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Methods and Applications in Networked Control and Feedback Control Design for Quantum Systems

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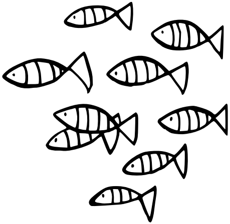
Definition of Networked Control Systems

Networked control systems comprise sensors, actuators and controllers, the operation of which is coordinated via a communication network.

These systems are typically spatially distributed, may operate in an asynchronous manner, but have their operation coordinated to achieve desired overall objectives.

The analysis and the design of such systems present multiple concurrent issues that cannot be addressed separately.



FeedNetback
co-design for networked control systems

Issues in Networked Control Systems

- Multi-agent architecture - NCS are made of a large number of agents, each one provided with computing, sensing and actuating capabilities.
- Scalability - the number of agents can be very large and unknown; the computational complexity of the algorithms, their memory and communication requirement, and their performances, must scale well with the size of the systems.
- Distributed approach - the large amount of information in a NCS is not supposed to be collected by any central controller, but handled locally by the agents to reduce communication and complexity and to improve robustness, privacy, and security of data.

Issues in Networked Control Systems (cont.)

- Partial knowledge of the system - the entire system state, structure, and parameters, is unknown to every agent, while each of them has access to (or store) part of it.
- Communication constraints - issues affecting communication among agents (quantization, data losses, data rate limitations, communication graphs) cannot be separated from the control design because they are severe and because they are correlated with the system state.
- Interaction with the underlying physical system - the agents of a NCS are usually deployed on a system governed by its own dynamics, that must be considered in the design of an effective control law, which usually consists in a sequence of measurement, communication/computation, and actuation steps.

Issues in Networked Control Systems (cont.)

- Robustness to system changes - agents are subject to failures, insertion / removal, reconfiguration; the behavior of the NCS cannot be endangered by these events, and its performances must be only partially affected by them.

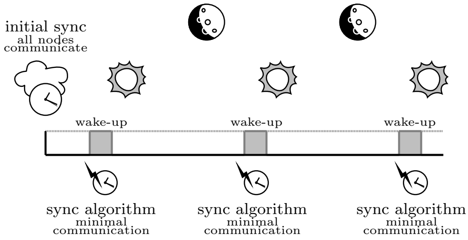
NCS analysis and design methodologies

The simultaneous presence of these issues in a NCS requires the development of specific methodologies for the analysis of such systems and the design of appropriate algorithms, control laws, and architectures.
By examining three specific applications, in which some of these issues are present, a series of methods have been studied, developed, and presented in the thesis.

Large scale distributed clock synchronization

Consider a large (unknown) number of electronic devices.

- Each device has its own clock, implemented via some low-cost low-performance circuitry
- devices can communicate according to a given communication graph, for example depending on how they are deployed in a large environment
- their clocks must be sufficiently synchronized, to allow simultaneous duty cycling to save battery life.



Problem formulation

We are interested in a synchronization algorithm that allows the analysis of its convergence rate even in noisy/faulty scenarios. We proposed a PI-like controller with gossip communication.

Bolognani, S., Carli, R., and Zampieri S. (2009).
A PI consensus controller with gossip communication for clock synchronization in wireless sensor networks.
NECSYS 09, Venice, Italy.

Clock free dynamics: $x_h(t+1) = x_h(t) + w_h(t) + d_h$

Synchronizing pair:
$$x_i(t+1) = \frac{1}{2}(x_i(t) + x_j(t)) + w_i(t) + d_i$$
$$x_j(t+1) = \frac{1}{2}(x_i(t) + x_j(t)) + w_j(t) + d_j$$

Auxiliary variables:
$$w_i(t+1) = \frac{\alpha}{2}(x_j(t) - x_i(t)) + w_i(t)$$
$$w_j(t+1) = \frac{\alpha}{2}(x_i(t) - x_j(t)) + w_j(t)$$

Mean square synchronization error analysis

The resulting time-varying discrete-time system has the form

$$\begin{bmatrix} x(t+1) \\ w(t+1) \end{bmatrix} = F_{e(t)} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix} + \begin{bmatrix} d \\ 0 \end{bmatrix},$$

where $F_{e(t)}$ depends on the edge e of the communication graph which is activated.

By introducing $\bar{x}(t) = \Omega x(t)$ and $\bar{v}(t) = \Omega(w(t) + d)$, we can study the covariance of the synchronization error:

$$P(t) = \mathbb{E}\left[\begin{bmatrix} \bar{x}(t) \\ \bar{v}(t) \end{bmatrix} \begin{bmatrix} \bar{x}(t) & \bar{v}(t) \end{bmatrix} \right],$$

which evolves according to $P(t+1) = \mathbb{E}\left[F_{e(t)} P(t) F_{e(t)}^T \right]$.

Convergence in variance

The convergence of the N^2-dimensional linear system $P(t+1) = \mathbb{E}\left[F_{e(t)} P(t) F_{e(t)}^T\right]$ depends on the set of edges $\mathcal{E}$ and on their probability of being chosen p_e.

For the particular case of a complete graph with uniform probability, the convergence in variance can be characterized as a condition on the only design parameter α.

Convergence result

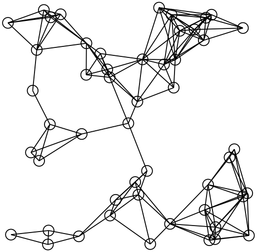
The synchronization error converges in variance if and only if

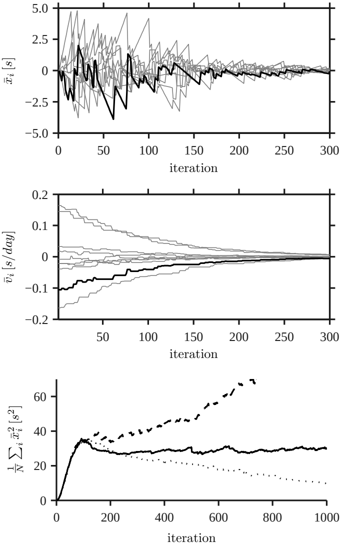
$$\alpha < \frac{3}{2} - N + \frac{1}{2}\sqrt{4N^2 - 12N + 17},$$

which is verified when $\alpha \leq 1/(N-1)$.

Simulations

Simulation confirm that the behavior of the system is qualitatively different according to the value of α.
Numerical experiments show that very similar behaviors are also observed in random geometric graphs.

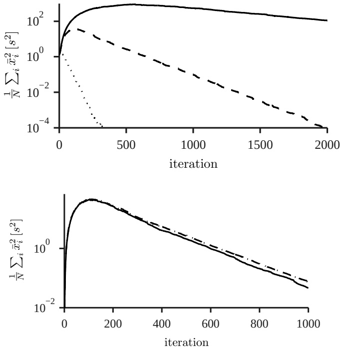




Scalability issues

The algorithm's scalability is therefore an issue:

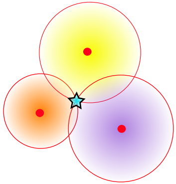
- as the network size grows, smaller α's have to be chosen to ensure stability, resulting in slower convergence;
- by allowing multiple simultaneous updates, the system behavior becomes independent from N, and therefore robust in the number of devices.



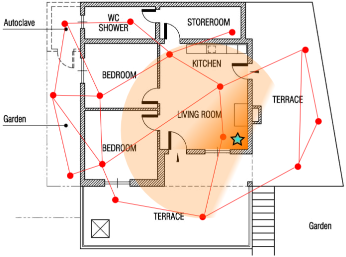
Average square clock errors with a stabilizing α for different network sizes. Original algorithm (above) and multiple symmetric gossip (below).

Channel identification in wireless networks

One of the methods for localization in WSNs is based on triangulation, based on the distance of the target from some anchor nodes deployed in the environment.



A convenient way to estimate the relative distance between wireless nodes relies on the received signal strength, which is a decreasing function of the nodes' separation.



$$P_{ij}^{rx} = P_j^{tx} + \beta - 10\gamma \log_{10} d_{ij} + w_{ij} + o_i$$

Parametric identification

Wireless channel and sensor parameters need to be estimated to be able to infer the nodes' distance from the RSS.
They consist in

- two channel parameters β and γ, which depend on the environment and change slowly during the day
- N measurement offsets o_i, which depends on the specific circuitry employed in the RSS sensor.

A distributed algorithm based on consensus for the identification of these parameters has been proposed.

Bolognani, S., Del Favero, S., Schenato, L., and Varagnolo D. (2008).
Distributed sensor calibration and least-square parameter identification in WSNs using consensus algorithms.
Allerton 08, Monticello, Illinois, USA.

Bolognani, S., Del Favero, S., Schenato, L., and Varagnolo D. (2010).
Consensus-based distributed sensor calibration and least-square parameter estimation in WSNs.
International Journal of Robust and Nonlinear Control, vol. 20, no. 2.

Offset compensation algorithm

Average consensus algorithms

For the following results, it suffices to define average consensus algorithms as distributed, scalable, leader-less, methods that allow nodes to agree on the average of their initial conditions.

Compensating measurement offset consists in estimating $\hat{o}_i$ such that $o_i - \hat{o}_i = \alpha$ for every node i.
By tackling this agreement problem via a linear average consensus algorithm we have

$$(o_i - \hat{o}_i)^+ = o_i - \hat{o}_i + \sum_{j \in \mathcal{N}(i)} p_{ij}((o_j - \hat{o}_j) - (o_i - \hat{o}_i)),$$

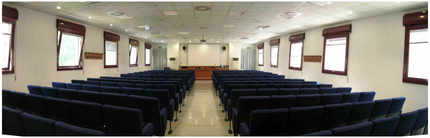
which under some mild conditions is guaranteed to satisfy $o_i - \hat{o}_i(t) \to \frac{1}{N} \sum_j (o_j - \hat{o}_j(0))$, for every i, as $t \to \infty$.

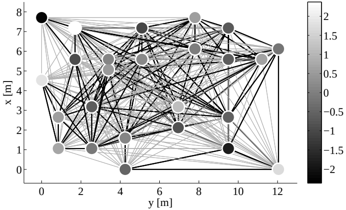
Offset compensation algorithm

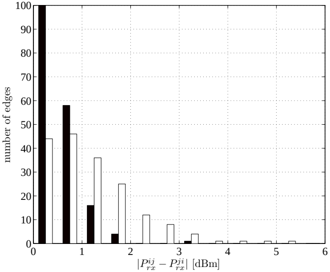
The consensus update law can be rewritten to obtain an update law for $\hat{o}_i$

$$\hat{o}_i^+ = \hat{o}_i - \sum_{j \in \mathcal{N}(i)} p_{ij}(P_{ji}^{rx} - P_{ij}^{rx} + \hat{o}_i - \hat{o}_j)$$

The model and the compensation algorithm have been validated on an experimental testbed, allowing the use of RSS for localization.







Least square identification

Once offsets are compensated, and fast fading components are averaged out, we have for every edge (i, j)

$$\bar{P}_{ij}^{\mathrm{rx}} = [1 \quad -10\log_{10} d_{ij}] \begin{bmatrix} \beta \\ \gamma \end{bmatrix} + w_{ij},$$

or equivalently $b_{ij} = a_{ij}^T \theta + w_{ij}$.

System-wide, this corresponds to the problem of solving the overdetermined system

$$b = A\theta + w$$

of $|\mathcal{E}|$ equations in the two variables $\theta = [\beta \ \gamma]^T$.

Least square solution

$$\hat{\theta} = \arg\min_{\theta} \|A\theta - b\| = (AA^T)^{-1}A^T b$$

Least square identification

Consider a set of virtual nodes (one for each edge (i, j) in $\mathcal{E}$). By inizializing a consensus algorithm with

$$x_{ij}(0) = a_{ij} a_{ij}^T \quad y_{ij}(0) = a_{ij} b_{ij},$$

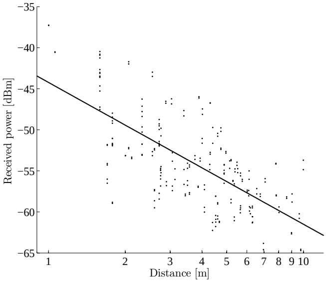
we have agreement on

$$x_{ij}(t) \to \frac{1}{|\mathcal{E}|} a_{ij} a_{ij}^T \quad y_{ij}(t) \to= \frac{1}{|\mathcal{E}|} a_{ij} b_{ij}$$

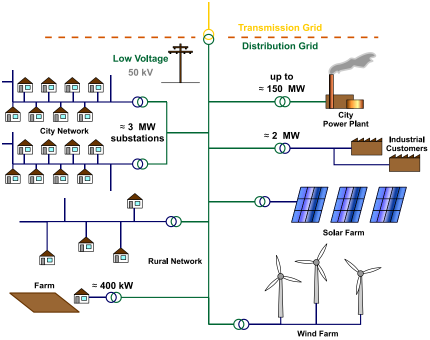
from which local estimates of the channel parameters can be obtained

$$\hat{\theta}_{ij}(t) = (x_{ij}(t))^{-1} y_{ij}(t)$$

$$\to \left(\sum_{\mathcal{E}} a_{ij} a_{ij}^T \right)^{-1} \left(\sum_{\mathcal{E}} a_{ij} b_{ij} \right) = \hat{\theta}$$



Distributed control of smart micro grids



Smart microgrid

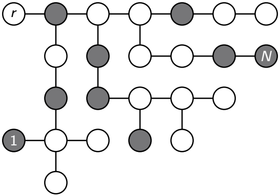
A portion of the electrical power distribution network that connects to the transmission grid in one point and that is managed autonomously from the rest of the network.

Reactive power compensation:

- users in the microgrid require reactive power
- reactive power can be produced by the electronic interfaces of microgenerators in the grid
- flows of reactive power should be minimized, as larger flows correspond to quadratically larger power losses.

Formulation of the optimization problem

We want to command compensators to minimize power losses while matching the total demand



$$\min \sum_e k_e f_e^2 \text{ subject to } \sum_i q_i = c.$$

This convex quadratic problem has a closed form solution. Computing it requires, however,

- complete knowledge of the system structure and state
- data collection, coordination and communication among all agents
- management of a large number of agents, subject to partial availability, disappearance, new insertion.

Gradient estimate via distributed sensing

Because of the physics of the system, nodes can estimate element-wise the gradient of the cost function by measuring node voltages.

Gradient descent methods

$$\mathbf{q}_{t+1} = \mathbf{q}_t - \boldsymbol{\Gamma}_t \mathbf{g}(\mathbf{q}_t)$$

The resulting algorithm consists of iterative repetitions of

- estimation of the gradient via measurements on the system
- computation of the next update step
- actuation of the system into the new state.

Bolognani, S., and Zampieri S. (2010).
Distributed quasi-Newton method and its application to the optimal reactive power flow problem.
NECSYS 2010, Annecy, France.

Gossip-like algorithm

Another approach consists in decomposing the optimization problem into subproblems that can be solved asynchronously.

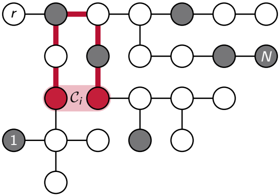
Bolognani, S., and Zampieri S. (2011).
A gossip-like distributed optimization algorithm for reactive power flow control.
IFAC World Congress 2011, Milano, Italy.

Problem decomposition

Nodes are clustered into sets $\mathcal{C}_i$, with associated subproblems

$$\min_{\Delta \mathbf{q}} J(\mathbf{q} + \Delta \mathbf{q}) \quad \text{subject to } \mathbf{1}^T \Delta \mathbf{q} = 0,\ \Delta q_j = 0\ \forall j \notin \mathcal{C}_i.$$

The i-th subproblem can be solved via local knowledge of the network, local measurements, and coordination among the nodes in $\mathcal{C}_i$.



Gossip-like algorithm

Convergence result

Under some mild conditions on the randomized sequence of subproblem execution, the algorithms converges if and only if the sets $\mathcal{C}_i$ are the edges of a connected hyper-graph.

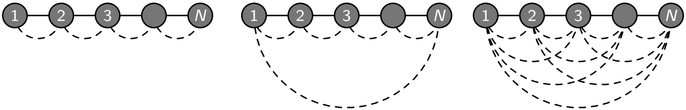
We are interested in the rate of convergence, corresponding to the slowest dynamics of the observable and reachable part of the linear system

$$\Delta(t+1) = \mathbb{E}[F^T \Delta(t) F]$$

where F belongs to a family of projectors describing the solution of the different subproblems.
An easier-to-compute bound on the convergence rate has also been derived.

Gossip-like algorithm

The convergence rate has been evaluated (analytically and numerically) for different communication topologies.



Fastest achievable convergence

The optimal strategy consists in choosing a clustering hyper-graph which coincides with the graph describing the physical interconnection of the electric network.

This result is interesting in the fact that it contrasts with the phenomena observed in gossip consensus algorithms, where long-distance communications improve the rate of convergence.

Conclusions

By considering these applications of networked control systems, different issues have been addressed, resulting in a series of methodologies that can be applied in much broader scenarios.

application	characteristic issues
Clock Synchronization	scalability, control of unstable dynamics, large scale synchronization.
Wireless Sensor Networks	distributed sensing and estimation, data fusion, parallel computation.
Smart Micro Grids	distributed optimization, algorithm performance analysis, interaction with underlying system, distributed identification

Other contributions

Another research topic has been covered in the thesis:
feedback control design for quantum systems.

Attractivity and invariance of a quantum subspace can be achieved by designing an appropriate feedback, discrete-time, control law.
Methodologies for the analysis of the stability of a quantum subsystem have been derived, together with an algorithm to check feasibility of the control problem and to return a stabilizing unitary control.

Bolognani, S., and Ticozzi, F. (2010).
Pure state stabilization with discrete-time quantum feedback.
ISCCSP 2010, Limassol, Cyprus.

Ticozzi, F., and Bolognani, S. (2010).
On a canonical QR decomposition and feedback control of discrete-time quantum dynamics.
MTNS 2010, Budapest, Hungary.

Bolognani, S., and Ticozzi, F. (2010).
Engineering Stable Discrete-Time Quantum Dynamics via a Canonical QR Decomposition.
IEEE Transactions on Automatic Control, vol. 55, no. 12.

Thanks!

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