



THE AUSTRALIAN NATIONAL UNIVERSITY

# Bias Correction in Localization Problem

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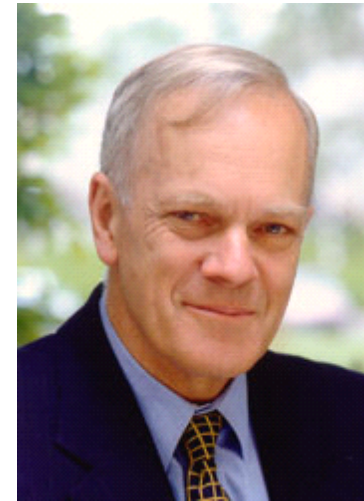
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## Collaborators



**Dr. Changbin (Brad) Yu**



**Professor Brian D. O. Anderson**

Assistance of Dr. Sam Drake of Australian Defence Science and Technology Organization (DSTO) with original problem formulation and provision of trial data is gratefully acknowledged

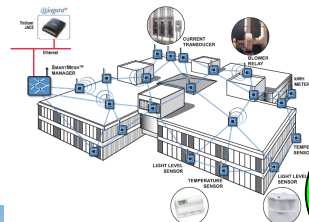
# Outline

- **Motivation**
- Bias in Localization Problem
- Taylor-Jacobian Bias Correction Method
- Performance Evaluation and Simulation
- Conclusion

# Motivation

**Industry**

- *Process control*
- *Automation*
- *Predictive maintenance*



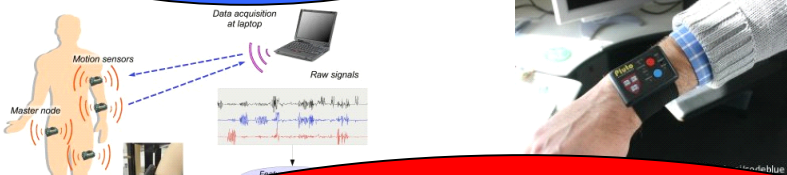
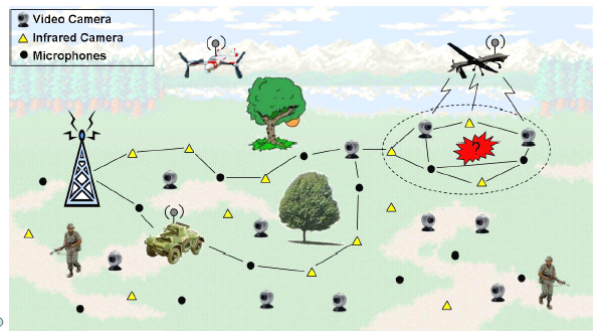
**Scientific Research**

- *High spatial and temporal density sampling*
- *Habitat monitoring*
- *Event detection*



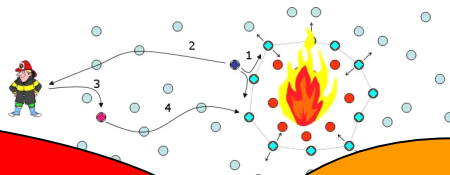
**Health Care**

- *Location aware patient monitoring*
- *Patient vital signals*



**Disaster Management**

- *Event detection (natural disasters – fire, earthquake)*
- *Location awareness (fire fighters looking for survivors)*
- *Emergency response*



**Military**

- *Battlefield surveillance*
- *Target tracking*



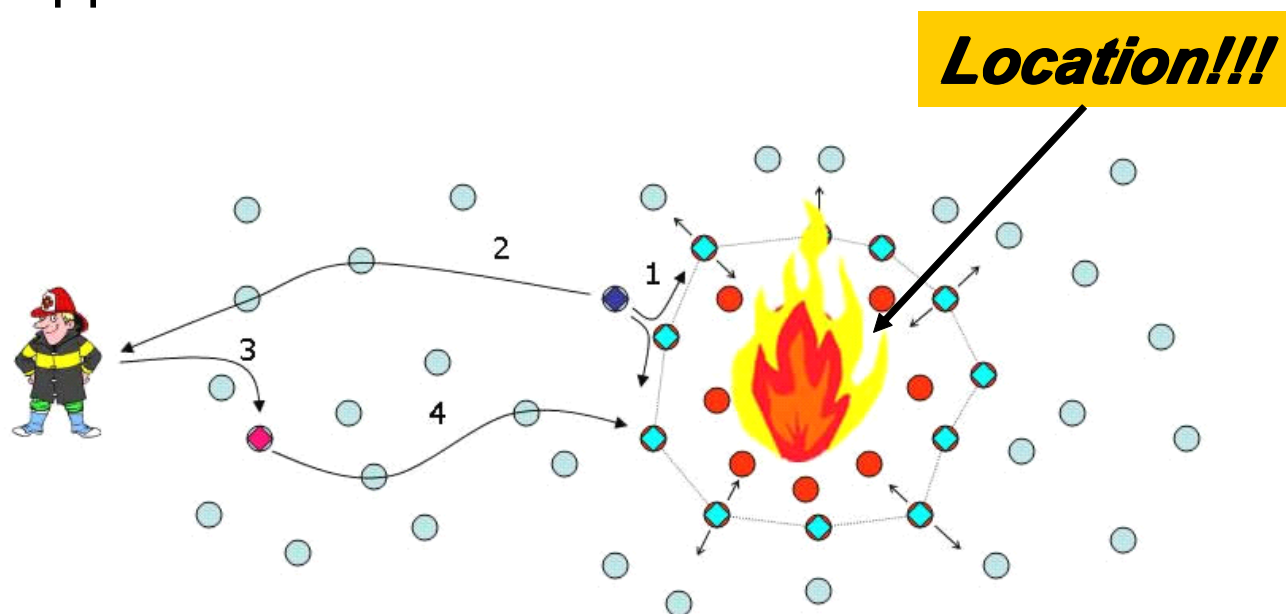
# Motivation

- Accurate location of sensors plays a vital role in various network applications



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## Motivation

- Many self-localization algorithms are proposed
- Generally, localization results are imprecise
  - Environment: noise, non line of sight
  - Hardware: range or angle measuring devices
  - Localization algorithms
- Many enhancement techniques have been proposed to improve the accuracy of localization
  - Geometric Constraints
  - Error Control Mechanisms (Baoqi Huang)
  - **Bias Correction Methods**

## Motivation

- Many self-localization algorithms are proposed
- Generally, localization results are imprecise
  - Environment: noise, non line of sight
  - Hardware **Why we choose bias?**
  - Localization algorithms
- Many enhanced techniques have been proposed to improve the accuracy of localization
  - Geometric Constraints
  - Error Control Mechanisms
  - **Bias Correction Methods**



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## What is Bias

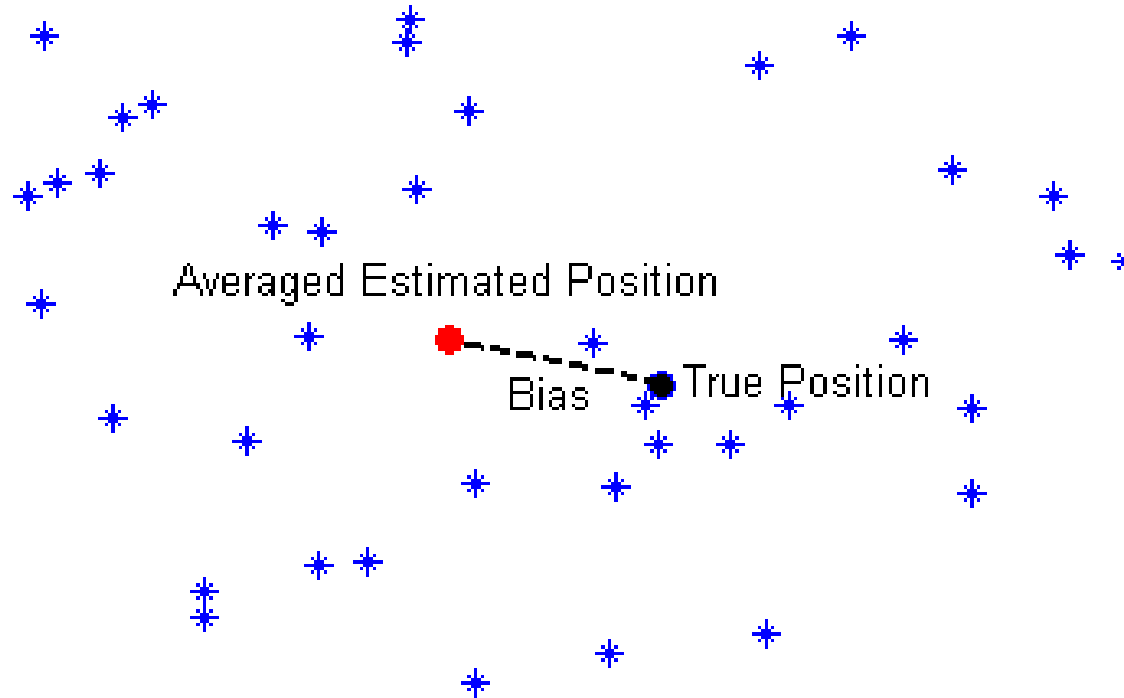
Bias is a term in estimation theory and is defined as the difference between the expected value of a parameter estimate and the true value of the parameter [1].

[1] J. L. Melsa and D. L. Cohn. Decision and Estimation Theory. McGraw-Hill Inc, 1978.

# What is Bias

Bias is a  
the difference  
parameter  
parameter

defined as  
of a



[1] J. L. Melsa

Inc, 1978.

## Bias in Localization Problem

In the noisy situation, we assume  $\mathbf{g} = (g_1, g_2, \dots, g_n)$  denotes the localization mapping from the measurements to the target position estimates. We have:

$$\tilde{\mathbf{x}} = \mathbf{g}(\Theta + \delta\Theta) = \mathbf{g}(\tilde{\Theta})$$

where  $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)^T$  denotes the inaccurate estimates of the target location,  $\tilde{\Theta} = (\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_N)$  denotes the noisy measurements and  $\delta\Theta = (\delta\theta_1, \delta\theta_2, \dots, \delta\theta_N)^T$  denotes the measurement noise.

## Bias in Localization Problem

In practice the measurement process will be repeated  $M$  times.  
As  $M \rightarrow \infty$ , we would expect the estimate to go to :

$$E[\tilde{x}_i] = E[g_i(\tilde{\Theta})]$$

Because  $g_i$  is nonlinear we have:

$$\begin{aligned} E[\tilde{x}_i] &= E[g_i(\tilde{\Theta})] \\ &\neq g_i(E[\tilde{\Theta}]) \\ &= g_i(\Theta) \\ &= x_i \end{aligned}$$

Therefore the bias appears in the estimation process:

$$Bias_{x_i} = E[\tilde{x}_i] - x_i \quad i = 1, 2, \dots, n$$

## Bias in Localization Problem

In practice the measurement process will be repeated  $M$  times.  
As  $M \rightarrow \infty$ , we would expect the estimate to go to :

The bias will exist if two conditions are satisfied:

Because

1. the mapping function is nonlinear
2. the measurements are noisy

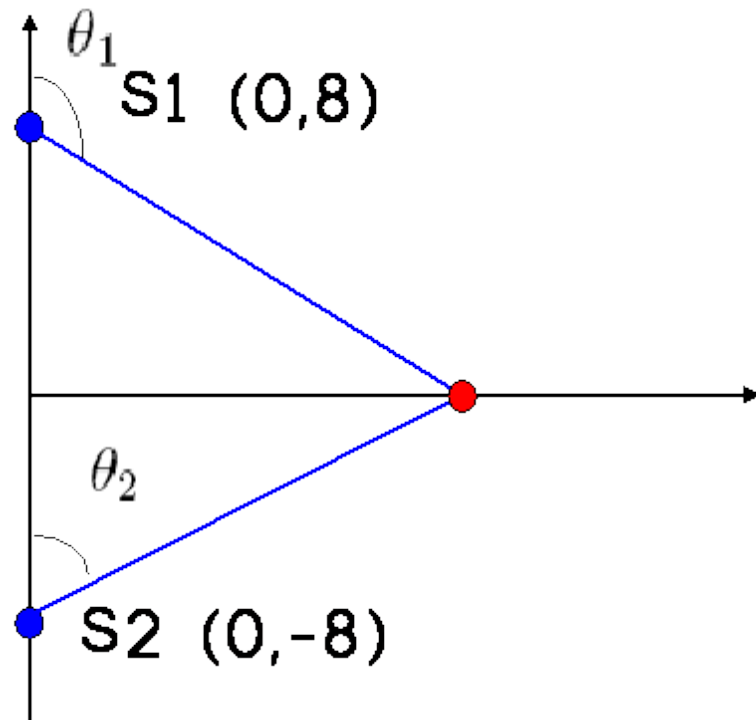
$$\begin{aligned} &= g_i(\Theta) \\ &= x_i \end{aligned}$$

Therefore the bias appears in the estimation process:

$$Bias_{x_i} = E[\tilde{x}_i] - x_i \quad i = 1, 2, \dots, n$$

## Significant Bias in Localization Problem

- Two sensors at  $(0, 8)$  and  $(0, -8)$
- $y$  value of the target is fixed at 0 while  $x$  value changes from 6 to 20
- Measurements are bearing-only
- Different variances used for measurement errors, which are zero mean.



# Significant Bias in Localization Problem

- Table 1 and 2 illustrate the bias of the x component compared to the standard deviation of the error in estimating x with different level of noise

Value of x	6	8	10	12
Percentage (%)	16.48	9.57	6.64	6.17
Value of x	14	16	18	20
Percentage (%)	6.52	7.87	8.51	9.72

TABLE I

THE COMPARISON OF THE BIAS AND THE STANDARD DEVIATION OF X COMPONENT ( $\sigma^2 = 1$ )

Value of x	6	8	10	12
Percentage (%)	20.92	18.3	14.14	12.5
Value of x	14	16	18	20
Percentage (%)	13.75	15.23	17.24	18.91

TABLE II

THE COMPARISON OF THE BIAS AND THE STANDARD DEVIATION OF X COMPONENT ( $\sigma^2 = 2$ )

Bias (before removal) can be a significant fraction of the errors.



## Significant Bias in Localization Problem

Normally the two conditions are satisfied easily:

- the mapping function is nonlinear
- the measurements are noisy

The bias can be a significant fraction of the error.

It is worth to analyse and remove the bias in localization.

# Outline

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# Taylor-Jacobian Bias Correction Method

Notations and Assumptions:

1.  $n$  denotes the number of dimensions of the ambient space
2.  $N$  denotes the number of obtained measurements
3.  $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$  denotes the position of the target
4.  $\Theta = (\theta_1, \theta_2, \dots, \theta_N)^T$  denotes the measurement set
5.  $\delta\Theta = (\delta\theta_1, \delta\theta_2, \dots, \delta\theta_N)^T$  denotes measurement errors
6.  $\mathbf{f} = (f_1, f_2, \dots, f_N)^T$  is the mapping from the target position to the measurements
7.  $\mathbf{g} = (g_1, g_2, \dots, g_n)^T$  is the localization mapping from the measurements to the target position

## Formulation of the bias

In the noisy case, errors in measurements are inevitable. Therefore the localization problem can be formulated as follows:

$$\mathbf{x} + \delta\mathbf{x} = \mathbf{g}(\Theta + \delta\Theta)$$

Next **a Taylor series** is used to expand the above equation truncating at second order:

$$\begin{aligned}x_i + \delta x_i &= g_i(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_N) \\&= g_i(\theta_1 + \delta\theta_1, \theta_2 + \delta\theta_2, \dots, \theta_N + \delta\theta_N) \\&\approx g_i(\theta_1, \theta_2, \dots, \theta_N) + \sum_{j=1}^N \frac{\partial g_i}{\partial \theta_j} \delta\theta_j \\&\quad + \frac{1}{2!} \sum_{j=1}^N \sum_{l=1}^N \delta\theta_j \delta\theta_l \frac{\partial^2 g_i}{\partial \theta_j \partial \theta_l}\end{aligned}$$

## Formulation of the bias

In the The approximate bias expression is immediate:  
 Therefore  
 follows

$$E(\delta x_i) = \frac{1}{2!} \sum_{j=1}^N \sigma_j^2 \frac{\partial^2 g_i}{\partial \theta_j^2}$$

Next *a Taylor series* is used to expand the above equation truncating at second order:

$$\begin{aligned} x_i + \delta x_i &= g_i(\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_N) \\ &= g_i(\theta_1 + \delta\theta_1, \theta_2 + \delta\theta_2, \dots, \theta_N + \delta\theta_N) \\ &\approx g_i(\theta_1, \theta_2, \dots, \theta_N) + \sum_{j=1}^N \frac{\partial g_i}{\partial \theta_j} \delta\theta_j \\ &\quad + \frac{1}{2!} \sum_{j=1}^N \sum_{l=1}^N \delta\theta_j \delta\theta_l \frac{\partial^2 g_i}{\partial \theta_j \partial \theta_l} \end{aligned}$$

## Formulation of the bias

However it is very difficult to compute the localization mapping  $\mathbf{g} = (g_1, g_2, \dots, g_n)^T$  and its derivatives.

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How to analytically express the bias in an easy way?

## Formulation of the bias

However it is very difficult to compute the localization mapping  $\mathbf{g} = (g_1, g_2, \dots, g_n)^T$  and its derivatives. **In contrast**  $\mathbf{f} = (f_1, f_2, \dots, f_N)^T$  **can be easily written down!**

The approximate bias expression is immediate:

$$E(\delta x_i) = \frac{1}{2!} \sum_{j=1}^N \sigma_j^2 \frac{\partial^2 g_i}{\partial \theta_j^2}$$



# Different Localization Techniques

## Range Measurements

$$d_1 = f_1(x, y) = \sqrt{(x - x_1)^2 + (y - y_1)^2}$$

$$d_2 = f_2(x, y) = \sqrt{(x - x_2)^2 + (y - y_2)^2}$$

## Bearing-Only Measurements

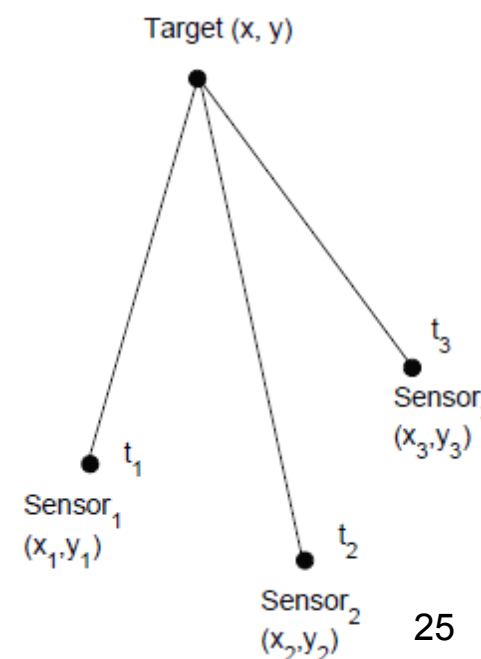
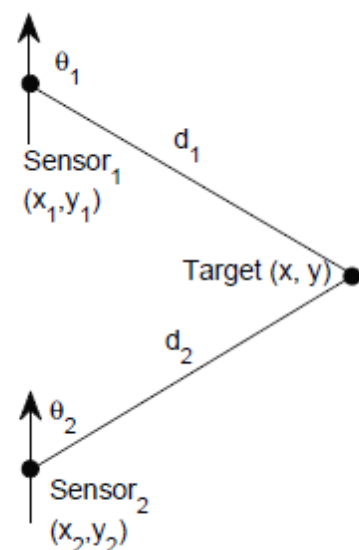
$$\theta_1 = f_1(x, y) = \pi + \text{actan}\left(\frac{x - x_1}{y - y_1}\right) \pmod{2\pi}$$

$$\theta_2 = f_2(x, y) = \text{actan}\left(\frac{x - x_2}{y - y_2}\right) \pmod{2\pi}$$

## TDOA Measurements

$$(t_2 - t_1) \times c = f_1(x, y) = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$

$$(t_3 - t_1) \times c = f_2(x, y) = \sqrt{(x_3 - x)^2 + (y_3 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$



# Different Localization Techniques

## Range Measurements

$$d_1 = f_1(x, y)$$

$$d_2 = f_2(x, y)$$

How to analytically express the bias in an easy way?

## Bearing-Only Measurements

$$\theta_1 = f_1(x, y) = \pi + \text{actan}\left(\frac{x - x_1}{y - y_1}\right) \pmod{2\pi}$$

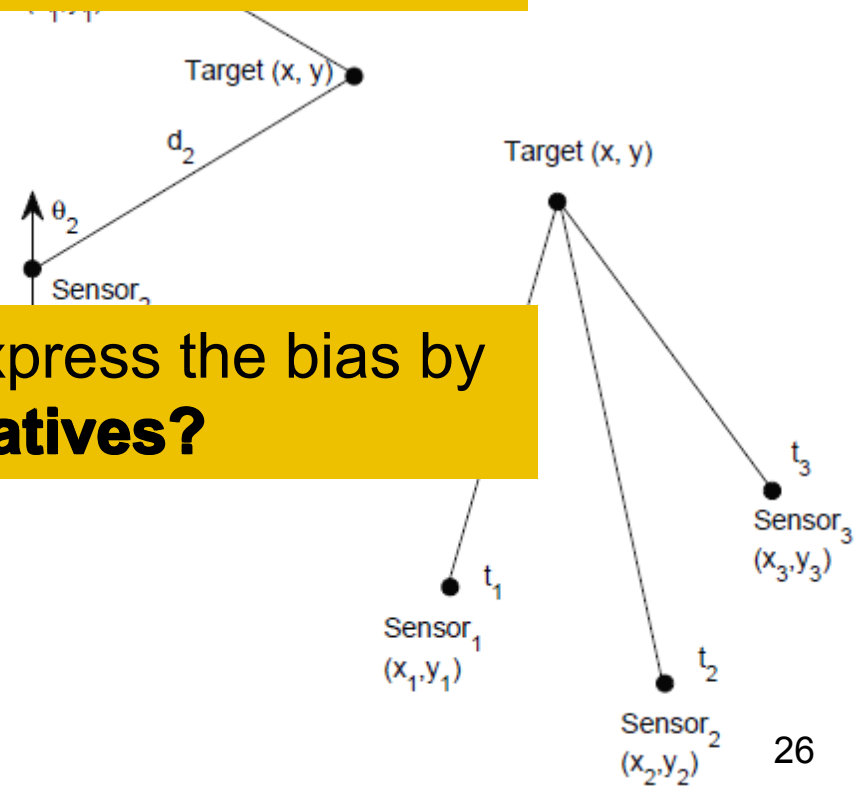
$$\theta_2 = f_2(x, y)$$

How to analytically express the bias by using ***f* and its derivatives?**

## TDOA Measurements

$$(t_2 - t_1) \times c = f_1(x, y) = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$

$$(t_3 - t_1) \times c = f_2(x, y) = \sqrt{(x_3 - x)^2 + (y_3 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$



## Taylor-Jacobian Bias Correction Method

**Jacobian matrix and one of its property** are used to calculate the derivatives of  $g$  in terms of the derivatives of  $f$ .

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial f_N}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial \theta_1} & \cdots & \frac{\partial g_1}{\partial \theta_N} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial g_n}{\partial \theta_1} & \cdots & \frac{\partial g_n}{\partial \theta_N} \end{bmatrix} = I_n$$

By solving the above equation set, we can obtain the analytical expression for

$$\frac{\partial g_i}{\partial \theta_j} \quad (i = 1, 2, \dots, n; j = 1, 2, \dots, N)$$

## Taylor-Jacobian Bias Correction Method

Here we take  $\frac{\partial g_1}{\partial \theta_1}$  for example. Assume  $\frac{\partial g_1}{\partial \theta_1} = g_{\theta_1}^1$ , differentiating the equation in respect to  $x_1, x_2, \dots, x_n$  respectively we can obtain the following equation set:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_1} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial f_1}{\partial x_n} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 g_1}{\partial \theta_1^2} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial^2 g_1}{\partial \theta_1 \partial \theta_N} \end{bmatrix} = \begin{bmatrix} \frac{\partial g_{\theta_1}^1}{\partial x_1} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial g_{\theta_1}^1}{\partial x_n} \end{bmatrix}$$

# Taylor-Jacobian Bias Correction Method

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$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial f_1}{\partial x_n} \end{bmatrix} \dots \begin{bmatrix} \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 g_1}{\partial \theta_1 \partial \theta_N} \end{bmatrix} \begin{bmatrix} \frac{\partial g_{\theta_1}^1}{\partial x_1} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial g_{\theta_1}^1}{\partial x_n} \end{bmatrix}$$

$$E(\delta x_i) = \frac{1}{2!} \sum_{j=1}^N \sigma_j^2 \frac{\partial^2 g_i}{\partial \theta_j^2}$$

Can be easily expressed analytically

# Taylor-Jacobian Bias Correction Method

How to analytically express the bias in an easy way?

**Solved!**

1. Taylor series
2. Jacobian matrix and its property

So we call the proposed method as ***Taylor-Jacobian*** bias correction method.

# Overdetermined Problem

## *Jacobian matrix and one of its property*

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial f_N}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial \theta_1} & \cdots & \frac{\partial g_1}{\partial \theta_N} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial g_n}{\partial \theta_1} & \cdots & \frac{\partial g_n}{\partial \theta_N} \end{bmatrix} = I_n$$

Important assumption:  **$N=n$**

# Overdetermined Problem

## *Jacobian matrix and one of its property*

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial f_N}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial \theta_1} & \cdots & \frac{\partial g_1}{\partial \theta_N} \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ \frac{\partial g_n}{\partial \theta_1} & \cdots & \frac{\partial g_n}{\partial \theta_N} \end{bmatrix} = I_n$$

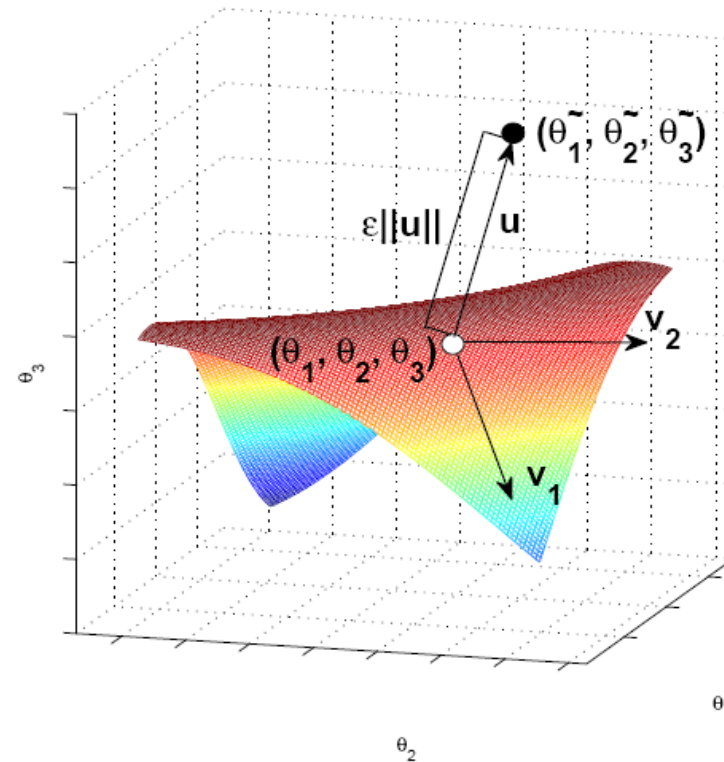
Important assumption:  **$N > n$**



# Overdetermined Problem ( $N=n+1$ )

Least squares method:

$$F_{\text{cost-function}}(\mathbf{x}, \tilde{\Theta}) = \sum_{i=1}^N (f_i - \tilde{\theta}_i)^2 = \sum_{i=1}^N \delta\theta_i^2$$



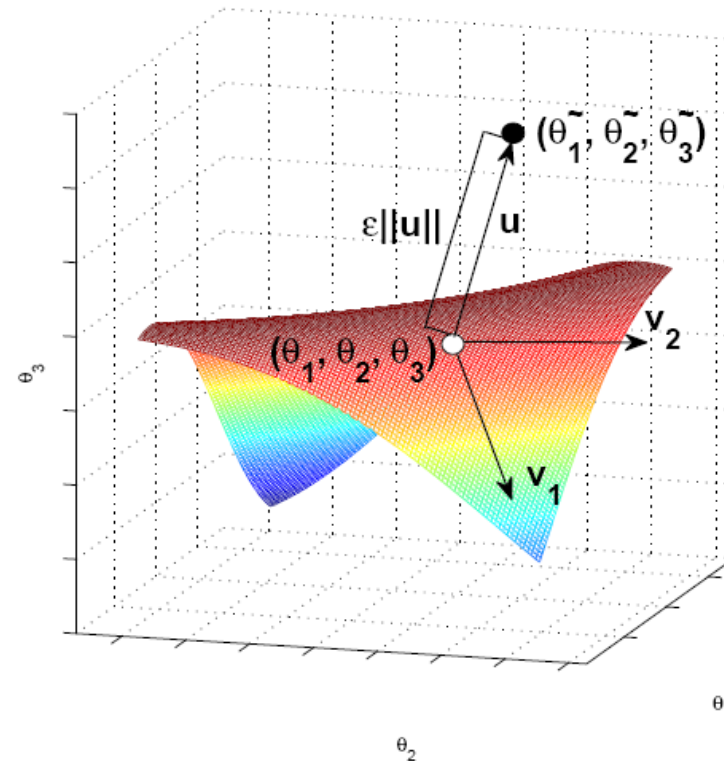
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Minimize the distance:

$$D_{\min} = \sqrt{\sum_{i=1}^N \delta\theta_i^2} = \varepsilon \|\mathbf{u}\|$$



# Overdetermined Problem ( $N=n+1$ )

Least squares method:

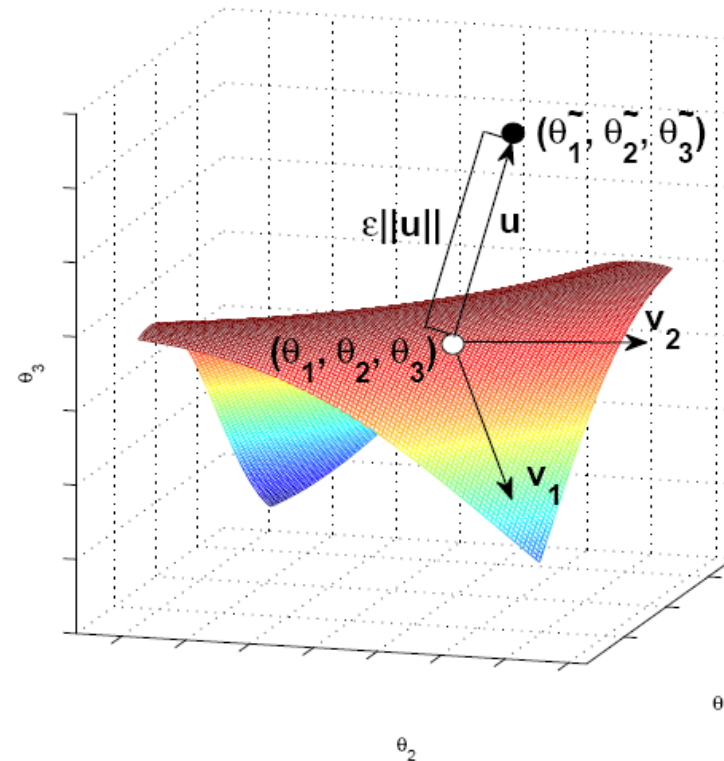
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Minimize the distance:

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For the white point:

$$\mathbf{v}_i = \left[ \frac{\partial f_1}{\partial x_i}, \frac{\partial f_2}{\partial x_i}, \dots, \frac{\partial f_N}{\partial x_i} \right]^T \quad i = 1, 2, \dots, n$$



# Overdetermined Problem ( $N=n+1$ )

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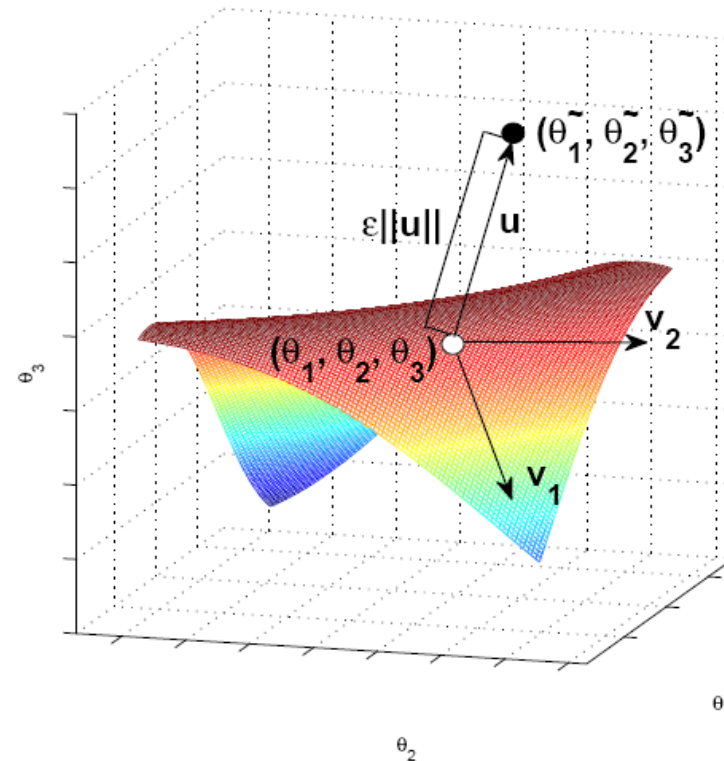
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The normal vector:

$$\mathbf{u} = [u_1, u_2, \dots, u_N]^T = \mathbf{v}_1 \times \mathbf{v}_2 \dots \times \mathbf{v}_n$$



# Overdetermined Problem ( $N=n+1$ )

Least squares method:

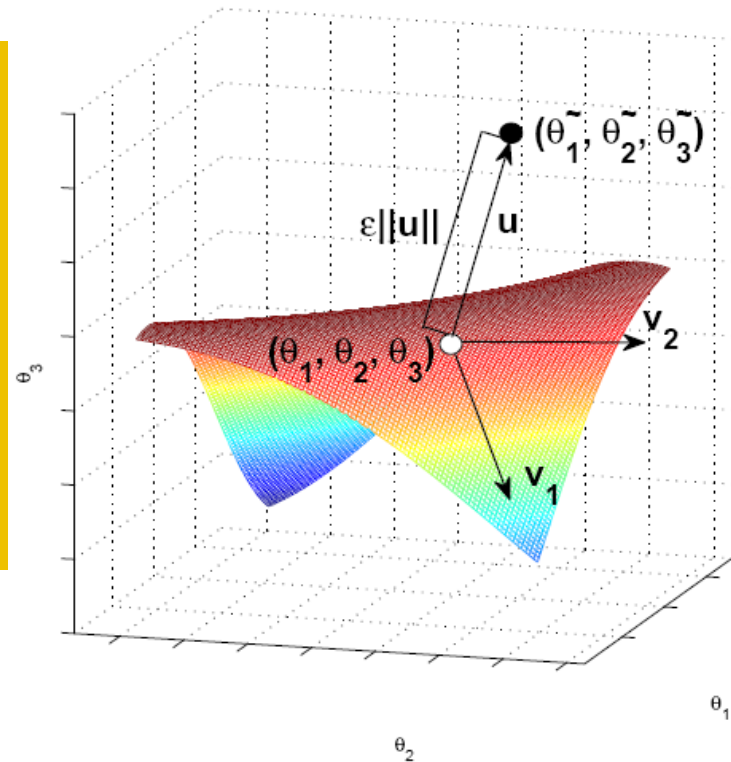
Finally we can obtain a new mapping:

$$\tilde{\Theta} = \mathbf{F}(\tilde{\mathbf{x}}, \varepsilon) = \mathbf{f}(\tilde{\mathbf{x}}) + \varepsilon \mathbf{u}$$

$$\mathbf{v}_i = \left[ \frac{\partial f_1}{\partial x_i}, \frac{\partial f_2}{\partial x_i}, \dots, \frac{\partial f_N}{\partial x_i} \right]^T \quad i = 1, 2, \dots, n$$

The normal vector:

$$\mathbf{u} = [u_1, u_2, \dots, u_N]^T = \mathbf{v}_1 \times \mathbf{v}_2 \dots \times \mathbf{v}_n$$



## Overdetermined Problem ( $N > n+1$ )

With  $N > n+1$ , the situation is similar to  $N = n+1$  case except that the extra variable is no longer a scalar. Instead, it is a vector which can be defined as follows:

$$\varepsilon = [e_1, e_2, \dots, e_{N-n}]^T$$

Where  $e_i$  ( $i = 1, 2, \dots, N-n$ ) denotes a coefficient to minimize the moved distance in each dimension of the normal.

# Overdetermined Problem ( $N > n + 1$ )

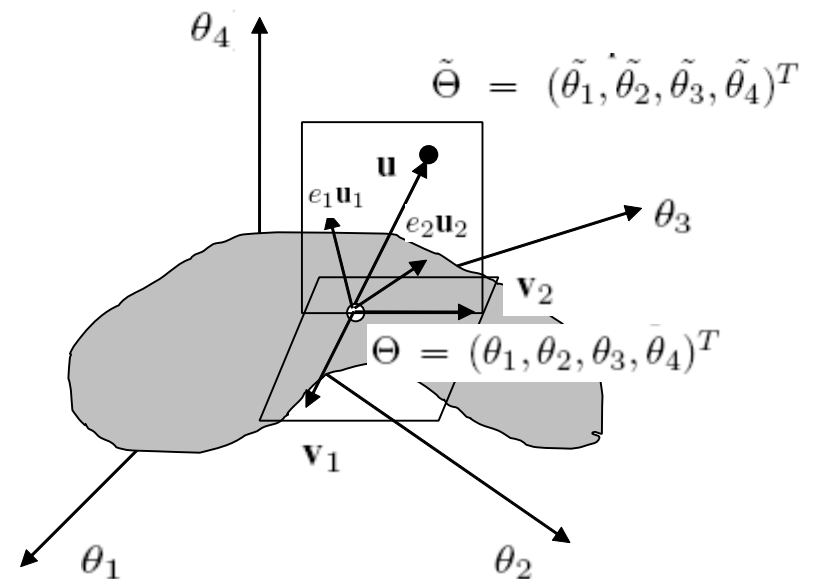
Assume  $N = 4$  and  $n = 2$ .

At the white point we can have:

$$\mathbf{v}_1 = \left[ \frac{\partial f_1}{\partial x_1}, \frac{\partial f_2}{\partial x_1}, \frac{\partial f_3}{\partial x_1}, \frac{\partial f_4}{\partial x_1} \right]^T$$

$$\mathbf{v}_2 = \left[ \frac{\partial f_1}{\partial x_2}, \frac{\partial f_2}{\partial x_2}, \frac{\partial f_3}{\partial x_2}, \frac{\partial f_4}{\partial x_2} \right]^T$$

These two tangent vectors define a tangent plane  $\mathbf{P}_{tangent}$



## Contributions and key mathematic tools of our work

### Contributions:

1. Express the bias in an easy way by using the function  $f$  (mapping from the target position to the measurements) and its derivatives
2. Adopt a method based on least-squares idea to solve the overdetermined problem

### Mathematic tools:

1. Taylor series
2. Jacobian Matrix



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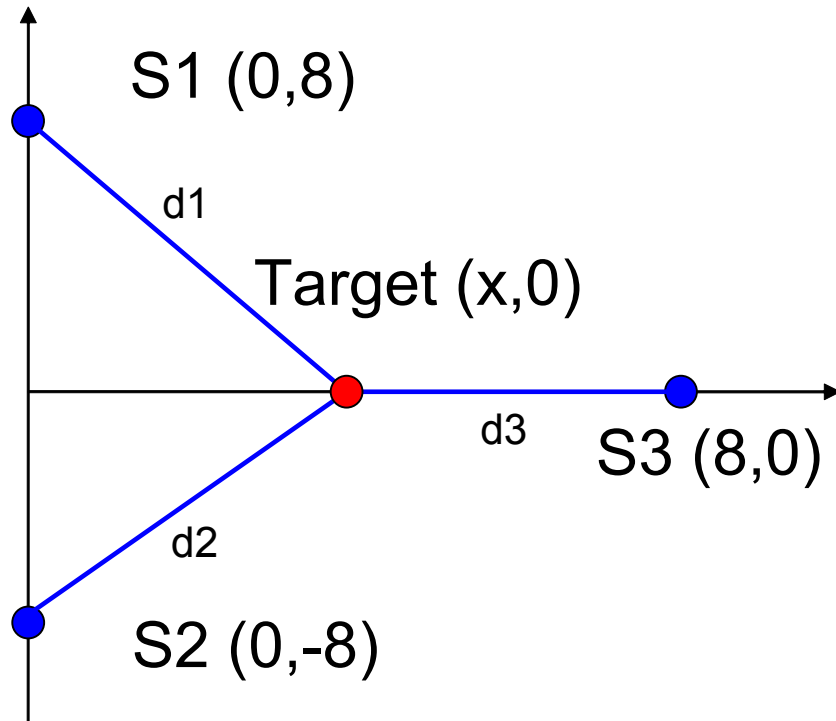
# Simulation

## Simulation Assumption

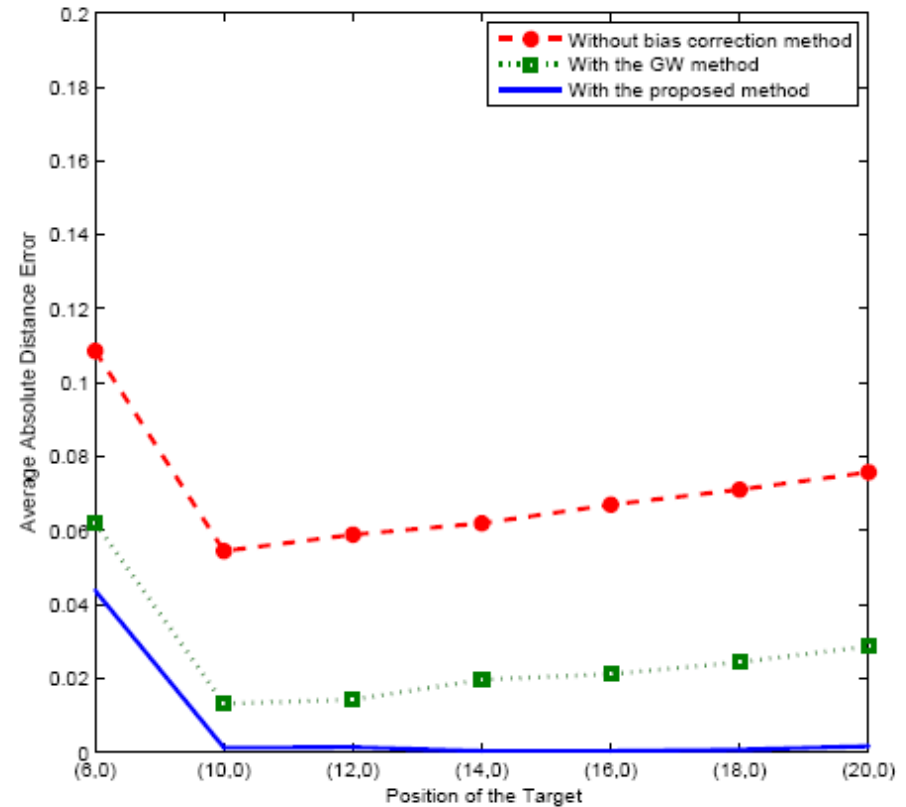
- All simulations are done in two-dimensional space
- The three sensors are fixed at  $(0, 8)$ ,  $(0, -8)$  and  $(8, 0)$
- The measurement noise for three sensors are produced by i. i. d. Gaussian with zero mean and variance  $\sigma^2 = 1$ .
- All the simulation results are obtained from 5000 Monte Carlo experiments.
- We compare our method with an well-cited bias-correction method GW method [1]

[1] M. Gavish and A. J. Weiss. Performance analysis of bearing-only target location algorithms. IEEE Transaction on Aerospace and Electronic Systems, 28(3): 817-827, 1992.

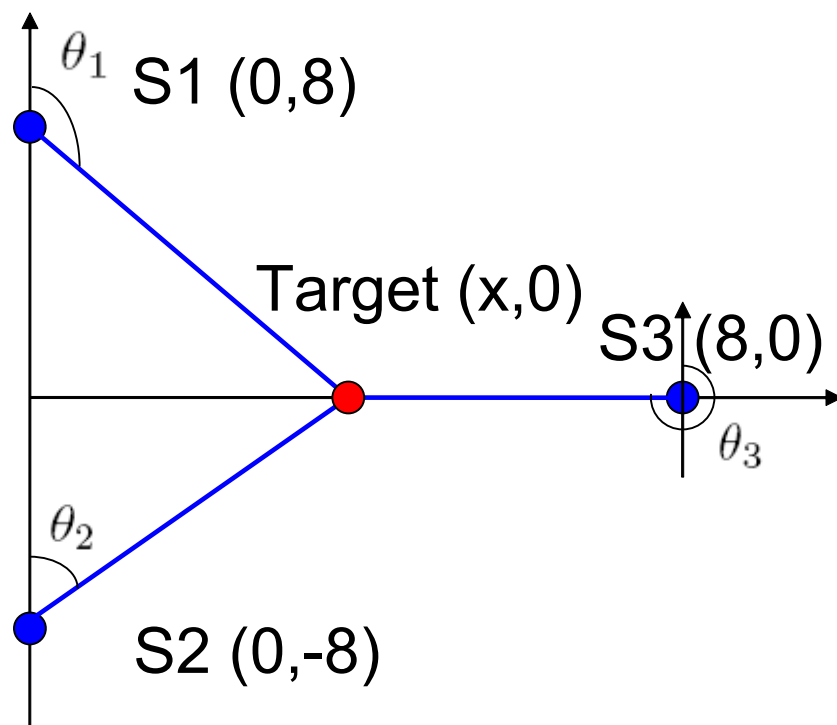
# Simulation Results - Range Measurement



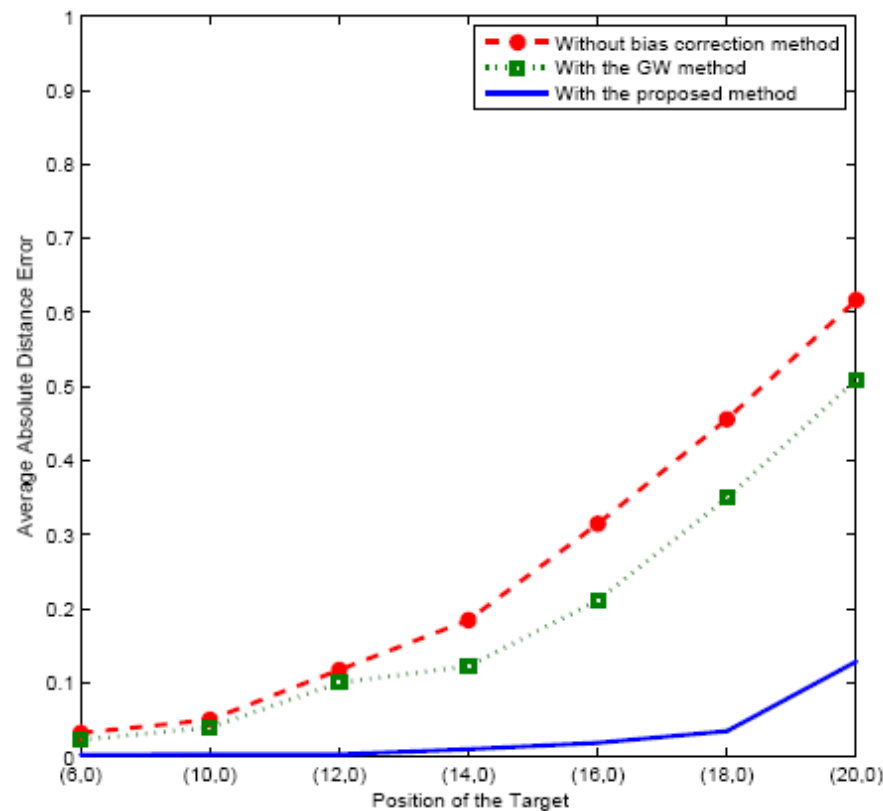
- Three sensors and a single target
- Range measurements only
- Measurement errors are  $\mathcal{N}(0,1)$
- The y value of the target is fixed at 0; x value is adjusted from 6 to 20



## Simulation Results – Bearing-Only Measurement

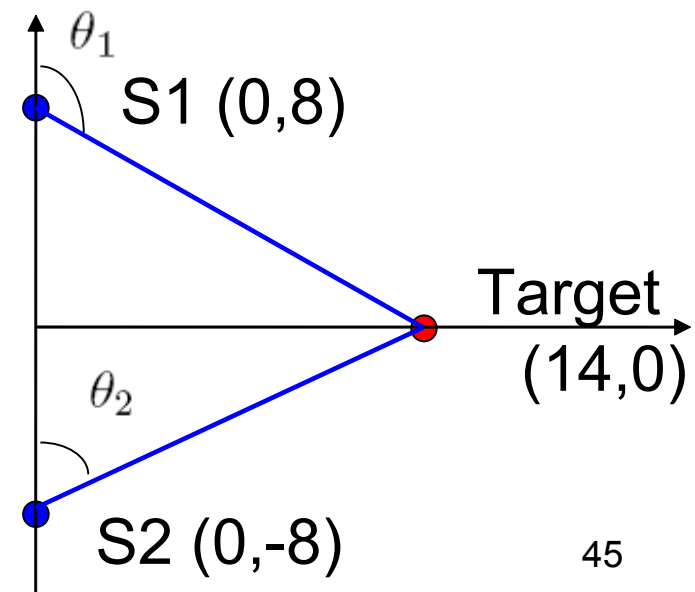


- Three sensors and a single target
- Bearing-only measurements
- Measurement errors are  $\mathcal{N}(0,1)$
- The y value of the target is fixed at 0; x value is adjusted from 6 to 20



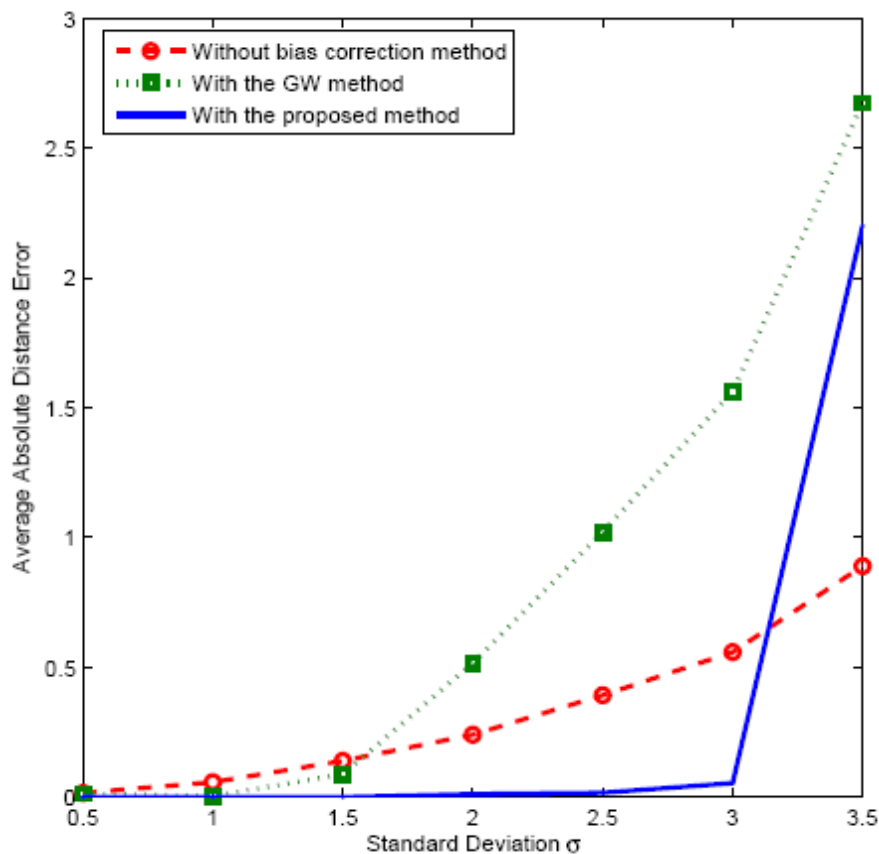
## Simulation Results - Different Level of Noise

- Truncation of Taylor series is not necessarily justified when the noise is large
- Noise level is adjusted over a large range via changing the standard deviation of measurement errors, from 0.5 to 3.5 in steps of 0.5
- Bearing-only measurements are used

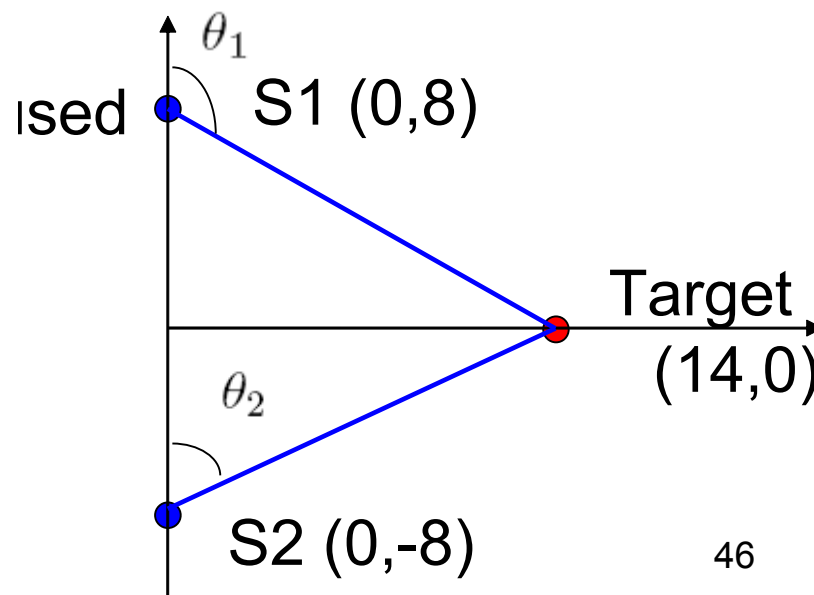


# Simulation Results - Different Level of Noise

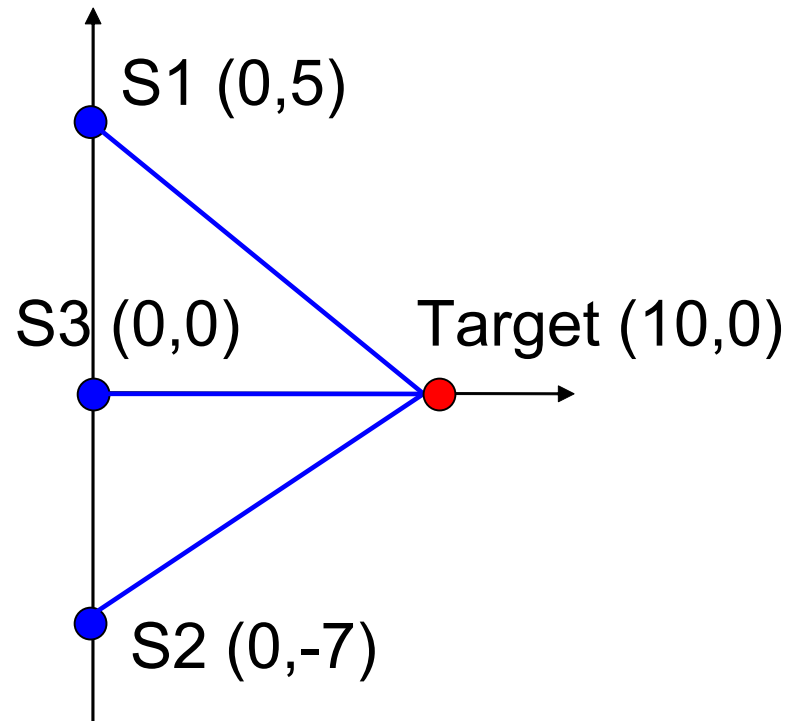
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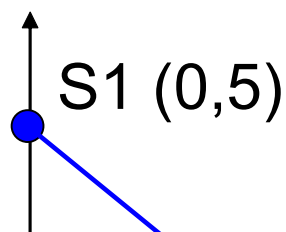


## Trial Data – Scan-based Measurement



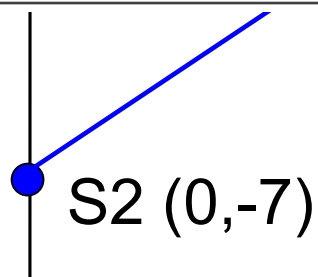
- Three physical sensors and a single target, which is a radar with a mechanically rotating antenna.
- Two usable sensor measurements are obtained.
- Noise in measurements is  $N(0,0.02)$

# Trial Data – Scan-based Measurement



- Three physical sensors and a single target, which is a radar

Localization errors without bias-correction method	Localization errors with bias-correction method
0.6610	0.0785
0.3435	0.0462
0.3454	0.0465



- measurements are obtained.
- Noise in measurements is  $N(0,0.02)$



## Performance of the Taylor-Jacobian Method

1. The Taylor-Jacobian method is generic
  - Range Measurement
  - Bearing-only Measurement
  - Scan-based Measurement
- The performance of the Taylor-Jacobian method is better than the GW method
- The Taylor-Jacobian method can be more robust to the level of noise than the GW method

# Outline

- Motivation
- Bias in Localization Problem
- Taylor-Jacobian Bias Correction Method
- Performance Evaluation and Simulation
- Conclusion

## Conclusion

- Bias arises due to simultaneous presence of noise and nonlinear transformations.
- In localization, the map need for computing the bias may not be analytically available; its inverse is available so the bias computation needs to be varied
- A generic Taylor-Jacobian bias correction method is proposed
- The simulation results demonstrate the performance of the proposed method

# Thank you!

## Publications:

- [1] Y. Ji, C. Yu and B. D. O. Anderson. Bias correction in localization algorithms. IEEE Global Communication Conference, pp. 1-7, 2009.
- [2] Y. Ji, C. Yu and B. D. O. Anderson. Geometric dilution of localization and bias-correction methods. International Conference on Control & Automation, pp. 578-583, 2010.
- [3] Y. Ji, C. Yu and B. D. O. Anderson. Localization bias correction in n-dimensional space. IEEE International Conference on Acoustics Speech and Signal Processing, pp. 2854-2857, 2010.
- [4] Y. Ji, C. Yu and B. D. O. Anderson. Bias-correction method in bearing-only passive localization. European Signal Processing Conference, Published, 2010.
- [5] Y. Ji, C. Yu and B. D. O. Anderson. Localization bias correction in n-dimensional space. Submitted to IEEE Transaction on Aerospace and Electronic Systems.

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