Lara Briñón Arranz

Cooperative Control Design of Multi-Agent Systems: Application to Underwater Missions

NeCS Team, INRIA Rhône-Alpes & GIPSA-lab

Padova, 24th July 2012

Context

NeCS Team
- GIPSA-lab / INRIA
Grenoble, France

PhD advisors
- Carlos Canudas de Wit
- Alexandre Seuret

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FeedNetBack Project
- *Networked Control Systems*
- Partners: Università di Padova, Universidad de Sevilla, KTH, ETH, INRIA Grenoble

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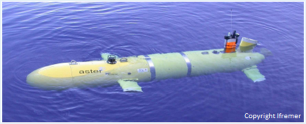
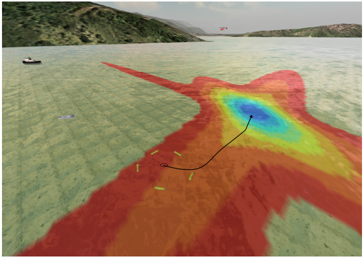
Case Study: Autonomous Underwater Vehicles (AUVs)

Source-seeking task
To locate and follow the source of the scalar field of interest



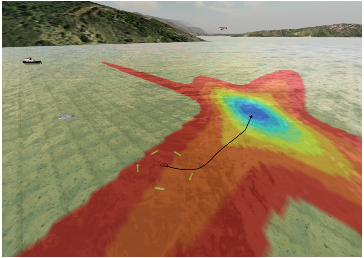
Case study



Final Objective

To design **collaborative control strategies** to steer a **fleet of AUVs** (Autonomous Underwater Vehicles) toward the **source localization** of a scalar field

Case study



Final Objective

To design **collaborative control strategies** to steer a **fleet of AUVs** (Autonomous Underwater Vehicles) toward the **source localization** of a scalar field

Proposed solution: **Mobile Sensor Networks**

- Fleet of AUVs ⇒ Formation control of multi-agent systems
- Exchange of information ⇒ Collaborative Control
- Underwater scenario ⇒ Communication constraints

Outline

2. Problem Statement: Control strategy

- Formation control of multi-agent systems: circular formation and other formations
- Collaborative Control: uniform distribution along the formation
- Communication constraints: Distributed algorithm for source-seeking

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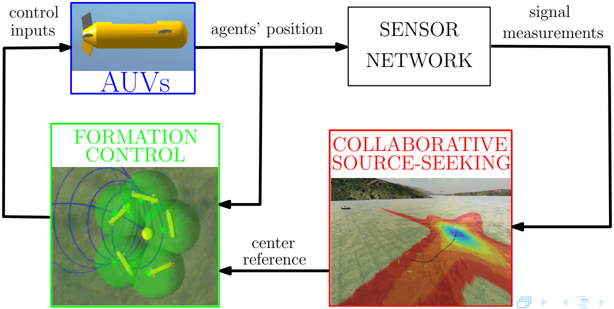
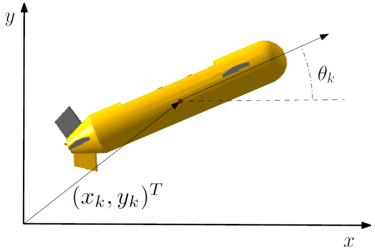


Model of the AUVs

Unicycle model

Fleet of N agents, in which each agent $k = 1, ..., N$ has the following constrained dynamics:

$$\dot{x}_k = v_k \cos\theta_k$$
$$\dot{y}_k = v_k \sin\theta_k$$
$$\dot{\theta}_k = u_k$$

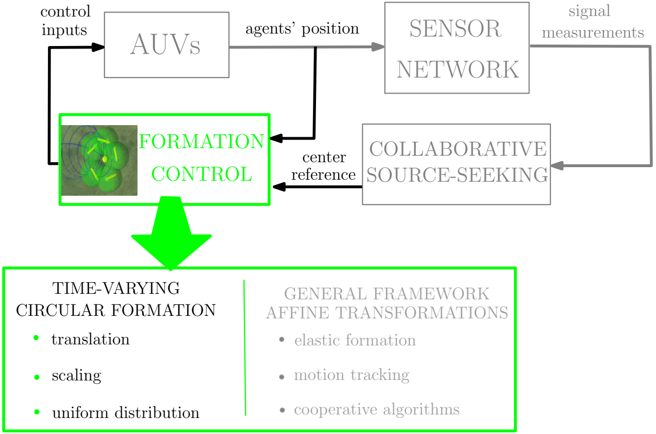


$\mathbf{r}_k = (x_k, y_k)^T$ is the position vector of agent k

θ_k is its heading angle

v_k, u_k are the control inputs

3. Time-varying Circular Formation Control



Previous works: Collective Circular Motion

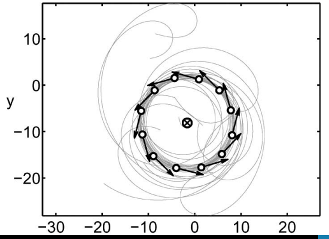
- Unicycle model with unit speed $v_k = 1 \quad \forall k$
- Cooperative approach: the vehicles only know relative distances $\mathbf{r}_k - \mathbf{r}_j$
- Formation center: results from a consensus algorithm

$$\tilde{\mathbf{r}}_k = \mathbf{r}_k - \mathbf{c}_m = \frac{1}{N}\sum_{j=1}^{N}(\mathbf{r}_k - \mathbf{r}_j)$$

Circular Formation Control Law

[Leonard et al. 2007, Sepulchre et al. 2007]

$$u_k = \omega_0 \left(1 + \kappa \tilde{\mathbf{r}}_k^T \dot{\mathbf{r}}_k\right)$$



Translation Control Design [Briñón-Arranz et al. CDC'09]

To stabilize each AUV to a circular motion with constant radius R tracking a time-varying center $\mathbf{c}(t)$.

Translation Control Design [Briñón-Arranz et al. CDC'09]

To stabilize each AUV to a circular motion with constant radius R tracking a time-varying center $\mathbf{c}(t)$.

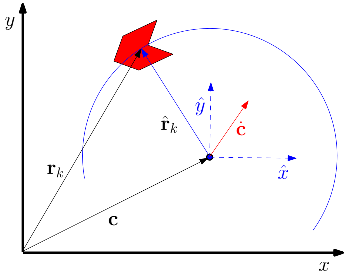
Coordinates transformation

$$\hat{\mathbf{r}}_k \triangleq \mathbf{r}_k - \mathbf{c}(t)$$

Transformed system

Imposed dynamics to $\hat{\mathbf{r}}_k$

$$\begin{aligned}\dot{\hat{x}}_k &= R|\omega_0|\cos\psi_k\\ \dot{\hat{y}}_k &= R|\omega_0|\sin\psi_k\\ \dot{\psi}_k &= \hat{u}_k\end{aligned}$$



$\hat{u}_k = \omega_0(1 + \kappa\dot{\hat{\mathbf{r}}}_k^T(\psi_k)(\mathbf{r}_k - \mathbf{c}))$ is a circular control law

Translation Control Design

Control strategy

- **Reference model:** relation between the original system (position vector of each agent) and the reference system (relative position vector)

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Translation Control Design

Control strategy

- **Reference model:** relation between the original system (position vector of each agent) and the reference system (relative position vector)
- **Fixed circular control law:** the reference system is stabilized to a circular motion with fixed center
- **Tracking approach:**
  - Transformed system (with imposed closed loop dynamics) is considered as a reference $\Longrightarrow$ Reference tracking
  - Aim: $\dot{\mathbf{r}}_k \rightarrow \dot{\hat{\mathbf{r}}}_k + \dot{\mathbf{c}}$ and $\ddot{\mathbf{r}}_k \rightarrow \ddot{\hat{\mathbf{r}}}_k + \ddot{\mathbf{c}}$
  - Control inputs $(\dot{v}_k, u_k)$

Theorem: Translation of a circular motion

Translation Control Law

$$\dot{v}_k = -\beta v_k + \frac{\dot{\psi}_k \dot{\mathbf{r}}_k^T \mathbf{R}_{\frac{\pi}{2}} \dot{\hat{\mathbf{r}}}_k + \dot{\mathbf{r}}_k^T (\ddot{\mathbf{c}} + \beta(\dot{\hat{\mathbf{r}}}_k - \dot{\mathbf{c}}))}{v_k}$$

$$u_k = \frac{\dot{\psi}_k \dot{\mathbf{r}}_k^T \dot{\hat{\mathbf{r}}}_k + \dot{\mathbf{r}}_k^T \mathbf{R}_{\frac{\pi}{2}}^T (\ddot{\mathbf{c}} + \beta(\dot{\hat{\mathbf{r}}}_k - \dot{\mathbf{c}}))}{v_k^2}$$

where $\beta > 0$ and $\mathbf{R}_{\frac{\pi}{2}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ makes the AUVs converge to a circular motion tracking the time-varying center $\mathbf{c}$.

- The center $\mathbf{c}(t)$ and its derivatives $\dot{\mathbf{c}}(t), \ddot{\mathbf{c}}(t)$ are external given references.
- $\dot{\psi}_k = \hat{u}_k = \omega_0(1 + \kappa \dot{\hat{\mathbf{r}}}_k^T(\psi_k)(\mathbf{r}_k - \mathbf{c}))$
- Singular point when $v_k = 0$

Proof

The convergence of the transformed system to a fixed circular motion is analyzed with the Lyapunov function:

$$S(\hat{\mathbf{r}},\psi) = \frac{1}{2}\sum_{k=1}^{N}\left\|\dot{\hat{\mathbf{r}}}_k - \omega_0 \mathbf{R}_{\frac{\pi}{2}}\hat{\mathbf{r}}_k\right\|^2 \geq 0$$

Equilibrium point when $S(\hat{\mathbf{r}},\psi) = 0$

$$\dot{\hat{\mathbf{r}}}_k = \omega_0 \mathbf{R}_{\frac{\pi}{2}}\hat{\mathbf{r}}_k \Rightarrow \dot{\hat{\mathbf{r}}}_k \perp \hat{\mathbf{r}}_k \quad \text{if} \underbrace{\Longrightarrow}_{\dot{\mathbf{r}}_k \rightarrow \dot{\hat{\mathbf{r}}}_k + \dot{\mathbf{c}}} \quad \dot{\mathbf{r}}_k = \dot{\mathbf{c}} + \omega_0 \mathbf{R}_{\frac{\pi}{2}}(\mathbf{r}_k - \mathbf{c})$$

Differentiating

$$\dot{S}(\hat{\mathbf{r}},\psi) = \sum_{k=1}^{N}\omega_0 \dot{\hat{\mathbf{r}}}_k^T \hat{\mathbf{r}}_k(\omega_0 - \dot{\psi}_k) = -\kappa \sum_{k=1}^{N}(\omega_0 \dot{\hat{\mathbf{r}}}_k^T \hat{\mathbf{r}}_k)^2 \leq 0$$

Proof

The control inputs of the original/real system are defined by a reference tracking process. The tracking error is denoted by:

$$e_k = \dot{\mathbf{r}}_k - (\dot{\hat{\mathbf{r}}}_k + \dot{\mathbf{c}})$$

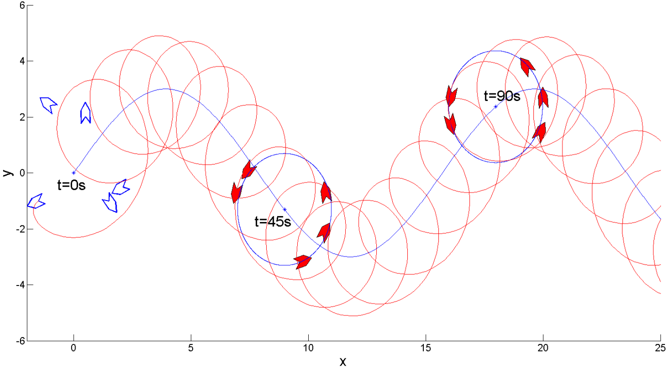
We impose the following error dynamics to make the error e_k converge to zero:

$$\dot{e}_k = -\beta e_k$$

And this equation determines the control inputs $(\dot{v}_k, u_k)$ because:

$$\frac{\dot{v}_k}{v_k}\dot{\mathbf{r}}_k + u_k \mathbf{R}_{\frac{\pi}{2}}\dot{\mathbf{r}}_k - \dot{\psi}_k \mathbf{R}_{\frac{\pi}{2}}\dot{\hat{\mathbf{r}}}_k - \ddot{\mathbf{c}} = -\beta(\dot{\mathbf{r}}_k - \dot{\hat{\mathbf{r}}}_k - \dot{\mathbf{c}})$$

Simulation



Scaling Control Design [Briñón-Arranz et al. ACC'10]

To stabilize each AUV to a circular motion centered at a fixed point **c** whose radius tracks the time-varying reference $R(t)$.

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To stabilize each AUV to a circular motion centered at a fixed point $\mathbf{c}$ whose radius tracks the time-varying reference $R(t)$.

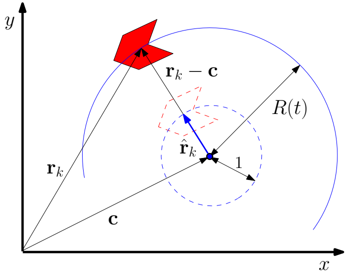
Coordinates transformation

$$\hat{\mathbf{r}}_k \triangleq \frac{\mathbf{r}_k - \mathbf{c}}{R(t)}$$

Transformed system

Imposed dynamics to $\hat{\mathbf{r}}_k$

$$\begin{aligned} \dot{\hat{x}}_k &= |\omega_0| \cos \psi_k \\ \dot{\hat{y}}_k &= |\omega_0| \sin \psi_k \\ \dot{\psi}_k &= \hat{u}_k \end{aligned}$$



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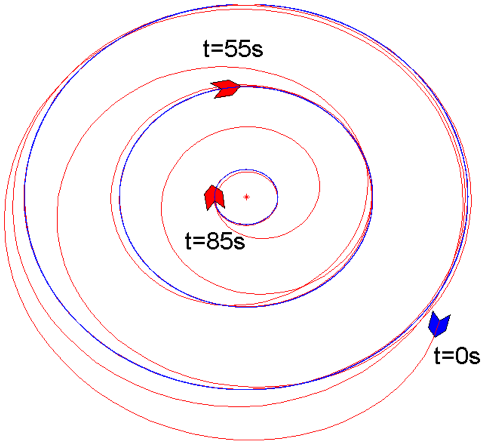


Uniform distribution along a circular formation

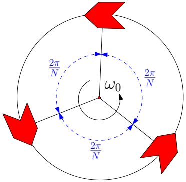
Motivations

- Formation control: previous translation/scaling control laws are not cooperative.
- Phase arrangement of vehicles is arbitrary
- Uniform distribution of a circular formation is appropriate for a source-seeking mission (**Lemma**: gradient approximation)

Definition

$$\phi_{kj} = \frac{2\pi}{N}$$

where $\phi_{kj} = \phi_k - \phi_j$ represents the angular difference between two adjacency vehicles.



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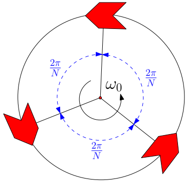
Definition

$$\dot{\hat{\mathbf{r}}}_k \perp \hat{\mathbf{r}}_k \quad \Rightarrow \quad \phi_k = \psi_k - \frac{\pi}{2}$$

Therefore $\phi_{kj} = \psi_{kj}$



Uniform Distribution Control Design

Previous works [Paley et al. 2005, Sepulchre et al. 2007/08] are based on the ideas from synchronization of coupled oscillators.

Potential function $U(\psi)$

- Invariant to rotations $\nabla U \mathbf{1} = 0$
- Heading angles of transformed system $\mathbf{B}_m = (\cos m\psi_1, \sin m\psi_1, \ldots, \cos m\psi_N, \sin m\psi_N)^T$
- Communication constraints: Laplacian matrix $\bar{\mathbf{L}} = \mathbf{L} \otimes \mathbf{I}_2$

$$U(\psi) = \frac{K}{N} \sum_{m=1}^{\lfloor N/2 \rfloor} \frac{1}{2m^2} \mathbf{B}_m \bar{\mathbf{L}} \mathbf{B}_m$$

Complete graph $\Rightarrow$ Uniform distribution is the only equilibrium point of $U(\psi)$

Theorem

Circular formation control law with uniform distribution

- Translation/scaling control law +

$$\dot{\psi}_k = \omega_0(1 + \kappa \dot{\hat{\mathbf{r}}}_k^T(\mathbf{r}_k - \mathbf{c})) - \frac{\partial U}{\partial \psi_k}$$

$$\frac{\partial U}{\partial \psi_k} = -\frac{K}{N} \sum_{j \in \mathcal{N}_k} \sum_{m=1}^{\lfloor N/2 \rfloor} \frac{\sin m\psi_{kj}}{m}$$

Theorem

Circular formation control law with uniform distribution

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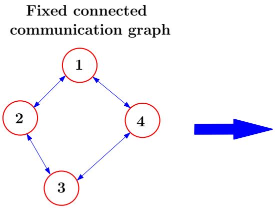
$$\frac{\partial U}{\partial \psi_k} = -\frac{K}{N} \sum_{j \in \mathcal{N}_k} \sum_{m=1}^{\lfloor N/2 \rfloor} \frac{\sin m\psi_{kj}}{m}$$

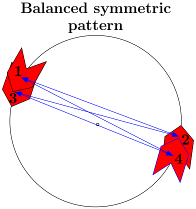
Proof:

$$V(\hat{\mathbf{r}}, \psi) = \kappa S(\hat{\mathbf{r}}, \psi) + U(\psi) \geq 0$$

$$\dot{V}(\hat{\mathbf{r}}, \psi) = \sum_{k=1}^{N} \left(\kappa \omega_0 \hat{\mathbf{r}}_k^T \dot{\hat{\mathbf{r}}}_k - \frac{\partial U}{\partial \psi_k} \right) (\omega_0 - \dot{\psi}_k) \leq 0$$

Limited Communication Range [Briñón-Arranz et al. ACC'10]





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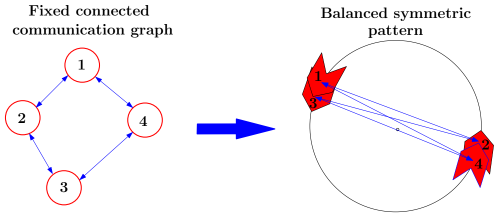
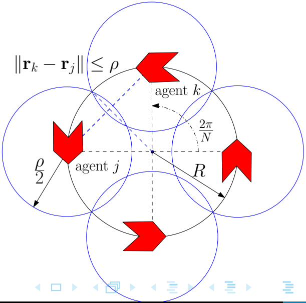


Critical communication distance ρ

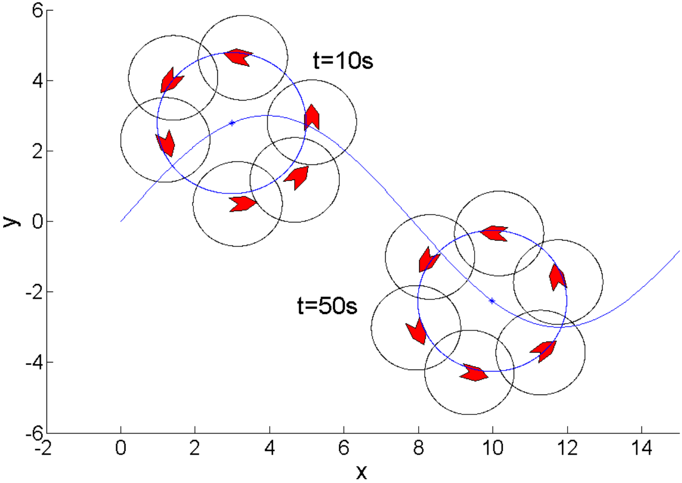
$$j \in \mathcal{N}_k \quad \Rightarrow \quad \|\mathbf{r}_k - \mathbf{r}_j\| \leq \rho$$

Geometrical condition:

$$\rho > 2R \sin \frac{\pi}{N}$$



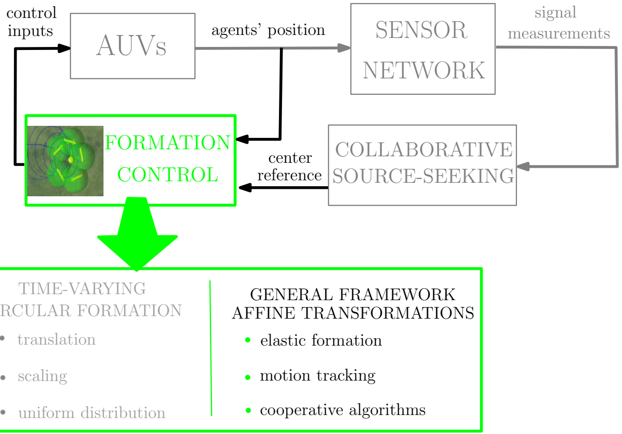
Simulations



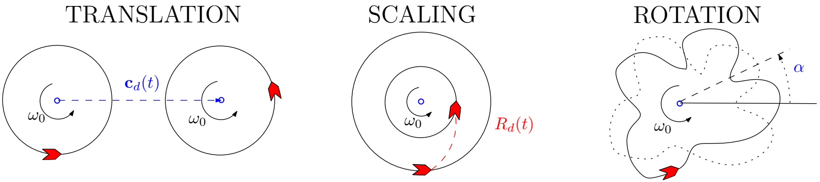
Conclusions

- Stabilization of a single vehicle to a circular motion which tracks a time-varying center $\mathbf{c}(t)$ or a time-varying radius $R(t)$.
- $\mathbf{c}(t)$ and $R(t)$ are external given references
- Uniform distribution of vehicles along the time-varying **circular formation**.
- Limited communication range: to avoid other phase arrangement

4. Elastic Formation Control Design



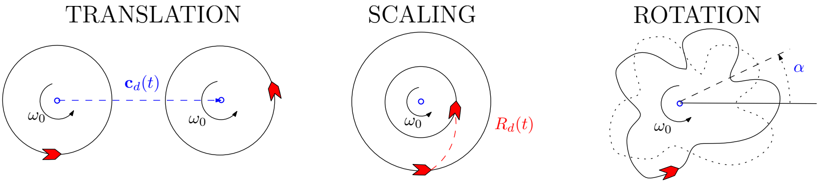
Affine Transformations



$$\mathbf{T}_c = \begin{pmatrix} 1 & 0 & c_x \\ 0 & 1 & c_y \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{S} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \mathbf{R}_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{T}_c^{-1} = \mathbf{T}_{-c} \qquad s_x > 0, \quad s_y > 0 \qquad \mathbf{R}_\alpha^{-1} = \mathbf{R}_\alpha^T$$

Affine Transformations



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$$\mathbf{T}_c^{-1} = \mathbf{T}_{-c} \qquad s_x > 0, \quad s_y > 0 \qquad \mathbf{R}_\alpha^{-1} = \mathbf{R}_\alpha^T$$

Homogeneous Coordinates

The homogeneous coordinates of a vector $\mathbf{z} \in \mathbb{R}^2$ are defined by $\mathbf{z}^h = (z_x, z_y, 1)^T$.

Elastic Formation

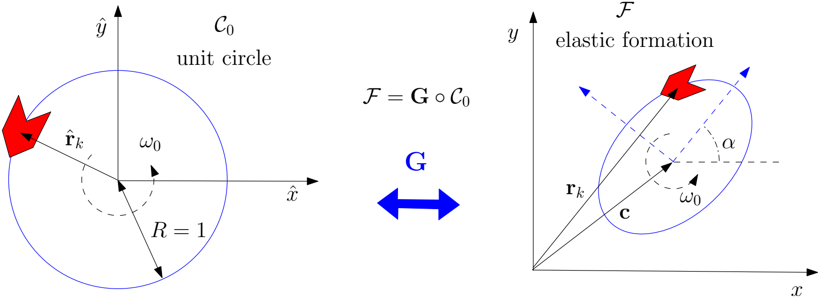
General transformation $\mathbf{G}$

$$\mathbf{G} = \prod_{i}^{I}\prod_{j}^{J}\prod_{k}^{K}\mathbf{S}_i\mathbf{R}_{\alpha_j}\mathbf{T}_{c_k}$$

Elastic Formation $\mathcal{F}$

$\mathcal{F}$ is a curve which results of applying $\mathbf{G}$ to the unit circle $\mathcal{C}_0$

$$\mathcal{F} = \mathbf{G} \circ \mathcal{C}_0$$



Elastic Motion Control Design [Briñón-Arranz et al. ACC'11]

To stabilize each AUV to an elastic motion $\mathcal{F} = \mathbf{G} \circ \mathcal{C}_0$.

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Coordinates transformation

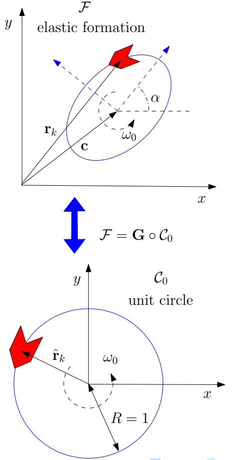
$$\hat{\mathbf{r}}_k \triangleq \mathbf{G}^{-1}\mathbf{r}_k$$

Transformed system

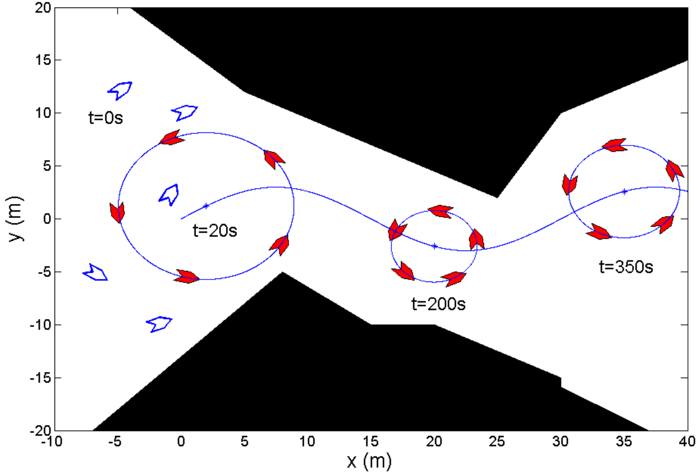
Imposed dynamics to $\hat{\mathbf{r}}_k$

$$\begin{aligned} \dot{\hat{x}}_k &= |\omega_0| \cos\psi_k \\ \dot{\hat{y}}_k &= |\omega_0| \sin\psi_k \\ \dot{\psi}_k &= \hat{u}_k \end{aligned}$$

$\hat{u}_k$ is a circular control law



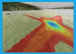
Simulation

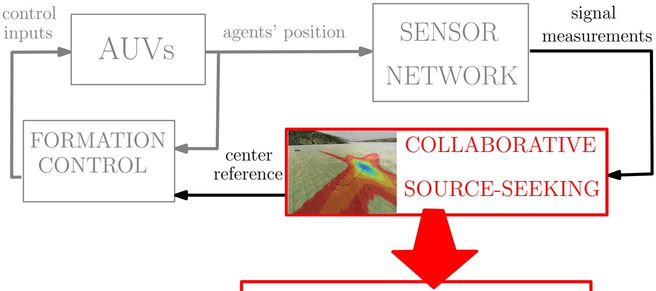

t=0s
t=20s
t=200s
t=350s
y (m)
x (m)
20
15
10
5
0
-5
-10
-15
-20
-10
-5
0
5
10
15
20
25
30
35
40

Conclusions

- Definition of Elastic Formation based on affine transformations.
- Stabilization of a single vehicle to an elastic motion which tracks several time-varying parameters.
- Desired motion parametrized by a few number of parameters.
- Uniform distribution of vehicles along the time-varying **elastic formation**.

5. Collaborative Source-Seeking

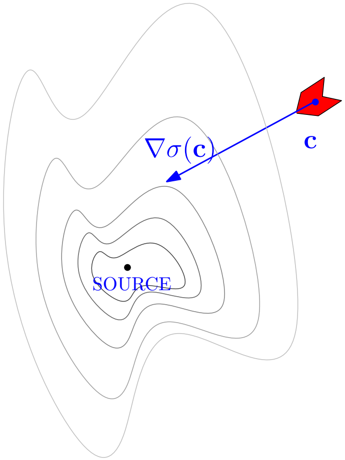




- approximation of gradient direction
- distributed estimation algorithm
- collaborative source-seeking

Problem Formulation

Scalar field: continuous signal distribution $\sigma(\mathbf{r}_k)$



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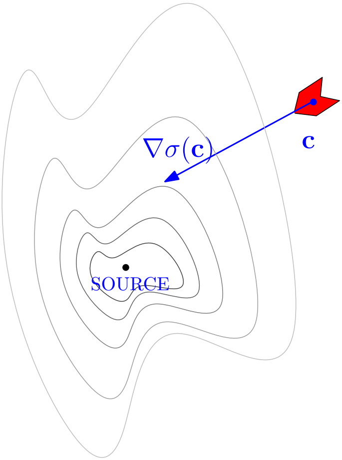
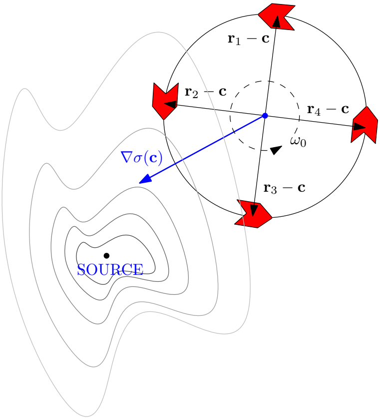
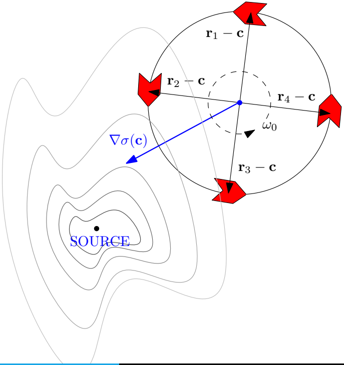




Approximation of the gradient of a scalar field

Lemma: Gradient Approximation [Briñón-Arranz et al. CDC'11]

$$\frac{1}{N}\sum_{k=1}^{N}\sigma(\mathbf{r}_k)(\mathbf{r}_k-\mathbf{c}) = \frac{R^2}{2}\nabla\sigma(\mathbf{c}) + o(R^2)$$



Proof:
Based on multi-variable Taylor series expansion of σ at $\mathbf{c}$:

$$\sigma(\mathbf{r}_k)-\sigma(\mathbf{c}) = \nabla\sigma(\mathbf{c})(\mathbf{r}_k-\mathbf{c})+o(R)$$

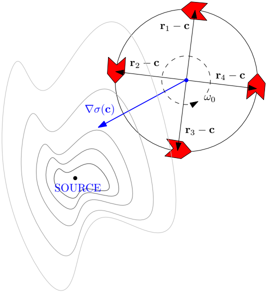
and applying trigonometric proprieties.

Distributed solution

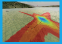
- Each agent estimates its own gradient direction $\mathbf{z}_k$
- Each agent receives the estimated direction of its neighbors
- Distributed algorithm to obtain the same estimated direction (to keep the circular formation)

This estimated direction will be the reference velocity of the formation center in order to steer the group of agents to the source location.

In this work, we consider a fixed center



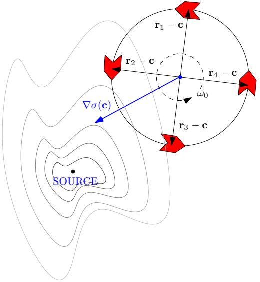
Distributed solution



- Each agent estimates its own gradient direction $\mathbf{z}_k$
- Each agent receives the estimated direction of its neighbors
- Distributed algorithm to obtain the same estimated direction (to keep the circular formation)

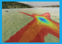
The objective is to make all estimated directions $\mathbf{z}_k$ converge to the mean direction defined as:

$$\mathbf{g}^* = \frac{1}{N}\sum_{k=1}^{N}\mathbf{g}_k; \quad \mathbf{g}_k = \sigma_k(\mathbf{r}_k - \mathbf{c})$$



$\mathbf{g}^*$ approximates the gradient direction of signal distribution at $\mathbf{c}$

Theorem: Distributed estimation [Briñón-Arranz et al. CDC'11]



Distributed Algorithm based on Consensus Filters

$$\dot{\mathbf{z}}_k = -\kappa \sum_{j \in \mathcal{N}_k} (\mathbf{z}_k - \mathbf{z}_j) + \sum_{j \in \mathcal{J}_k} (\mathbf{g}_j - \mathbf{z}_k)$$

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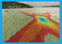


Distributed Algorithm based on Consensus Filters

$$\dot{\mathbf{z}}_k = -\kappa \sum_{j \in \mathcal{N}_k} (\mathbf{z}_k - \mathbf{z}_j) + \sum_{j \in \mathcal{J}_k} (\mathbf{g}_j - \mathbf{z}_k)$$

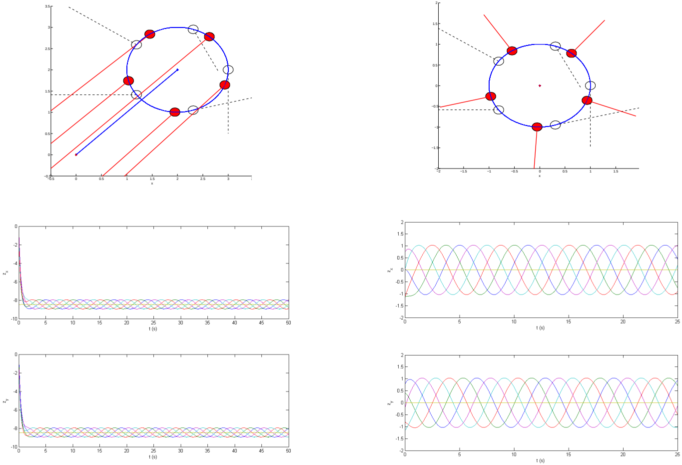
If $\mathbf{g}^*$ satisfies $\|\dot{\mathbf{g}}^*\| \leq \nu$, then $\mathbf{z}^* = \mathbf{1} \otimes \mathbf{g}^*$ is a globally asymptotically ϵ-stable equilibrium with

$$\epsilon = \frac{(\nu\sqrt{2N}(1 + d_{max}) + \alpha\gamma)\lambda_{max}^{\frac{1}{2}}(\mathbf{A}_\kappa)}{\lambda_{min}^{\frac{5}{2}}(\mathbf{A}_\kappa)}$$

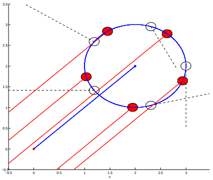
Proof:

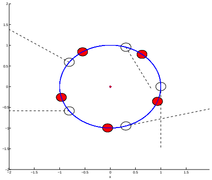
- error equation $\eta = \mathbf{z} - \mathbf{1} \otimes \mathbf{g}^*$
- error dynamics $\dot{\eta} = \dot{\mathbf{z}} - \mathbf{1} \otimes \dot{\mathbf{g}}^* = -\mathbf{A}_\kappa \mathbf{z} + \mathbf{B}\mathbf{g} - \mathbf{1} \otimes \dot{\mathbf{g}}^*$
  where $\mathbf{A}_\kappa = (\mathbf{I}_N + \Delta + \kappa \mathbf{L}) \otimes \mathbf{I}_2$ and $\mathbf{B} = (\mathbf{I}_N + \mathcal{A}) \otimes \mathbf{I}_2$
- Lyapunov function $V = \frac{1}{2}\eta^T \mathbf{A}_\kappa \eta \geq 0$

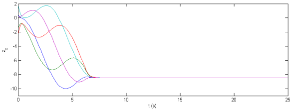
Simulations

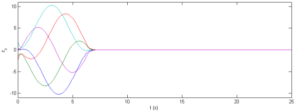


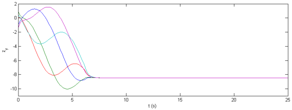
Simulations (Input-average Consensus Algorithm)

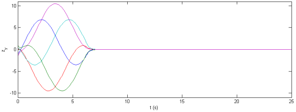




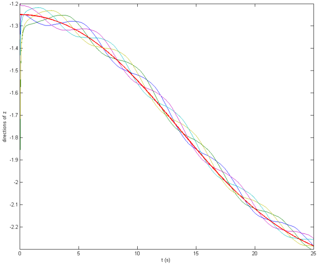


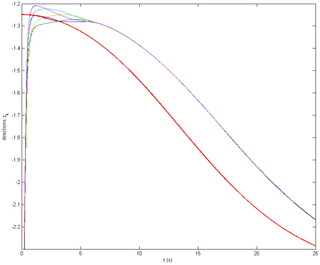






Simulations with time-varying source





6. Conclusions

Formation Control

- Stabilization of a fleet of AUVs to a time-varying circular motions (based on ideas from collective circular motions)
- **Main idea: coordinates transformation + reference tracking**
- Generalization to stabilize the AUVs to elastic formations
- Uniform distribution of vehicles along the formation

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Collaborative Source-Seeking

- Lemma: approximation of the gradient
- Distributed algorithm to estimate the gradient direction
- Analysis of the algorithm with a time-varying source

Perspectives

Formation Control

- Generalization of proposed methodology to collective motions
- Time-varying formation in a flowfield
- Extension to 3-dimensions?
- Consider obstacle avoidance techniques

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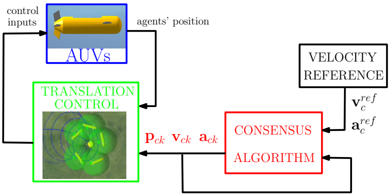
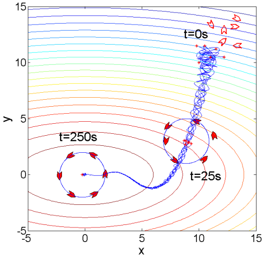
Collaborative Source-Seeking

- Lemma in the case of time-varying circular formation?
- Source-seeking algorithm: time-varying formation control + distributed estimation of the gradient
- Other communications problems (noise, packet drops, time delays)

Ongoing research

Cooperative Translation Control based on Consensus with Reference Velocity: a Source-seeking Application with a Fleet of AUVs





Grazie per la vostra attenzione