Localization and optimization problems for camera networks

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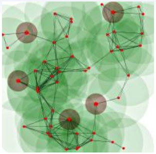
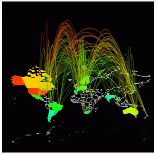
Outline

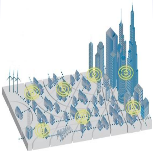
Cooperative multi-agent systems



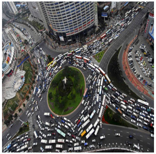
WSN



Communication Networks



Smart Grids



Traffic control

Robotics coordination

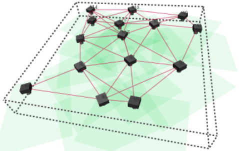
Biological Networks

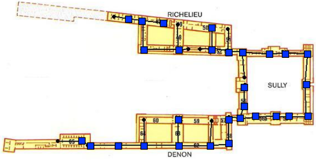
Opinion dynamics

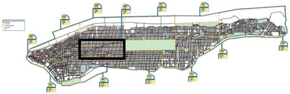
Coverage control

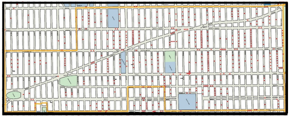
Economics

Camera network applications









Where:

- civil and military buildings
- outdoor environments *e.g. NYC*

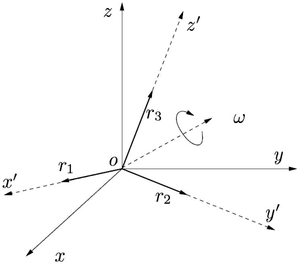
Tasks:

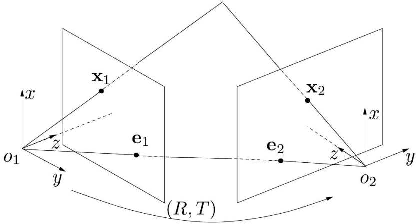
- surveillance
- tracking

Frame localization

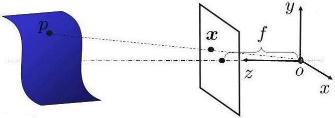
Given a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$: from relative measurements to absolute ones

$$\{R_{ij}, T_{ij}\}_{(i,j)\in\mathcal{E}} \Rightarrow \left\{R_i^1, T_i^1\right\}_{i\in\mathcal{V}}$$





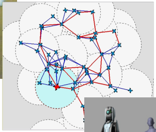
Camera pinhole model

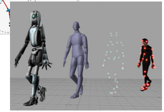


Calibration problem

Why calibration is important?

- intrusion detection/vehicle tracking
- coverage
- motion capture





Why distributed?

- low power small devices
- adaptivity
- periodical re-calibration
- reliability of multi-hops and agents

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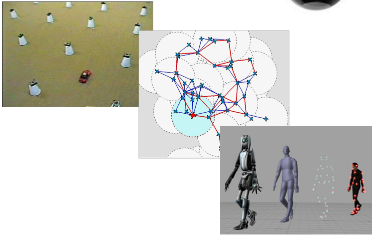

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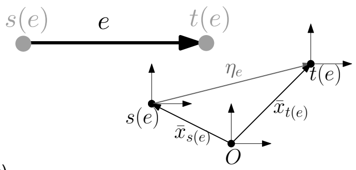
Outline

Position localization

- Given
  - $\mathcal{G} = (\mathcal{V}, \mathcal{E}), |\mathcal{V}| = N, |\mathcal{E}| = M$
  - $\eta_e = \bar{x}_{s(e)} - \bar{x}_{t(e)} - \epsilon_e \in \mathbb{R}$

    noisy relative measurements



- Goal: estimate $\bar{x} \in \mathbb{R}^N$ (up to global translations)
- Cost function:

$$V(x) = \sum_{e \in \mathcal{E}} (x_{s(e)} - x_{t(e)} - \eta_e)^2 = \|Ax - \eta\|_2^2$$

$A \in \{\pm 1, 0\}^{M \times N}$ incidence matrix of $\mathcal{G}$

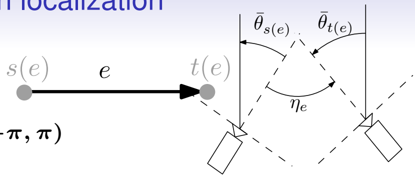
- Minimization problem: $\hat{x} := \arg\min_x V(x)$

[**Barooah-Hespana (2005)**]

2D Orientation localization

- Given
  - $\mathcal{G} = (\mathcal{V}, \mathcal{E}), |\mathcal{V}| = N, |\mathcal{E}| = M$
  - $\eta_e = (\bar{\theta}_{s(e)} - \bar{\theta}_{t(e)} - \epsilon_e)_{2\pi} \in [-\pi, \pi)$

    noisy relative measurements



- Goal: estimate $\bar{\theta} \in \mathbb{R}^N$ (up to global translations $\mod 2\pi$)
- Cost function:

$$V(\theta) = \|(A\theta - \eta)_{2\pi}\|_2^2 = \|A\theta - \eta - 2\pi K(\theta)\|_2^2 , \quad K(\theta) \in \mathbb{Z}^M$$

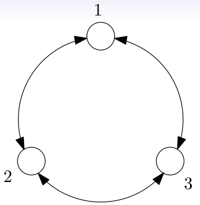
$\bar{K} = K(\bar{\theta})$ s.t.

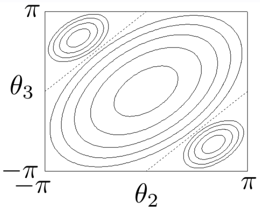
$$\eta = A\bar{\theta} - \epsilon - 2\pi\bar{K} \in [-\pi, \pi)$$

- Minimization problem: $\hat{\theta} := \arg\min_\theta V(\theta)$

[Piovan et al. (2011)], [Sarlette-Sepulchre (2009)], [Tron-Vidal (2009)]

A simple example

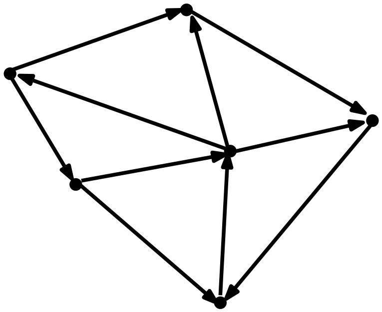




1. True orientations: $\bar{\theta}_1 = \bar{\theta}_2 = \bar{\theta}_3 = 0$; Anchor: $\theta_1 = 0$
2. Noise: $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0$; Variables: $\theta_2, \theta_3 \in [-\pi, \pi)$
3. Cost function: $V(0, \theta_2, \theta_3) = \theta_2^2 + \theta_3^2 + (\theta_2 - \theta_3)^2_{2\pi}$

How to avoid local minima/maxima?

First approach: restriction to a spanning tree $\mathcal{T} \subset \mathcal{G}$



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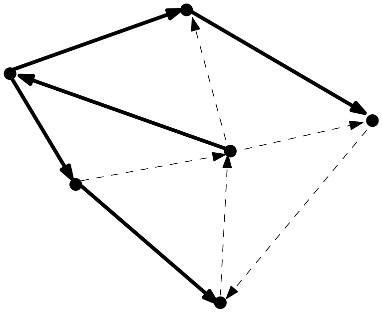

LOSS OF DATA!

How to avoid local minima/maxima?

Second approach: 2-step hybrid algorithm

1) estimate $\bar{K}$

$\Rightarrow$ reshape the cost function V

2) estimate $\bar{\theta}$

Cost function: $V(\theta) = \|(A\theta - \eta)_{2\pi}\|_2^2 = \|A\theta - \eta - 2\pi K(\theta)\|_2^2$

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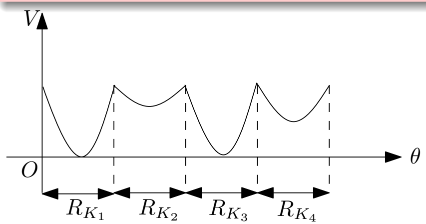

Convex regions: $R_K(\eta) := \{\theta \in \mathbb{R}^N : |A\theta - \eta - 2\pi K| \leq \pi \mathbf{1}\}$

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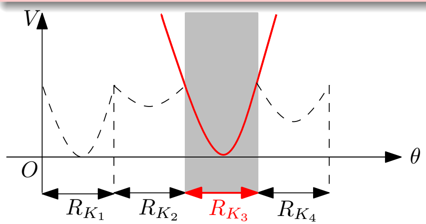
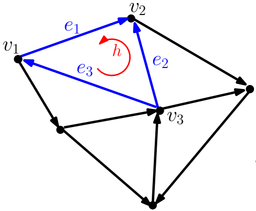


Convex regions: $R_K(\eta) := \{\theta \in \mathbb{R}^N : |A\theta - \eta - 2\pi K| \leq \pi \mathbf{1}\}$

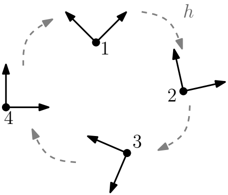
Reshaped cost function: $V_{\hat{K}}(\theta) := \|A\theta - \eta - 2\pi\hat{K}\|_2^2$

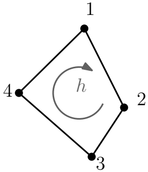
Main idea: cycle constraints



Cycle vector

$$r_h = \begin{array}{ccccc} & e_1 & e_2 & e_3 & \\ [& -1 & +1 & -1 & 0\dots0] \end{array}$$





$$\sum_{e\in h}(\bar{\theta}_{s(e)} - \bar{\theta}_{t(e)}) = \mathbf{0}, \text{ for any cycle } h$$

Two proposed algorithms

Given the relative measurements

$\eta = A\bar{\theta} - \epsilon - 2\pi\bar{K} \qquad \Rightarrow r_h\bar{K} = -q_{2\pi}(r_h\eta) - q_{2\pi}(r_h\epsilon)$

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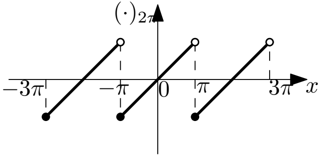
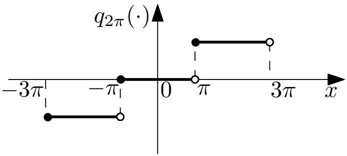
$$\begin{array}{rcccl} (\cdot)_{2\pi}: & \mathbb{R} & \rightarrow & [-\pi,\pi) \\ & x & \mapsto & (x)_{2\pi} = x - 2\pi q_{2\pi}(x) \end{array} \qquad \begin{array}{rcccl} q_{2\pi}: & \mathbb{R} & \rightarrow & \mathbb{Z} \\ & x & \mapsto & q_{2\pi}(x) = \left\lfloor \frac{x+\pi}{2\pi} \right\rfloor \end{array}$$





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$$\eta = A\bar{\theta} - \epsilon - 2\pi\bar{K} \qquad \Rightarrow r_h\bar{K} = -q_{2\pi}(r_h\eta) - q_{2\pi}(r_h\epsilon)$$

If $|r_h\epsilon| < \pi$ then

$$r_h\bar{K} = -q_{2\pi}(r_h\eta) \quad \Leftrightarrow \quad \sum_{e\in h} r_h(e)\bar{K}_e = -q_{2\pi}\left(\sum_{e\in h} r_h(e)\eta_e\right)$$

Used to implement the algorithms!

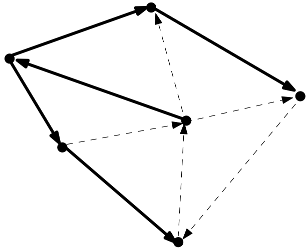
- $M - N + 1$ independent equations
- M unknowns

Find $\bar{K}$ up to $Im_{\mathbb{Z}}A$:

$dimIm_{\mathbb{Z}}A = N - 1 \Rightarrow \bar{K}$ uniquely found!

Main difference: choice of the cycle basis

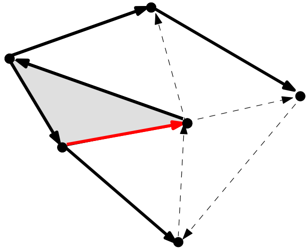
Fundamental-cycles-basis:



Minimal-cycles-basis:

Cycle space basis construction

Fundamental-cycles-basis:



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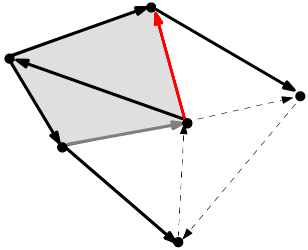

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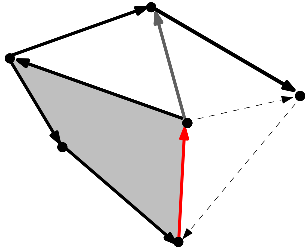

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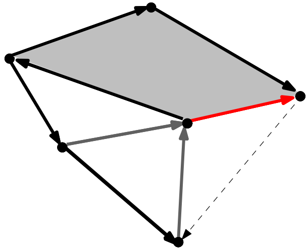

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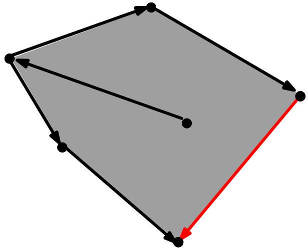

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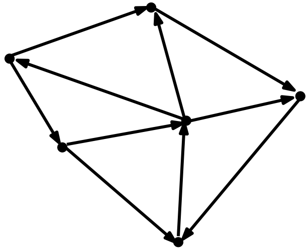

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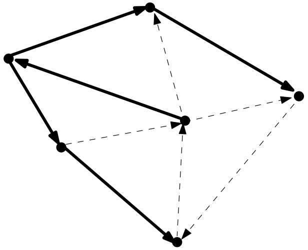

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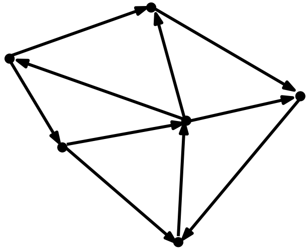

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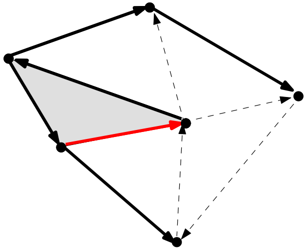

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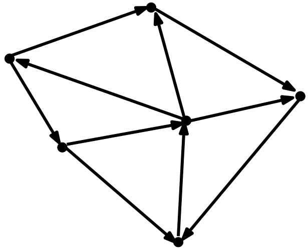

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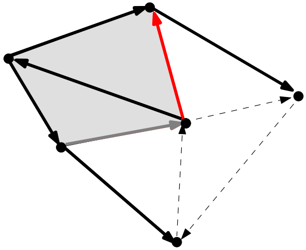

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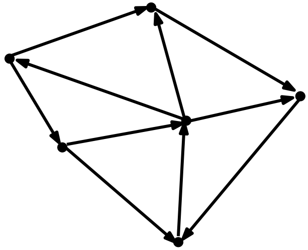

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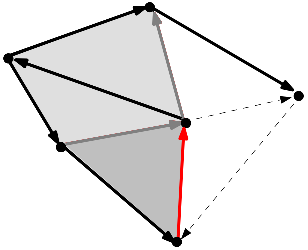

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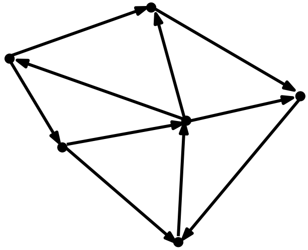

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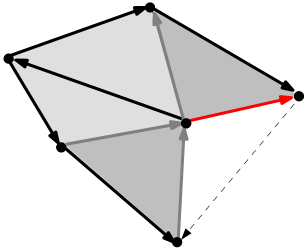

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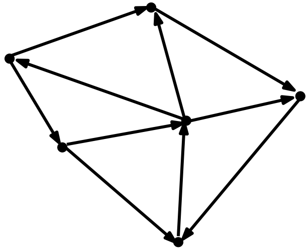

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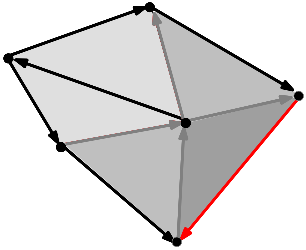
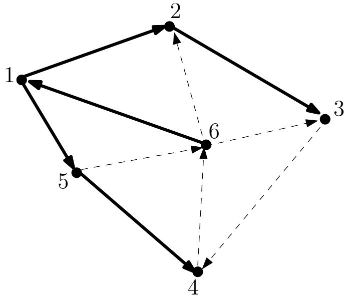


Minimal Cycles-Algorithm (MC)



$$\hat{K}_e^M = 0, e \in \mathcal{E}_{\mathcal{T}}$$
$$\sum_{e \in h} r_{h_0}(e)\hat{K}_e^M = -q_{2\pi}\left(\sum_{e \in h} r_h(e)\eta_e\right)$$

How the algorithm works

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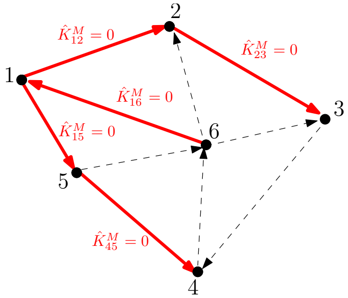

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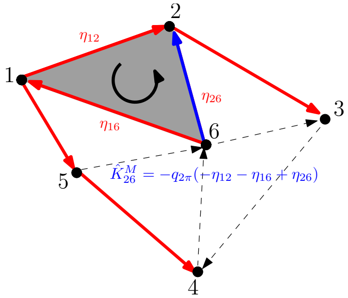

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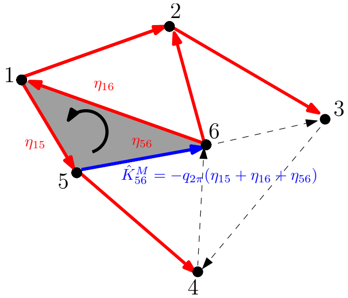

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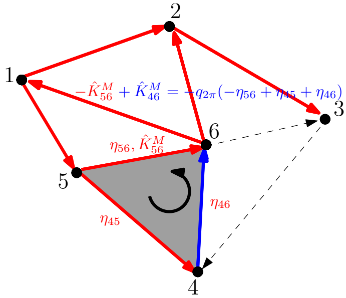

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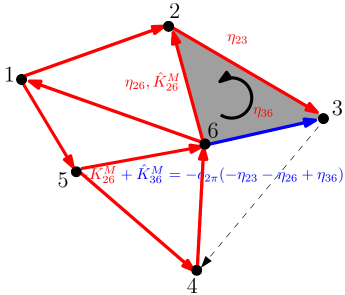
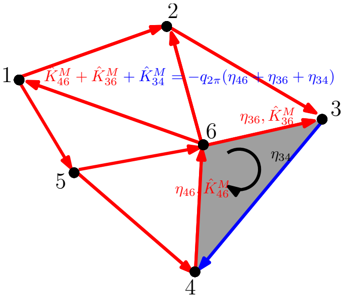
$$\sum_{e \in h} r_{h_0}(e)\hat{K}_e^M = -q_{2\pi}\Big(\sum_{e \in h} r_h(e)\eta_e\Big)$$

Minimal Cycles-Algorithm (MC)



$$\hat{K}^M_e = 0, e \in \mathcal{E}_{\mathcal{T}}$$
$$\sum_{e \in h} r_{h_0}(e)\hat{K}^M_e = -q_{2\pi}(\sum_{e \in h} r_h(e)\eta_e)$$

Minimal Cycles-Algorithm (MC)



$$\hat{K}_e^M = 0, e \in \mathcal{E}_{\mathcal{T}}$$
$$\sum_{e \in h} r_{h_0}(e)\hat{K}_e^M = -q_{2\pi}\left(\sum_{e \in h} r_h(e)\eta_e\right)$$

Analysis and comparison

Outline:

1. sufficient conditions to have $\hat{K}^F - \bar{K} \in Im_{\mathbb{Z}}A$ and $\hat{K}^M - \bar{K} \in Im_{\mathbb{Z}}A$ (closed formulae for $\hat{K}$)
2. vector space case (closed formula for $\hat{\theta}$)
3. asymptotic behavior in N of $\mathbb{E}[\hat{\theta}]$, $Var(\hat{\theta})$

Analysis and comparison

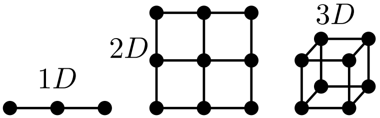
Proposition (Sufficient condition to correctly estimate $\bar{K}$)

Suppose $\mathbb{P}(|\epsilon_e| \geq \bar{\epsilon}) = 0, \forall e \in \mathcal{E}$. Then $\hat{K} = \bar{K} + Al, l \in \mathbb{Z}^N$ if

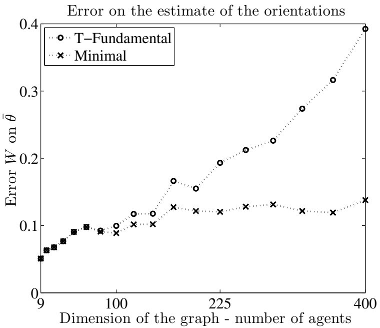
1. $\bar{\epsilon} < \frac{\pi}{L_{\mathcal{T}}}$, $L_{\mathcal{T}}$ max length of h_e's (*T-Algorithm*)
2. $\bar{\epsilon} < \frac{\pi}{L_0}$, L_0 max length of minimal cycles (*MC-Algorithm*)

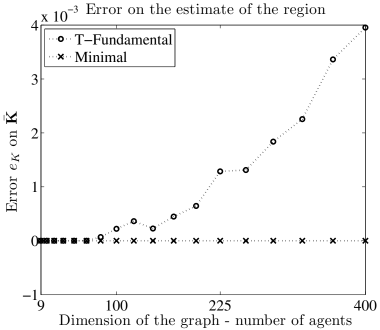
Proposition [Barooah-Hespana'05]

- $\mathbb{E}\hat{\theta} = \bar{\theta} \mod 2\pi$
- d-lattices
  1. $d = 1$: $\frac{1}{N}\mathrm{Var}(\hat{\theta} - \tilde{\theta}) \sim N$
  2. $d = 2$: $\frac{1}{N}\mathrm{Var}(\hat{\theta} - \tilde{\theta}) \sim \log(N)$
  3. $d = 3$: $\frac{1}{N}\mathrm{Var}(\hat{\theta} - \tilde{\theta}) \sim const$



Numerical results





Average error on $\hat{\theta}$

$$W(\hat{\theta}) = \frac{1}{N} \left\| (\hat{\theta} - \bar{\theta})_{2\pi} \right\|^2$$

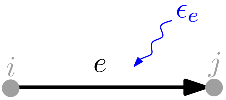
Average error on $\hat{K}$

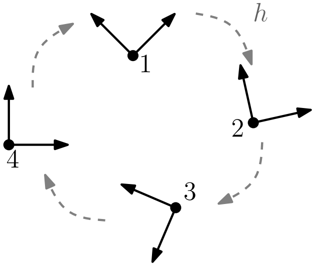
$$e_{\hat{K}} = \frac{1}{M} \left\| (\hat{K} - \bar{K})_{Im_{\mathbb{Z}} A} \right\|^2$$

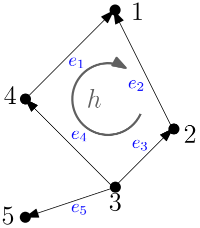
$$\frac{\pi}{L_0} = \frac{\pi}{4} > \frac{\pi}{8} = \bar{\epsilon} \Rightarrow OK!$$

Constrained non-linear minimization problem

- **Noisy data:** $\eta_e = (\bar{\theta}_i - \bar{\theta}_j + \epsilon_e)_{2\pi}$
- **Estimate:** $\hat{\psi}_e = \hat{\theta}_i - \hat{\theta}_j$
- **Cycle matrix:** $R \in \{0, \pm 1\}^{(M-N+1)\times M}$
- **Cycle error:** $c_h = (r_h \hat{\psi})_{2\pi}$







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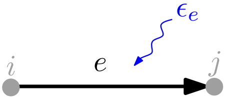

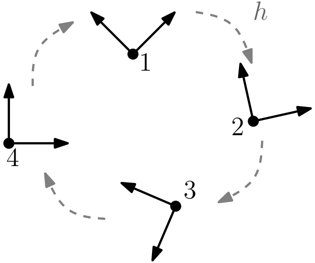

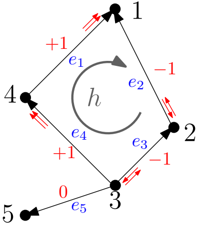

$r_h = (\ 1 \quad -1 \quad -1 \quad 1 \quad 0\ )$

- **Noisy data:** $\eta_e = (\bar{\theta}_i - \bar{\theta}_j + \epsilon_e)_{2\pi}$
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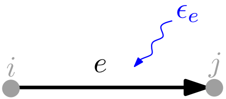
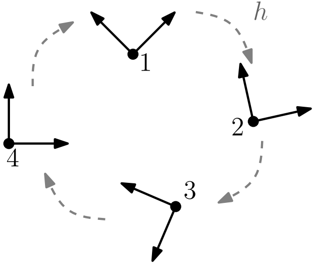
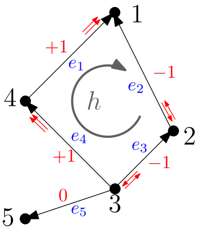






$$r_h = \begin{pmatrix} 1 & -1 & -1 & 1 & 0 \end{pmatrix}$$

$$\min_{\psi} \|(\psi - \eta)_{2\pi}\|_2^2$$

s.t. $c_h = 0$, for each cycle h

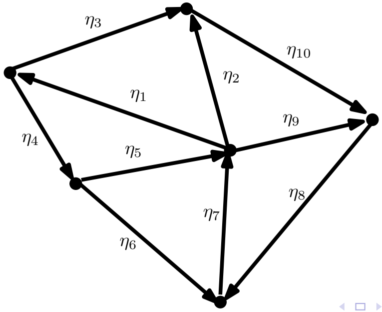
[Piovan et al. 2011]

An asynchronous gossip algorithm

Main idea: split the error c_h in equal components among its edges

1. fix spanning tree $\mathcal{T}$, and $M - N + 1$ independent cycles
2. fix stepsize $k \in (0, 1)$
3. initialize $\hat{\psi}(0) = \eta$
4. at each time t, choose a random $e(t) = e \in \mathcal{E}$

$$\hat{\psi}_e(t+1) = \hat{\psi}_e(t) - k(R^T c)_e$$



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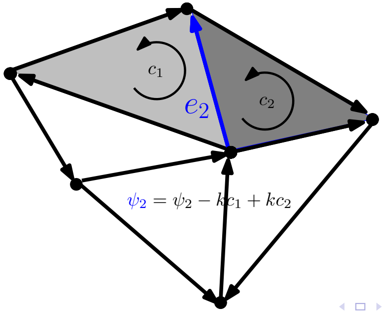

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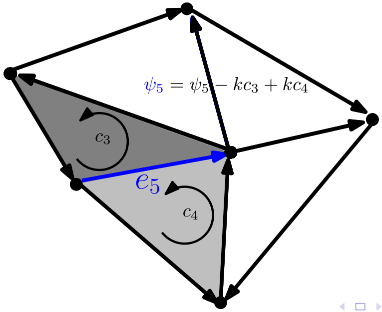


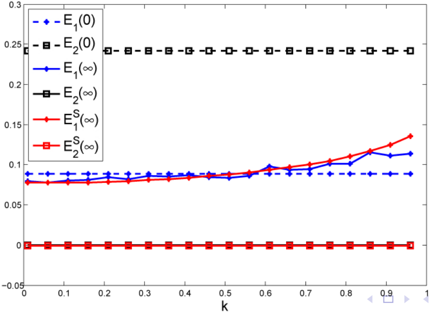
Exponential convergence I

Theorem (General planar graphs)

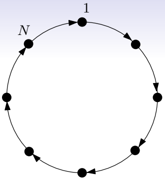
1. $\hat{\psi}(t)$ converges to $\hat{\psi}(\infty)$ almost surely and exponentially fast in mean square sense, that is

$$\|\hat{\psi}(t) - \hat{\psi}(\infty)\|_{L^2} \leq \frac{k\|RR^T\|^{1/2}}{M(1-\rho)}\rho^t \|\epsilon(0)\|_{L^2}, \quad \rho \in (0,1)$$

2. $\|\hat{\psi}(\infty) - \eta\|_{L^2} \leq \text{const}$
3. $c(t) := (R\hat{\psi}(t))_{2\pi} \to 0$



Exponential convergence II



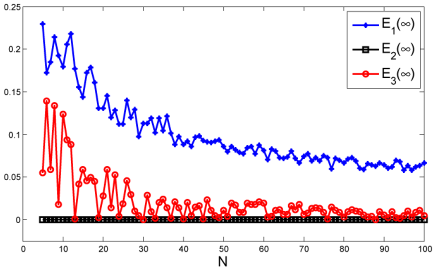
- Gossip update

$$\begin{cases} \hat{\psi}(0) = \eta \\ \hat{\psi}_e(t+1) = \hat{\psi}_e(t) - k c_1 \end{cases}$$

- Optimal solution [Kackmarz 1993]: $\psi^* = \eta - \frac{1}{N} r^T (r\eta)_{2\pi}$

Theorem (Ring graphs)

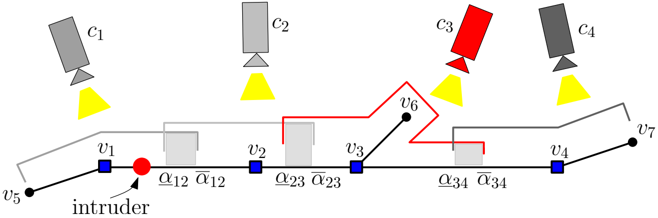
- $\hat{\psi}(t) \to \hat{\psi}(\infty)$ for each realization
- $N^{-1} \|\hat{\psi}(\infty) - \psi^*\|_{L^2} = O(N^{-1})$



Outline

Setup

- Roadmap $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
- Cameras in $\mathcal{V}_c \subseteq \mathcal{V}$
- **Smart** Intruder trajectory p
- Cameras trajectory X



Performance index

- Worst-case Detection Time

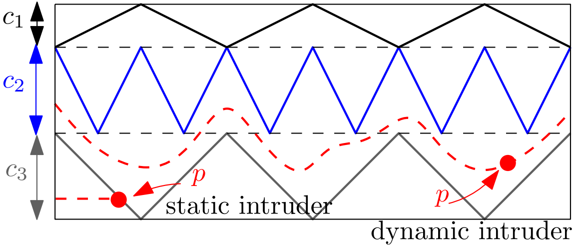
$$\mathrm{WDT}(X) := \sup_{p,\ t_0 \in [0,T]} t^*(t_0, p, X),$$

$$t^*(t_0, p, X) = \min\{\{t - t_0 > 0 \mid p(t) \in X(t)\} \cup \{\infty\}\}.$$

Problem definition

Detection: Design optimal periodic cameras trajectories that **minimize** the **detection time** of intruders/events, in a **distributed** way.

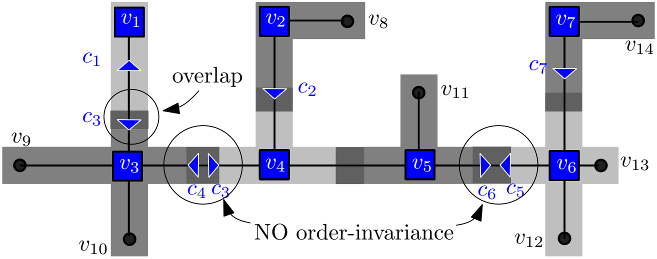
Find X^* s.t. $\text{WDT}(X^*) = \min_X \text{WDT}(X) = \text{WDT}^*$.



[Alberton 2012], [Baseggio 2010], [Carli 2011], [Spindler 2012]

Equivalent cameras trajectories

- Order-invariant trajectories (general roadmap)
- Non-overlapping trajectories (tree and ring)

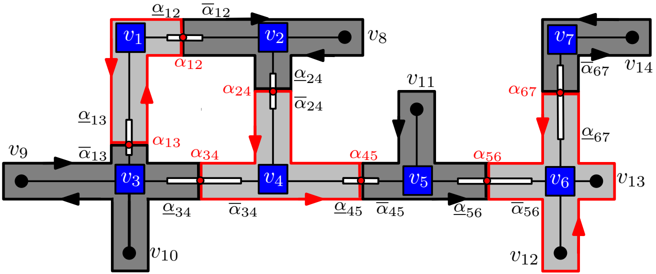


$$\mathcal{V} = \{v_1, \dots, v_{14}\}, \mathcal{V}_c = \{v_1, \dots, v_7\}$$

Cameras trajectory properties & Depth-First Trajectory

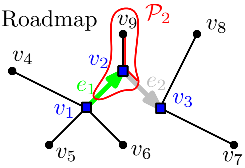
Equivalent cameras trajectories

- Order-invariant trajectories (general roadmap)
- Non-overlapping trajectories (tree and ring)



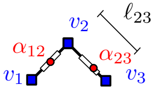
Continuous graph partitions

- $\boldsymbol{A}$ incidence matrix, $\boldsymbol{b}$ leaves-term, $\boldsymbol{\alpha} \in [\mathbf{0}, \mathbf{1}]^{\mathcal{E}_c}$
- Partition $\mathcal{P} = \{\mathcal{P}_i\}_{i=1}^n, \mathcal{P}_i \subseteq \mathcal{E}, |\mathcal{P}_i| = L_i$



$$A = \begin{array}{cc} e_1 & e_2 \\ \left[\begin{array}{c} \ell_{12} \\ -\ell_{12} \\ 0 \end{array}\right. & \left.\begin{array}{c} 0 \\ \ell_{23} \\ -\ell_{23} \end{array}\right] \end{array}$$

$$b = \begin{bmatrix} \ell_{14} + \ell_{15} + \ell_{16} \\ \ell_{29} + \ell_{23} \\ \ell_{37} + \ell_{39} \end{bmatrix}$$



Camera network

(1) Min-max partition: (Non-diff.) $\quad \|A\boldsymbol{\alpha}_\infty^* + \boldsymbol{b}\|_\infty = \min_{\underline{\boldsymbol{\alpha}} \leq \boldsymbol{\alpha} \leq \overline{\boldsymbol{\alpha}}} \|A\boldsymbol{\alpha} + \boldsymbol{b}\|_\infty$

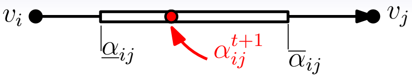
(2) Min partition: (Diff.) $\quad \|A\boldsymbol{\alpha}_2^* + \boldsymbol{b}\|_2 = \min_{\underline{\boldsymbol{\alpha}} \leq \boldsymbol{\alpha} \leq \overline{\boldsymbol{\alpha}}} \|A\boldsymbol{\alpha} + \boldsymbol{b}\|_2$

$$\boldsymbol{\alpha}^* \longleftrightarrow \mathcal{P}* = \{\mathcal{P}_1, \dots, \mathcal{P}_n\}$$

Theorem (Min-max and min partitions)

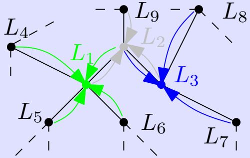
$(2) \Rightarrow (1)$, i.e. $\quad \|A\boldsymbol{\alpha}_2^* + \boldsymbol{b}\|_\infty = \min_{\underline{\boldsymbol{\alpha}} \leq \boldsymbol{\alpha} \leq \overline{\boldsymbol{\alpha}}} \|A\boldsymbol{\alpha} + \boldsymbol{b}\|_\infty$

Optimal partitions via distributed partitioning algorithms (PA)



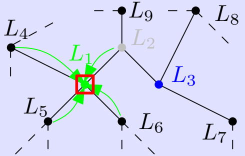
1) Synchronous Gradient PA

- every v_i is selected
- v_i receives α_{jk}^{t+1} from 1-hop neighbors
- $\alpha_{ij}^{t+1} \leftarrow F_{ij}(\alpha^t)$



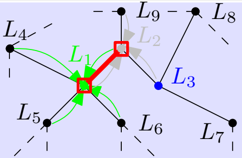
2) Asymmetric Broadcast PA

- select randomly v_i
- v_i receives α_{jk}^{t+1} from 1-hop neighbors
- $\alpha_{ij}^{t+1} \leftarrow F_{ij}(\alpha^t)$



3) Symmetric Gossip PA

- select randomly $\{v_i, v_j\}$
- v_i and v_j receive α_{kl}^{t} from 1-hop neighbors
- $\alpha_{ij}^{t+1} \leftarrow G_{ij}(\alpha^t)$



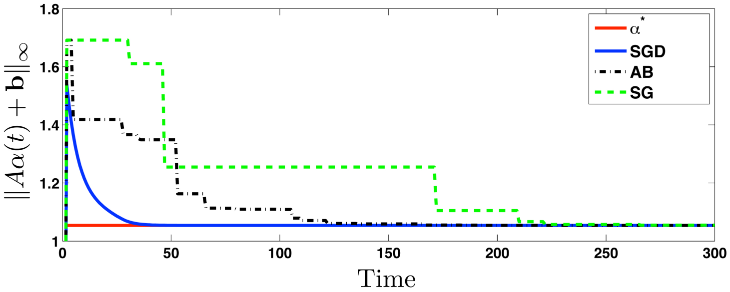
Analytical and numerical results

Theorem (Convergence of algorithms)

- Algorithms SGD, AB, SG asymptotically converge
- estimators are min partitions **(2)**

⇓

- estimators min-max partitions **(1)**



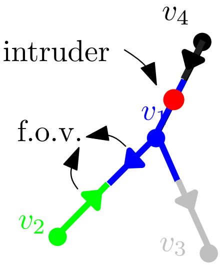
DF-Trajectory and Static/Dynamic intruders

Theorem (DF-Trajectory against STATIC intruders)

Given $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, $\mathcal{V}_{\mathrm{c}}$, and $\mathcal{P}^*$ then

1. $\mathrm{WDT}_{\mathrm{s}}(X^*) = 2 \max_i \left|\mathcal{P}_i^*\right|$
2. $\mathrm{WDT}_{\mathrm{s}}(X^*) \leq 2\,\mathrm{WDT}_{\mathrm{s}}^*$

$\mathcal{G}$ tree or ring, then $\mathrm{WDT}_{\mathrm{s}}(X^*) = \mathrm{WDT}_{\mathrm{s}}^*$



Theorem
(Equivalent condition against DYNAMIC intruders)

$$\exists X \text{ with } \mathrm{WDT}_{\mathrm{d}}(X) < \infty$$

IFF

$\forall v_i \in \mathcal{V}_{\mathrm{c}}$ with $|\mathcal{N}_i| > 2$, $\exists v_j \in \mathcal{N}_i^{\mathrm{in}}$:

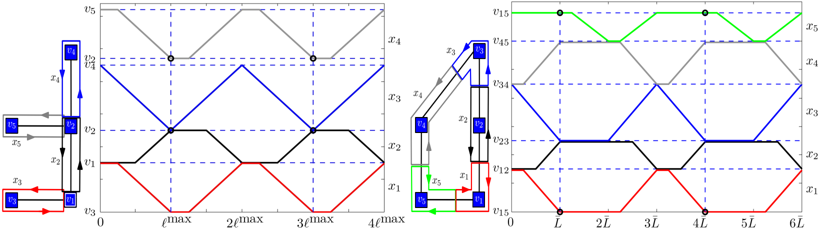
$$\begin{aligned} \underline{\alpha}_{ij} &= 0, \text{ if } i < j \\ \overline{\alpha}_{ij} &= 1, \text{ if } i > j \end{aligned}$$

Camera trajectories for dynamic intruders

Theorem (Tree-Sync-Trajectory X^s)

Given $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ tree, $\mathcal{V}_\mathrm{c}$ and $\underline{\alpha} = \mathbf{0}$, $\overline{\alpha} = \mathbf{1}$, then

$$\mathrm{WDT}_\mathrm{d}(X^s) = \mathrm{WDT}_\mathrm{s}(X^s) \leq 2\mathrm{WDT}^*_\mathrm{d}.$$



Theorem (Ring-Sync-Trajectory X^s)

Given $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ ring, $\mathcal{V}_\mathrm{c}$, then

1. if n is even, $\mathrm{WDT}_\mathrm{d}(X^s) = \mathrm{WDT}_\mathrm{s}(X^s) = \mathrm{WDT}^*_\mathrm{d}$;
2. if n is odd, $\mathrm{WDT}_\mathrm{d}(X^s) = \mathrm{WDT}_\mathrm{s}(X^s) \leq \frac{3}{2}\mathrm{WDT}^*_\mathrm{d}$.

Outline

Future research

Calibration 2D

- distributed algorithm to construct minimal cycles
- probabilistic characterization of the estimate $\hat{K}^F$, $\hat{K}^M$ (MAP, ML)
- coordinated broadcast vs gossip

Calibration 3D

- noise modeling
- identification of convex regions for V defined over $SO(3)$
- coordinated broadcast vs gossip

Graph partitioning problem

- optimal trajectories for general graphs (static case)
- optimal synchronized trajectories (dynamic case)

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Thank you for the attention!

Questions?