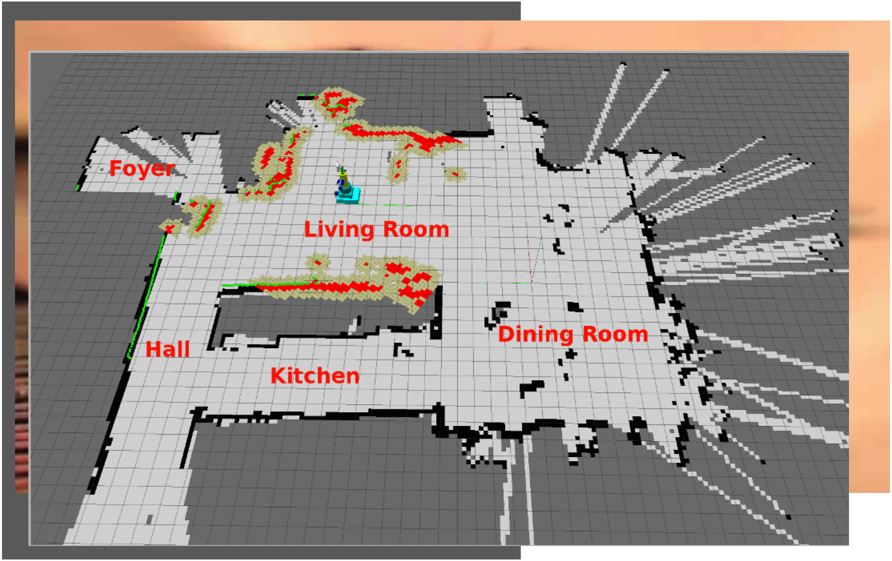

Why Multirobot?

1. Better localization (error $\frac{\sigma}{\sqrt{N}}$),
2. Map building can be N time faster.

But there are some difficulties:

1. Coordination
2. Integration of the information
3. Limited communication

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Andrew Howard. "Multi-robot Simultaneous Localization and Mapping using Particle Filters". In: *RSS 15*

S. Shen, N. Michael, and V. Kumar. "Autonomous multi-floor indoor navigation with a computationally constrained MAV". . In: *ICRA 11*

P. Newman, D. Cole, and K. Ho. "Outdoor SLAM using visual appearance and laser ranging". In: *ICRA 06*

How to localize the robots?

1. Sensors
2. Sensor fusion


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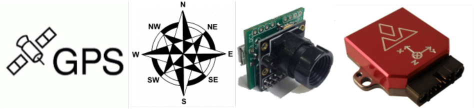

A. Carron et al. "Multi-Robot Localization via GPS and Relative Measurements in the Presence of Asynchronous and Lossy Communication". In: *ECC 16*
M. Todescato et al. "Distributed Localization from Relative Noisy Measurements: a Robust Gradient Based Approach". In: *ECC 15*
A. Carron et al. "An asynchronous consensus-based algorithm for estimation from noisy relative measurements". In: *IEEE TCNS* (2014)
A. Carron et al. "Adaptive consensus-based algorithms for fast estimation from relative measurements". In: *IFAC NecSys 13*

Thesis Contributions

Localization:

1. efficient
2. distributed
3. heterogeneous measurements

Mapping:

1. efficient
2. applied to coverage control
3. time-varying functions

Multi-robots Client-Server Gaussian Estimation and Coverage Control with Lossy Communications

Literature

M. Schwager, D. Rus, and J-J. Slotine. "Decentralized, adaptive coverage control for networked robots". In: *IJRR* (2009)

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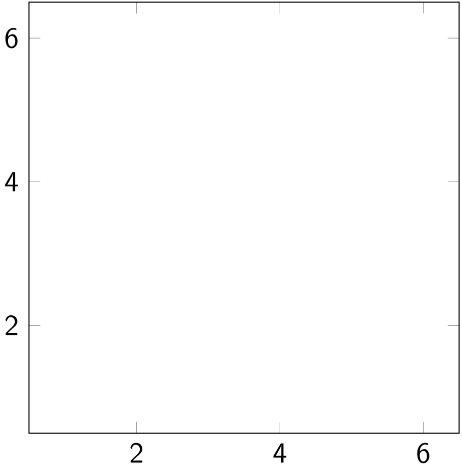
J. Cortés. "Distributed Kriged Kalman filter for spatial estimation". In: *IEEE Transactions on Automatic Control* (2009)

Contributions

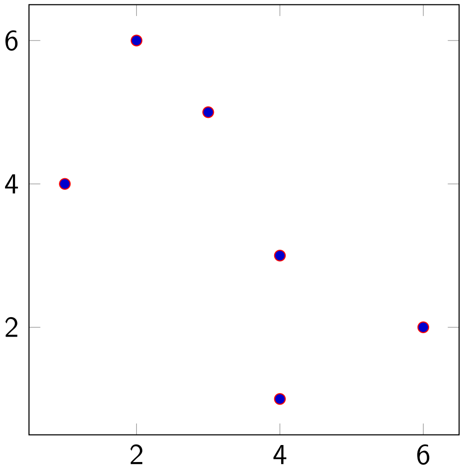
1. Estimation from noisy measurements
2. Bounds on the estimation error
3. Robustness

Voronoi Partitions and Coverage



- Environment $\mathcal{X}$

Voronoi Partitions and Coverage



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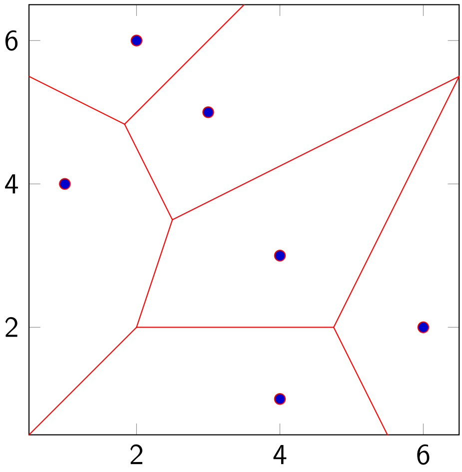
- Agents $x_1, \ldots, x_N$

Voronoi Partitions and Coverage



- Environment $\mathcal{X}$
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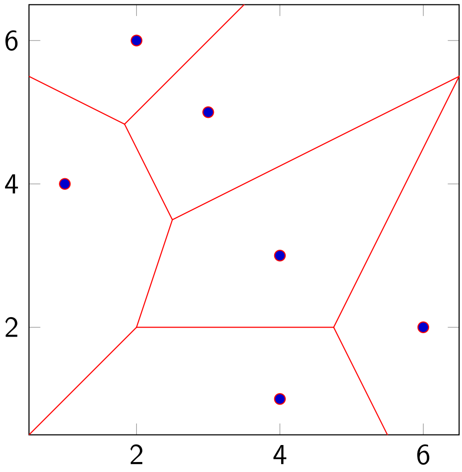
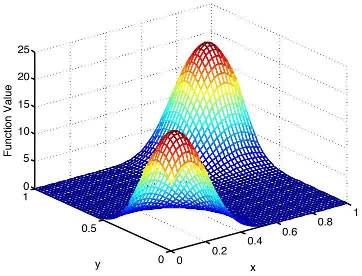
- Voronoi Partitions $\mathcal{P} = \mathcal{W}(x_1, \ldots, x_N)$

Voronoi Partitions and Coverage



- Environment $\mathcal{X}$
- Agents $x_1, \ldots, x_N$
- Voronoi Partitions $\mathcal{P} = \mathcal{W}(x_1, \ldots, x_N)$
- Density Function μ and Centroids $c(\mathcal{P}, \mu)$



Coverage Goal and the Lloyd Algorithm

Goal

Dispatch the N robots to **optimally cover** the environment $\mathcal{X}$, namely we want to have many robots where $\mu(x)$ is large and few where it is small.

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$$\min_{\mathcal{P}} H(\mathcal{P}, \mu) = \min_{\mathcal{P}} \sum_{i=1}^{N} \int_{\mathcal{P}_i} ||q - c_i(\mathcal{P}_i)||^2 \mu(q) dq$$

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Solution: Classical Lloyd algorithm

1. compute the centroids of the current partition, e.g. $c(\mathcal{P})$
2. update $\mathcal{P}$ to the partition $\mathcal{W}(c(\mathcal{P}))$

Or more briefly $\mathcal{P}^L(k+1) = \mathcal{W}\left(c(\mathcal{P}^L(k))\right)$.

- **Unknown** function $\mu : \mathcal{X} \subset \mathbb{R}^2 \mapsto R_+$
- μ is a zero-mean Gaussian random field with covariance $K : \mathcal{X} \times \mathcal{X} \mapsto \mathbb{R}_+$
- Radial Mercer Kernels
- $K(x, x) = \lambda$

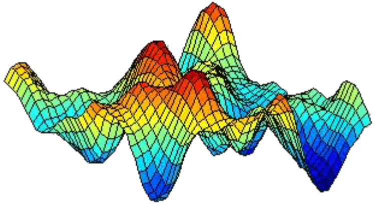

Figure : Gaussian Process

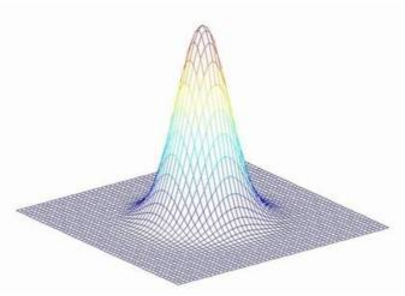

Figure : Gaussian Kernel

Minimum Variance Estimate

The set $I_k = \{x_i, y_i\}_{i=0}^{m_k}$ represents the complete information set available at the BS at iteration k and $m_k = |I_k|$ is its cardinality.

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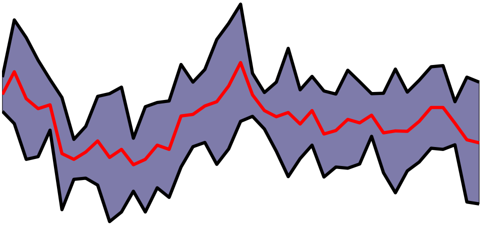
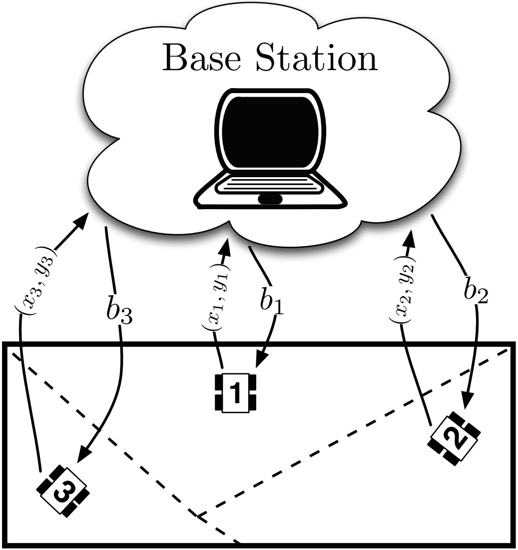
An index of the **accuracy** of the estimate is given by the posterior variance

$$V_k(x) = \text{Var}\left[\mu(x)|I_k\right]$$



Problem Formulation



Exploration and Exploitation Dilemma

Goal

The ultimate goal is to position the N robots in the centroids of a good partition that minimizes $H(\mathcal{P}, \mu)$. To do so we need to:

1. have a good estimate $\hat{\mu}$ of the sensory function $\rightarrow$ exploration
2. minimize the cost function $H(\mathcal{P}, \mu) \rightarrow$ exploitation

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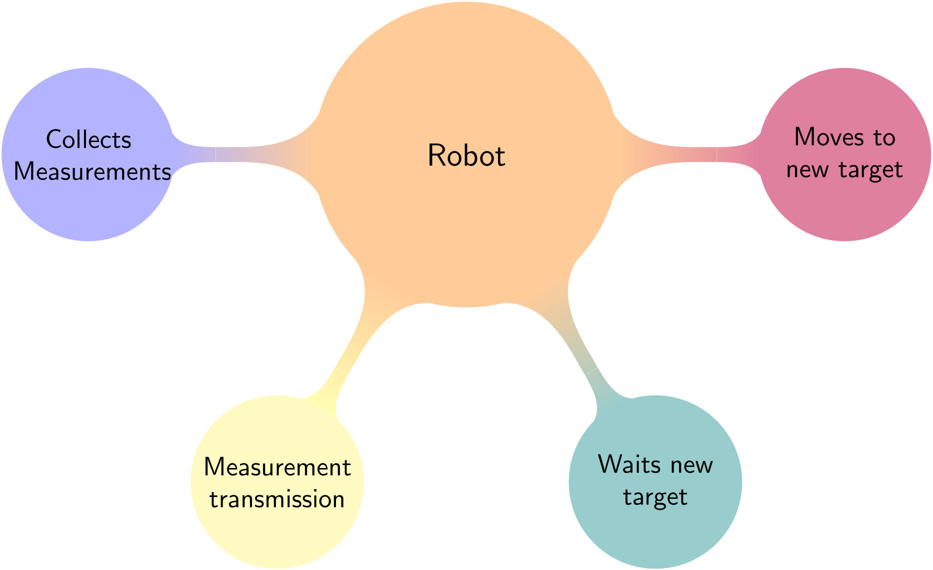
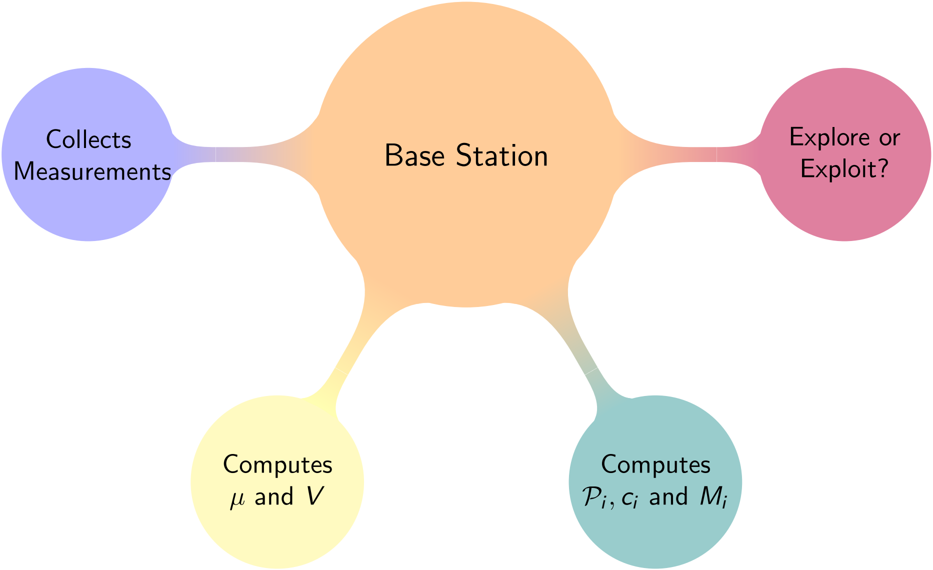
Strategy

1. initially promote exploration
2. when the estimate is more accurate transit to exploitation
3. random approach based on the maximum of the posterior variance

rEC - Robots


Robot
Collects Measurements
Moves to new target
Measurement transmission
Waits new target

rEC - Base Station



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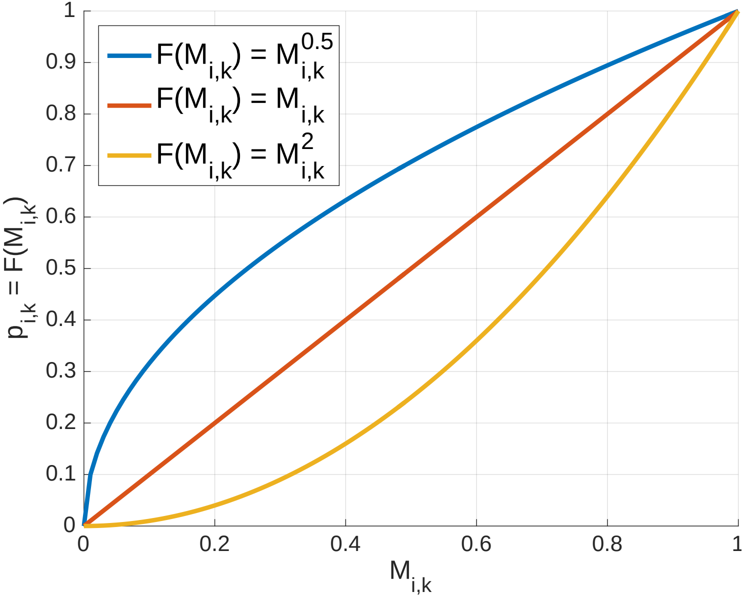

Convergence Analysis - Sensory Function

Proposition 1 - Sensory Function Convergence

If:

1. $F(M) : [0, 1] \to [0, 1]$ continuous and monotonically increasing,
2. $F(M) > 0$ for $M > 0$,
3. $\mathbb{P}[\beta_{i,k} = 1] = \overline{\beta} > 0$,
4. $\mathbb{P}[\gamma_{i,k} = 1] = \overline{\gamma} > 0$.

Then

$$\widehat{\mu}_k \xrightarrow{P} \mu.$$

Online Gaussian Estimation

What is the most expensive operation?

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What is the most expensive operation?

$$(\bar{K}_{k+1} + \sigma^2 I)^{-1} = \left(\begin{bmatrix} \bar{K}_k & \bar{K}_{k+1,12} \\ \bar{K}_{k+1,12}^T & \bar{K}_{k+1,22} \end{bmatrix} + \sigma^2 I \right)^{-1}$$

How much is its computational complexity?

Online Gaussian Estimation

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How much is its computational complexity?

Naive: $\left(\bar{K}_k + \sigma^2 I\right)^{-1} \rightarrow O(k^3)$

Schur: $\left(\bar{K}_{k+1,22} - \bar{K}_{k+1,12}^T * (\bar{K}_k + \sigma^2 I)^{-1} * \bar{K}_{k+1,12}\right)^{-1} \rightarrow O(k^2)$

Grid Based Approximation

Consider

$$\mathcal{X}_{\mathrm{grid}} := \{x_{\mathrm{grid,1}}, \ldots, x_{\mathrm{grid,m}}\} \subset \mathcal{X}.$$

Given the scalar $\Delta > 0$, $\mathcal{X}_{\mathrm{grid}}$ forms a *sampled space* of resolution Δ if

$$\min_{i=1,\ldots,m} \|x_{\mathrm{grid,i}} - x\| \leq \Delta, \quad \forall x \in \mathcal{X}.$$

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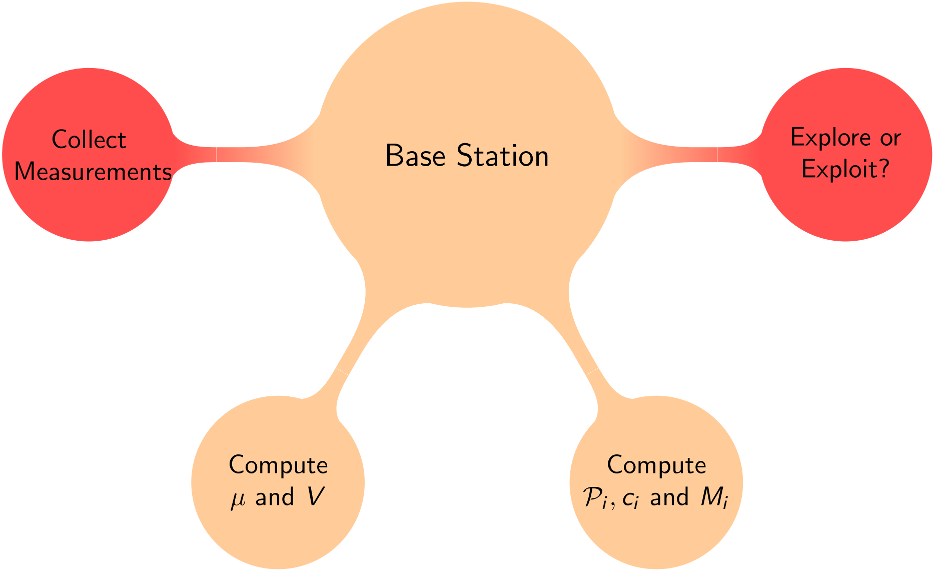
We define the projector operator

$$\mathcal{X} \longmapsto \mathcal{X}_{\mathrm{grid}} \ : \ x \longmapsto \mathrm{proj}(x) = \arg \min_{a \in \mathcal{X}_{\mathrm{grid}}} \|x - a\| \, .$$

rEC-grid - Base Station



Convergence Analysis - Sensory Function

Proposition 2 - Posterior Variance

If the assumptions of Proposition 3 holds then:

$$\lim_{k \to \infty} V_k(x) = \lambda - k_{\mathrm{grid}}(x) K_{\mathrm{grid}}^{-1} k_{\mathrm{grid}}(x)^\top .$$

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The following simple Δ dependent bound holds

$$\lim_{k \to \infty} V_k(x) \leq \lambda - \frac{K^2(\Delta)}{\lambda}.$$

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If K is the Gaussian kernel with $K(a,b) = \lambda e^{-\frac{\|a-b\|^2}{\zeta^2}}$, for small Δ we have

$$\lim_{k \to \infty} V_k(x) \leq \lambda - \frac{K^2(\Delta)}{\lambda} \approx \frac{\lambda \Delta^2}{\zeta^2}.$$

Simulations Setup

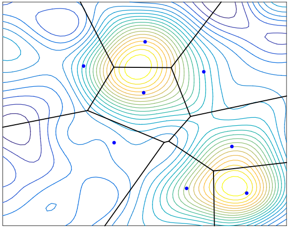
- Team of $N = 8$ robots
- Domain $\mathcal{X} = [0,1] \times [0,1]$
- Gaussian kernel $K(x, x') = e^{-\frac{\|x-x'\|^2}{0.002}}$
- Exploration-Exploitation trade-off: $F_\alpha(M) = M^\alpha$
- Sensory function

$$\mu(x) = 5\left(e^{\frac{-\|x-\mu_1\|^2}{0.04}} + e^{\frac{\|x-\mu_2\|^2}{0.04}}\right)$$

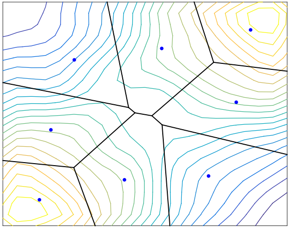
where

$$\mu_1 = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix}$$

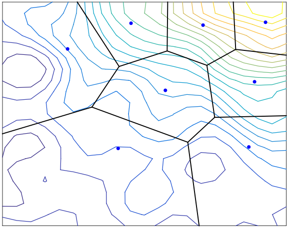
Coverage



$$5\left(e^{\frac{-\|x-\mu_1\|^2}{0.04}} + e^{\frac{\|x-\mu_2\|^2}{0.04}}\right)$$



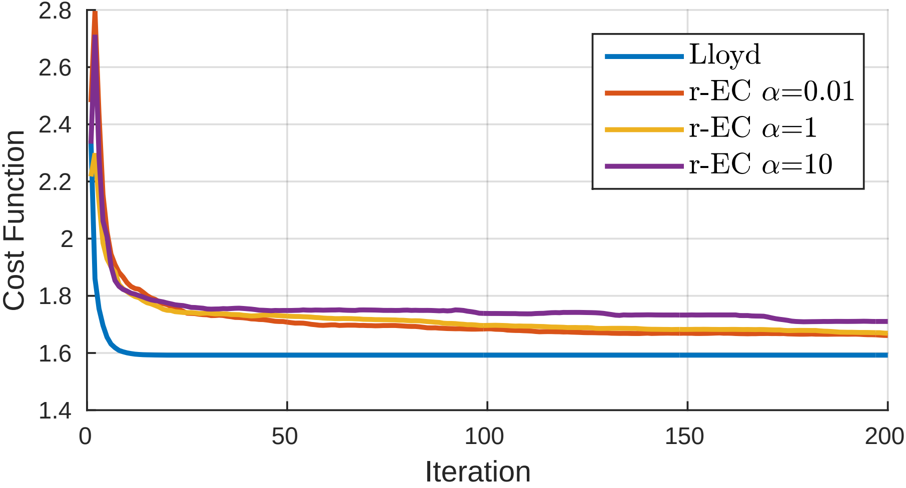
$$10\cos(\pi x_1)\cos(\pi x_2)+10$$



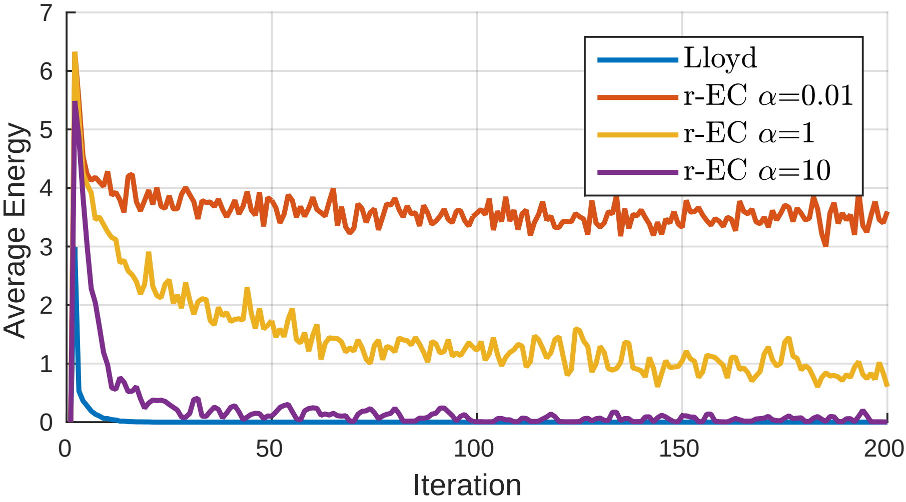
$$10\sin\left(x_1 x_2^2\right)$$

Voronoi partitions computed using the r-EC Algorithm (black lines) for different sensory function $\mu(x)$. Blue dots indicate the locations of the centroids obtained with the r-EC algorithm.

Cost Function



Average Energy



rEC vs rEC-grid

p^2	$\frac{\Delta}{\zeta}$	Max. Posterior (after 400 iterations)	Min. Max. Posterior Achievable	Exe. Time [sec.]
9	1.65	0.9836	0.9836	2.3
16	1.25	0.8418	0.8418	2.4
25	1	0.5769	0.5766	3.9
36	0.83	0.3489	0.3481	4.7
r-EC	–	0.1988	0	865.4

Comparison between the original r-EC algorithm and the grid based approximation for different total number of points p^2. The table reports the steady state value after 400 iterations and the execution times obtained using the grid based approximation and classic algorithms.

rEC in action!

Conclusions and on-going works

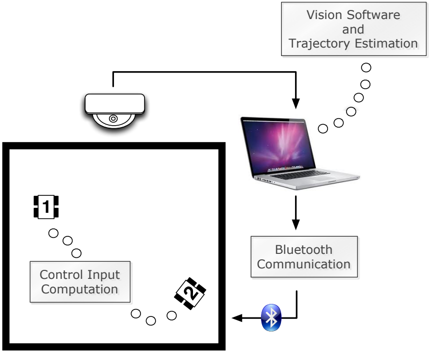
The r-EC/r-EC-grid are shown to be:

1. capable to converge to the optimal estimate of μ,
2. robust to packet losses,
3. efficient.

What else can be done?

1. consider time varying μ,
2. consider localization errors.

Competitive - Cooperative RHC game



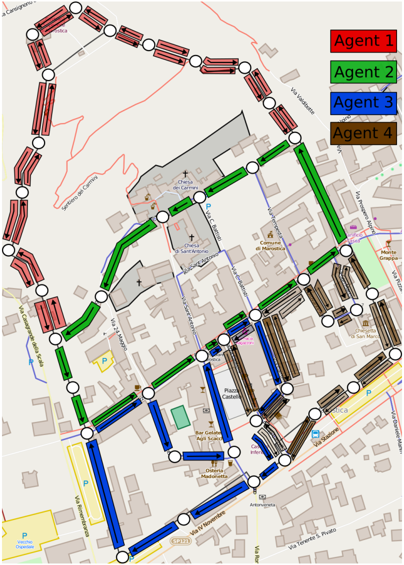
GOAL: minimize a cost function which depends on your state, your input and the opponent input.

RESULTS: closed form solution given the control parameters and stability analysis.

A. Carron and E. Franco. "Receding Horizon Control of a two-agent system with competitive objectives". In: *ACC 13*

A. Carron and E. Franco. "Analytical Solution of a Two Agent Receding Horizon Control Problem with Auto Regressive State Predictions". In: *Automatica [submitted]*

Hitting Time of Multiple Random Walker



R. Patel, A. Carron, and F. Bullo. "The Hitting Time of Multiple Random Walks". In: *SIAM Journal on Matrix Analysis and Applications [submitted]*

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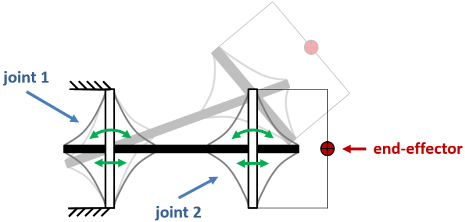
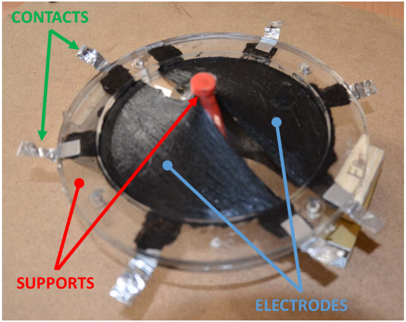
Robotics for Space Applications





A. Antonello et al. "A Novel Approach to the Simulation of On-Orbit Rendezvous and Docking Maneuvers in a Laboratory Environment Through the Aid of an Anthropomorphic Robotic Arm". In: *MetroAeroSpace 14*

F. Branz et al. "Kinematics and control of redundant robotic arm based on Dielectric Elastomer Actuators". In: *SPIE Smart Structure*

F. Branz et al. "Dielectric Elastomer space manipulator: design and testing". In: *IAC 15*

Thank you

Thank you for your attention!