Simulation and Multitarget Tracking

Corso di Progettazione di Sitemi di Controllo — A.A. 2009/2010

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Outline

1. Introduction
   - Goals and Beyond
   - Results
2. Literature and State of the Art
   - Bayesian Approach
   - non-Bayesian Approach
3. Camera Simulator
   - Camera Model
   - Possible Improvements
4. Markov Chain Monte Carlo Data Association
   - Model Formulation
   - Algorithm
   - Possible Improvements
   - Simulation Results
5. Stable Marriage Problem
   - Definition & Algorithm
   - Application to Data Association
   - Simulation Results
   - Possible Improvements
6. Summary

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Applications

Civilian Area

- Surveillance-related systems
- Computer Vision (motion capture)
- Network and computer security (process query systems)
- Sensor networks (coordination of multiple agents)

Military Area

- Ballistic missile defense (reentry vehicles)
- Air defense (enemy aircraft)
- Ocean surveillance (surface ships and submarines)
- Battlefield surveillance (ground vehicles and military units)

Multi-Target Tracking

Purpose

- Accurate data association
- Real-time computation
- Robust to false alarms and corrupted or missing reports

Possible Improvements

Recognize and take into account shape constraints

Simulator

Purpose

- Generate synthetic data
- Introduce various types of distortion

Possible Improvements

Implement an hidden point feature

Implementations

Simulator

A routine based on pin-hole model camera

Multiple-Target Tracking

- MCMCDA algorithm
- SMP algorithm

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Joint Probabilistic Data Association

PDA extension to a known number of targets in clutter.
The derivation of joints events is done using Bayes' rule

$$P[\theta(k)|Y^k] = P[\theta(k)|Y(k), Y^{k-1}] = \frac{1}{c}P[Y(k)|\theta(k), Y^{k-1}]P[\theta(k)]$$

Advantages and Drawbacks

- **PROS** single-scan, optimal solution
- **CONS** known number of targets, marginal association by summing over all the joint events

Track-Splitting Filter

Algorithm Procedure

1. Measurement Validation
2. Track Splitting
3. Kalman Filtering
4. Hypothesis Testing

The Hypothesis Testing

The modified log-likelihood can be computed recursively

$$\lambda(k) = \lambda(k-1) + e^T(k)S^{-1}(k)e(k),$$

and is a chi-square with kn_y degree of freedom.
Thus the statistical hypothesis test is

$$P[\chi^2_{kn_y} > \delta] = \alpha \quad \text{(true track rejection probability)}$$

Multi Hypothesis Tracker

Features

- Multiple-scan correlation
- Clustering
- Recursiveness

Algorithm Subroutines

CLUSTER Form new clusters, identify which targets and measurements are associated with each cluster

HYPGEN Form new set of hypothesis, calculate their probability and perform a target measurement update for each hypothesis of each cluster

REDUCE Reduce number of hypothesis by elimination or combination

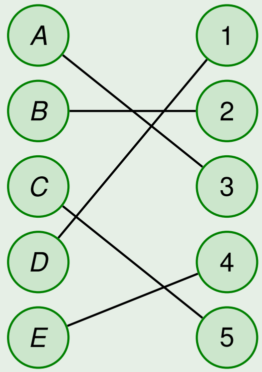
Linear Assignment Problem

Assignment problems deal with the question how to assign *n* items (e.g. jobs) to *n* machines (or workers) in the best possible way.

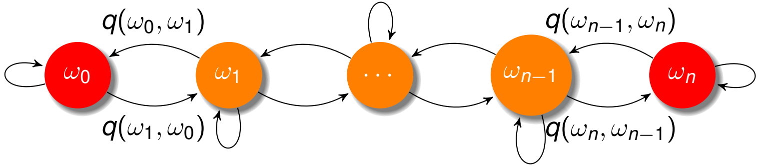
The problem may then be reformulated as an integer programming problem or, more precisely, as a multidimensional assignment problem.

THE GOAL is to make an hard measurement-to-track ASSOCIATION.

Representation



Monte Carlo Markov Chain Data Association



Single Scan

- JPDA ϵ-good approximation
- polynomial time execution

Multi Scan

- Handle unknown and changing over the time # targets
- On-line version for “real-time” processing

Stable Marriage Problem

The Problem

- 1 matchmaker
- 100 female clients
- 100 male clients
- 200 preference lists
- Arrange 100 happy marriages

Pair Satisfaction

When given two married pairs, (♂, ♀) and (⚣, ♀), if man ♂ prefers another woman ♀ more than his current wife ♀ and woman ♀ prefers ♂ more than her current husband ⚣, then (♂, ♀) is called a dissatisfied pair

The marriage is stable if there are no dissatisfied pairs

THE GOAL is to "marry" targets with measurement in a STABLE WAY

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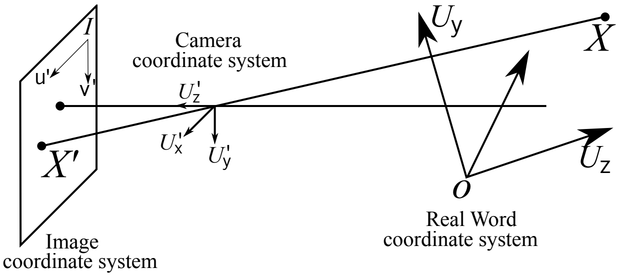
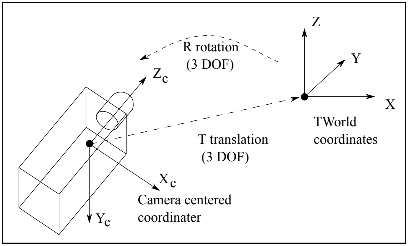
Pin-hole model

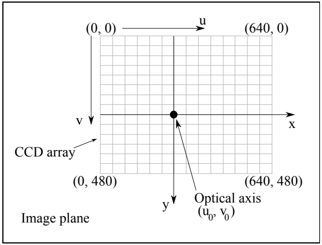


Projection equations:

$$x = \frac{fX_c}{Zc};\ y = \frac{fY_c}{Zc}$$

Pin-hole model

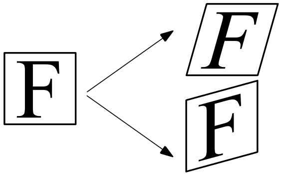


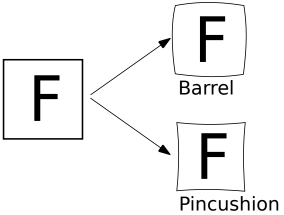


$$\begin{bmatrix} su_1 & su_2 & \cdots & su_n \\ sv_1 & sv_2 & \cdots & sv_n \\ s & s & \cdots & s \end{bmatrix} = K \cdot F \cdot RT \cdot P_{3dpts} = \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

Distortions





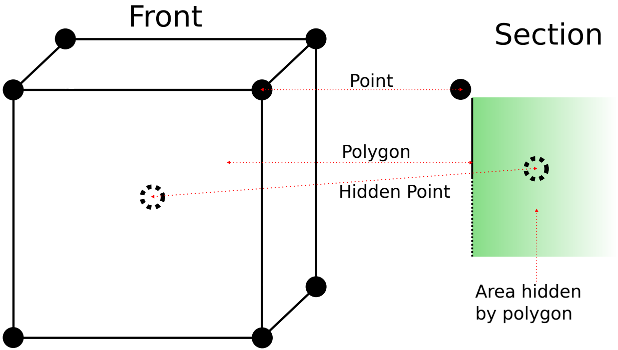
Skew

$$K' = \begin{bmatrix} k_u & s_w & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Radial distortion

$$x = x_d(1 + a_1 r^2 + a_2 r^4)$$
$$y = y_d(1 + a_1 r^2 + a_2 r^4)$$

Hidden Points



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Target Trajectories

We suppose exist an hidden Markov chain $\mathcal{M}$ *suitable for modelling track labelling*.

Model

$$\begin{cases} x_{t+1}^{q} = F^{k}\left(x_{t}^{q}\right) + w_{t}^{q} \\ y_{t}^{j} = \begin{cases} H^{j}\left(x_{t}^{k}\right) + v_{t}^{j} \\ u_{t} \end{cases} \end{cases}$$

Parameters

- p_d, target detection probability
- p_z, target disappearance probability
- λ_b target birth rate
- λ_f false alarms rate

Approach

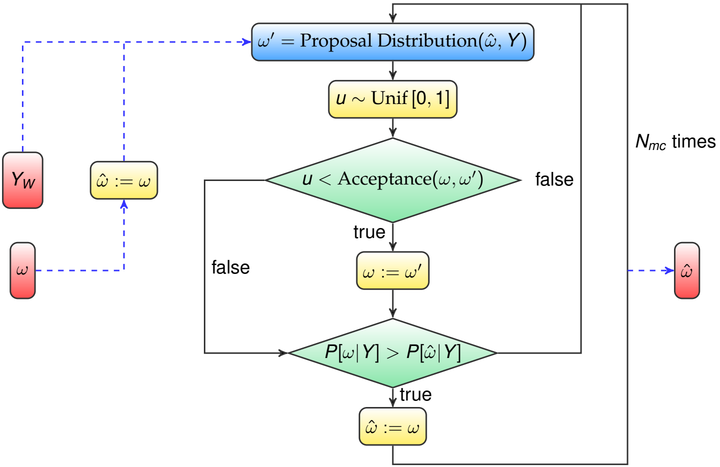
- sliding window of T frames, up to current time H
- $Y_W = \{Y_t \quad | \quad t = H - T + 1, \ldots, H\}$, 3D noisy data

$$\Downarrow$$

Labelling Structure

- $\tau_k = \tau_{[H-T,\ldots,H]_k} = \mathrm{Alg}(Y_W)$, target tracks
- $\omega = \{\tau_0, \tau_1, \ldots, \tau_K\}$, labelling & track history in the sliding window

Online Multi-Scan MCMCDA



Move selection

Constructing a suitable Markov chain

A random move m is chosen,
accordingly to the probability distribution $\xi_{K,H}(m)$,
for N_{mc} times every frame.

1) Birth Move

2) Death Move

3) Split Move

4) Merge Move

5) Extension Move

6) Reduction Move

7) Track Update Move

8) Track Switch Move

Acceptance Probability

$$A(\omega, \omega') = min\left(1, \frac{\pi(\omega')q(\omega', \omega)}{\pi(\omega)q(\omega, \omega')}\right)$$

- $\pi(\omega_a) := P(\omega_a | Y_W)$
- $q(\omega_a, \omega_b) := \xi_{K,H}(m_{\omega_a})$

A posteriori Probability

$$P(\omega|Y_W) = \frac{1}{Z_0} \prod_{\tau \in \omega \setminus \{\tau_0\}} \prod_{i=2}^{|\tau|} \mathcal{N}(\tau(t_i); \hat{y}_{t_i}(\tau), B_{t_i}) \cdot$$

$$\cdot \prod_{t=1}^{T} p_z^{z_t} (1 - p_z)^{m_{t-1} z_t} \cdot P_d^{d_t} (1 - P_d)^{u_t} \lambda_b^{a_t} \lambda_f^{f_t}$$

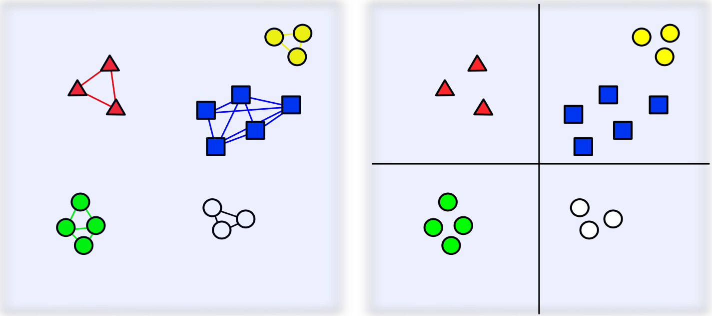
- m_t target at time t
- a_t new targets
- z_t terminated targets
- d_t detected targets
- u_t undetected targets
- f_t false alarms
- d_t measurements
- z_t missing relabelled measurements

Shaping $\xi_{K,H}(m)$

$$\xi_{K,H}(m) = \begin{cases} 0.033 & m = 1 \quad P[\text{birth}] = 1/30 \\ 0.066 & m = 2 \quad P[\text{death}] = 1/30 \\ 0.133 & m = 3 \quad P[\text{split}] = 2/30 \\ 0.2 & m = 4 \quad P[\text{merge}] = 2/30 \\ 0.5 & m = 5 \quad P[\text{extension}] = 9/30 \\ 0.666 & m = 6 \quad P[\text{reduction}] = 5/30 \\ 0.833 & m = 7 \quad P[\text{track switch}] = 5/30 \\ 1 & m = 8 \quad P[\text{track update}] = 5/30 \end{cases}$$

Two-by-Two Track Distance Bond Pruning

Volume or Distance Clusters Splitting up



Moving cube

Moving cube

Moving cube, vertices trajectories

$Proposal\ Distribution(\omega, Y)$ with perfect data

$Proposal\ Distribution(\hat{\omega}, Y)$ with perfect data

$N_{mc} = 30$ with noisy data

$N_{mc} = 80$ with noisy data

$N_{mc} = 1000$ with noisy data

Simulation results		
$N_{mc} = 30$	wrong associations	$\sim 36\%$
	generated tracks	12
	missing labelled markers	$\sim 10\%$
	runtime	40.70 minutes
$N_{mc} = 80$	wrong associations	$\sim 12\%$
	generated tracks	9
	missing labelled markers	$\sim 6\%$
	runtime	102.30 minutes
$N_{mc} = 1000$	wrong associations	$\sim 0.1\%$
	generated tracks	8
	missing labelled markers	$\sim 0.1\%$
	runtime	364.75 minutes

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Definition

The Stable Marriage Problem:

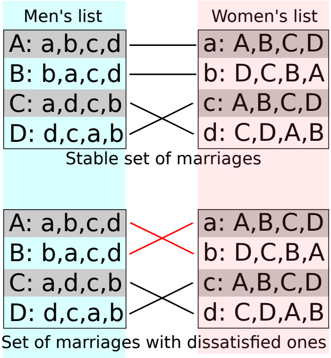
- We have N single men and an equal number of single women.
- Each man and each woman want to get marry, so express their own list of preferences.
- Our goal is to arrange N stable marriages

Men's list
A: a,b,c,d
B: b,a,c,d
C: a,d,c,b
D: d,c,a,b

Women's list
a: A,B,C,D
b: D,C,B,A
c: A,B,C,D
d: C,D,A,B

Definition

- A marriage is a match between one element of M and one of F (example: $A - b$).
- A pair $X - y$ is a dissatisfied one if in the solution S exists two marriages, $X - z$ and $W - y$ such that X prefer y more than is current partner z and woman y prefers X more than her current partner W.
- A set of marriage M is called stable if there are no dissatisfied pairs.



Algorithm

Analysis

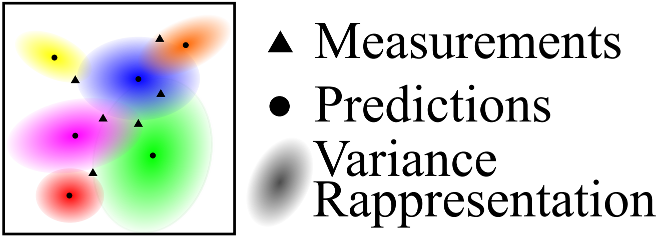
- Worst case complexity: $O(n^2)$
- Average case complexity: $O(n \cdot log(n))$
- Male-optimal

Optimality Criterion

A marriage between a man A and a woman B is feasible if there exists a stable pairing in which A and B are married. A pairing is male-optimal if every man is married with his highest ranked feasible partner.

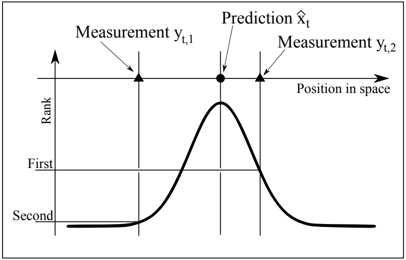
Data Association

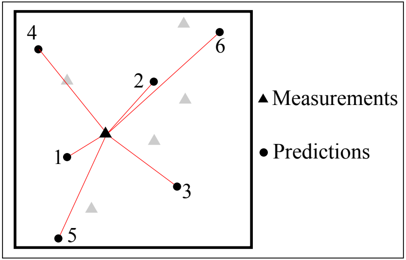
The SMP basically provides a way to pair the elements of two sets.
We adapted it to associate measurements to markers.



Preferences Criterion

To use SMP we have to define how the men and women rank their partners:





For males (markers) preferences list we evaluate the normal distribution $\mathcal{N}(y_i; \hat{y}_t, \Sigma_t)$, for females (measurements) preferences list we evaluate the distance from each prediction.

Simulation Results

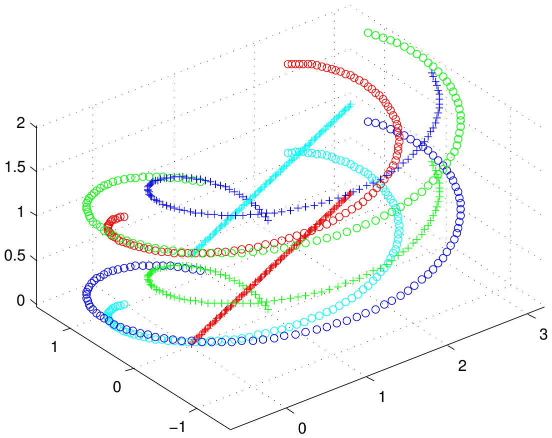
Three tests:

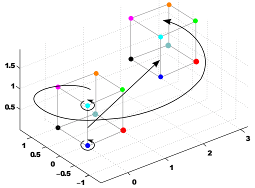
- 8 points on a translating and rotating cube
- 16 points on two cubes crossing each other
- Random Walk

In the first two case the reconstruction is correct.
In third case, designed specifically to put in trouble our algorithm:

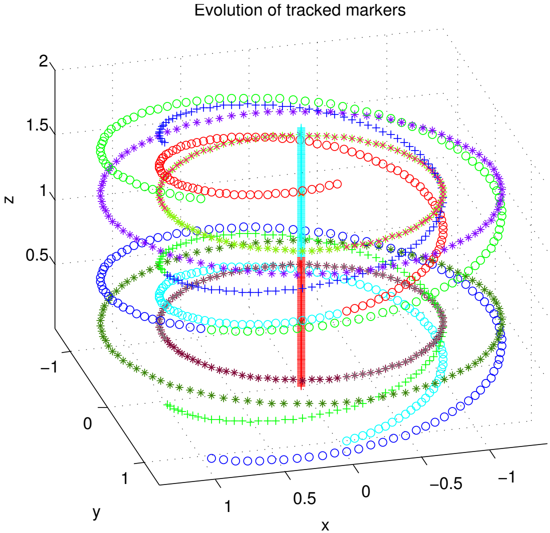
wrong associations	percentage
0 (all correct)	$\sim 82\%$
2	$\sim 11\%$
3	$\sim 2.5\%$
4	$\sim 2.5\%$
5	$\sim 0.5\%$
6	$\sim 0.5\%$
7	$\sim 0.5\%$
8	$\sim 0.5\%$

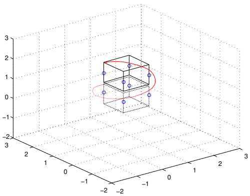
First Simulation



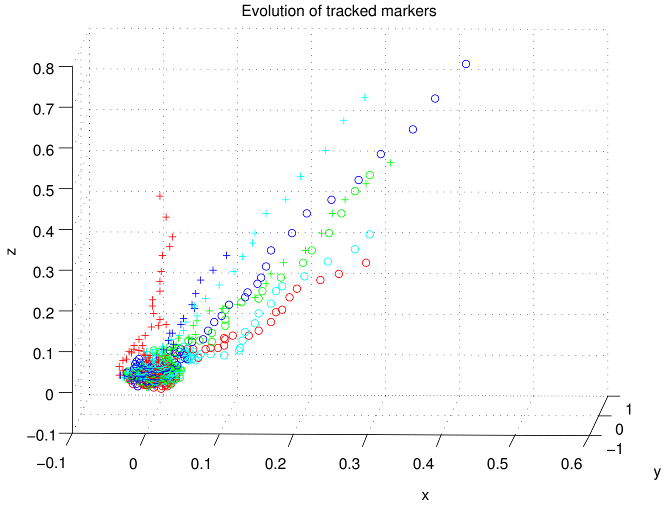


Second Simulation





Third Simulation



Possible Improvements

Variable number of targets

- decreasing measurements: factitious women
- increasing measurements: start new Kalman filter

False alarms / undetect measurements

Hospitals Problem: more than one patient (female) associated to one hospital (male)

Sliding window

Take into consideration the last n measurements to decide the preferences list

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Conclusions

MCMCDA

- Highly accurate
- Robust to missing and false reports
- Very slow

SMP

- Highly accurate
- Real-time
- Can't handle false alarms and undetections (at the moment...)

Correctness

- If there is a single man, there is a single woman
- If a man proposal is refused, he proposes to the next woman on his list until the last woman
- In a man list there are all the women

Therefore, all women receive at least one proposal

- A woman always accept her first proposal since she is unmarried

Therefore, all women get married.

Complexity

- Every time a man makes a proposal to a woman, she is removed from his list;
- There are N men and N woman

Therefore, the number of proposals is less then N^2.

Stability

Suppose there is a dissatisfied pair $X - b$, where in solution S there are the marriages $X - a$ and $Y - b$.

1. X prefers b over $a \Longrightarrow X$ have proposed to b before a
2. Since in S woman b is married with Y:
   - either b rejected X
   - or accepted but dropped him for a better man Y

Therefore b prefers Y to X contradicting the hypothesis that $X - B$ is a dissatisfied pair.

Optimality

Let M be a man which is just be rejected by his optimal parter w

- w have rejected M for $Z \Longrightarrow$ she prefers Z
- Z have proposed to w before $X \Longrightarrow$ her rank for Z is higher or equal of X
- If w is the optimal mate for X, there must exist a solution S including the marriage $X - w$

Optimality

In the a solution S including the marriage $X - w$:

- w prefers Z to M
- Z prefers w to his mate, since Z prefers w at least as much as X prefers his optimal partner w

Therefore The marriage is unstable, since $Z - w$ is a dissatisfied pair
$\Longrightarrow$ w cannot be M's optimal partner