A Multi-Agents Control Approach for the Optimal Power Flow Problem

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***Abstract*— We consider the problem of minimizing the power generation cost by exploiting the microgenerators dispersed in the power distribution network. The proposed strategy requires that the intelligent agents, located at the microgenerator buses, measure their voltage and then actuate the physical layer by adjusting the amount of injected power, according to a feedback control law derived from a projected gradient descent strategy. Simulations are provided in order to illustrate the algorithm behavior.**

I. Introduction

The ultimate goal of the optimal power flow (OPF) problem is to find an operating point of the power system that minimizes a cost function (tipically the generation cost or the line losses) satisfying the power demand and some operative constraints. In the past, algorithms for the solution of the OPF problem have been applied to the transmission networks, namely, the high voltage networks transporting the electrical power from the power plants to the distribution networks and finally to the users. The advent of distributed energy resources is drastically changing the actual power distribution scenario. Indeed, in the near future a massive number of small power generators are envisioned to be deployed in the low voltage and medium voltage power distribution grid yielding to a number of benefits for the electrical distribution system. On the other hand, power injection of several renewable energy sources could lead, if not properly regulated, to system instability, thus requiring the solution of OPF problems also in the low and medium voltage power distribution networks. Many algorithms solving the OPF problem have been designed. Many of them exploit powerful optimization techniques, like ADMM ([1]), primal or dual optimization ([2]), or convex relaxation. They typically require a large number of iterations and a high computational burden to converge and they are based on the standing assumptions that all the buses of the grid are monitored and all the grid parameters are perfectly known. The algorithm we propose extends the approach of [3] to the OPF problem, and can be considered as a feedback control strategy: its key feature is the alternation between measurement steps and actuation steps which are based on the measured data (phasorial voltages), and therefore it is inherently an online algorithm. This fact is particularly important as it allows to chase the power demand and the generation capability variation, that in presence of renewable energy sources are highly changing. Remarkably, the algorithm we propose is guaranteed to converge to an approximated optimal solution without monitoring all the grid nodes, but only the generators. In the OPF problem we consider, the goal is to minimize the global generation cost by controlling the amount of powers injected in the grid by the generators. The active and reactive powers are subject to box constraints modeling the generation capability of each generator, while the objective function is given by the sum of the generation cost functions associated to the generators. We tackle the problem via a projected gradient-based approach. In particular exploiting an approximated solution of the power flow nonlinear equations, we show the gradient of the objective function can be computed by the compensators just via local measurements of the phasorial voltages at their connection points. Applying at each iteration a projected gradient descent step, the algorithm is shown to be provably convergent to an appromixated optimal solution of the OPF problem.

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II. Mathematical Preliminaries and Notation

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \sigma, \tau)$ be a directed graph, where $\mathcal{V}$ is the set of nodes, $\mathcal{E}$ is the set of edges, with $n = |\mathcal{V}|, r = |\mathcal{E}|$. Moreover $\sigma, \tau : \mathcal{E} \to \mathcal{V}$ are two functions such that edge $e \in \mathcal{E}$ goes from the source node $\sigma(e)$ to the terminal node $\tau(e)$. In the paper we introduce complex-valued functions defined on the nodes and on the edges. These functions will also be intended as vectors in $\mathbb{C}^n$ and $\mathbb{C}^r$. Given a vector u, we denote by $\bar{u}$ its (element-wise) complex conjugate, and by u^T its transpose. We denote by $\Re(u)$ and by $\Im(u)$ the real and the imaginary part of u, respectively. Let $A \in \{0, \pm 1\}^{r \times n}$ be the incidence matrix of the graph $\mathcal{G}$, defined via its elements

$$[A]_{ev} = \begin{cases} -1 & \text{if } v = \sigma(e) \\ 1 & \text{if } v = \tau(e) \\ 0 & \text{otherwise.} \end{cases}$$

We define as $\mathbf{1}$ the column vector of all ones, while $\mathbf{1}_v$ is the vector whose value is 1 in position v, and 0 everywhere else. Given $u, v, w \in \mathbb{R}^\ell$, with $v_h \leq w_h, h = 1, \ldots, \ell$ we define the operator $\text{proj}(u, v, w)$ as the component wise projection of u in the set $\left\{x \in \mathbb{R}^\ell : v_h \leq x_h \leq w_h, h = 1, \ldots, \ell\right\}$, i.e.,

$$[\text{proj }(u, v, w)]_h = \begin{cases} u_h & \text{if } v_h \leq u_h \leq w_h \\ v_h & \text{if } u_h < v_h \\ w_h & \text{if } u_h > w_h \end{cases} \tag{1}$$

III. Cyber-physical model of a smart grid

In this work, we envision a *smart* power distribution network as a cyber-physical system, in which the *physical layer* consists of the power distribution infrastructure, including power lines, loads, microgenerators, and the point of connection to the transmission grid, while the *cyber layer* consists of intelligent agents, dispersed in the grid, and provided with actuation, sensing, communication, and computational capabilities. We model the physical layer as a directed graph $\mathcal{G}$, in which edges in $\mathcal{E}$ represent the power lines, and nodes in $\mathcal{V}$ represent both loads and generators that are connected to the microgrid.We limit our study to the steady state behavior of the system, where all voltages and currents are sinusoidal signals at the same pulsation ω_0, and can therefore be represented in phasorial notation.

The system state is described by the following system variables:

- $u \in \mathbb{C}^n$, where u_v is the grid voltage at node v;
- $i \in \mathbb{C}^n$, where i_v is the current injected at node v;
- $s = p + iq \in \mathbb{C}^r$, where s_v, p_v and q_v are the complex, the active and the reactive power injected at node v.

We assume that every microgenerator, and also the PCC, corresponds to an *agent* in the cyber layer. We denote this subset of the nodes of $\mathcal{G}$ by $\mathcal{C}$ (with $|\mathcal{C}| = m$). Each agent is provided with sensing capability in the form of a phasor measurement unit (PMU, i.e., a sensor measuring voltage amplitude and angle). Agents that correspond to microgenerators can command the amount of power injected in the grid. Moreover agents can communicate with each other, via some communication channels which could possibly via power line communication (PLC). We introduce the following block decomposition for the vectors of voltages u and powers s

$$u = \begin{bmatrix} u_0 & u_G & u_L \end{bmatrix}^T, \quad s = \begin{bmatrix} s_0 & s_G & s_L \end{bmatrix}^T, \qquad (2)$$

where u_0 is the voltage at the PCC, $u_G \in \mathbb{C}^{m-1}$ and $u_L \in \mathbb{C}^{n-m}$ are the voltages at the microgenerators and at the loads respectively. Similarly for $s_G = p_G + jq_G$ and $s_L = p_L + jq_L$. For every edge e of the graph, we define by z_e the impedance of the corresponding power line. We assume the following.

Assumption 1: All power lines in the grid have the same inductance/resistance ratio, i.e., $z_e = e^{j\theta}|z_e|$, for any e in $\mathcal{E}$ and for a fixed θ.

Assumption 1 is satisfied when the grid is relatively homogeneous, and is reasonable in most practical cases. We collect all the grid impedances absolute values in the matrix Z=diag$(|z_e|, e\in\mathcal{E})$. We label the PCC as node 0 and take it as an ideal sinusoidal voltage generator (*slack bus*) at the microgrid nominal voltage U_N, with arbitrary, but fixed, angle ϕ. We model all nodes but the PCC as *constant power* or *P-Q buses*. The powers s_v corresponding to grid loads are such that $p_v < 0$, meaning that positive active power is *supplied* to the devices. On the other hand, the complex powers corresponding to microgenerators are such that $p_v \geq 0$, as positive active power is *injected* into the grid. It is known that the system state satisfies the equations

$$u = e^{-i\theta} Y i \qquad (3)$$

$$u_0 = U_N e^{i\phi} \qquad (4)$$

$$u_v i_v = p_v + iq_v \qquad v \neq 0 \qquad (5)$$

where $Y := A^T Z^{-1} A$ is the matrix collecting the absolute values of the bus admittance matrix of the grid. The following Lemma will be useful in the sequel.

Lemma 2: Given $Y := A^T Z^{-1} A$, there exists a unique symmetric, positive semidefinite matrix $X \in \mathbb{R}^{n\times n}$ such that

$$\begin{cases} XY = I - \mathbf{1}\mathbf{1}_0^T \\ X\mathbf{1}_0 = 0. \end{cases} \qquad (6)$$

The matrix X depends only on the topology of the grid power lines and on their impedances and, adopting the same block decomposition as in (2), we can write

$$X = \begin{bmatrix} 0 & 0 & 0 \\ 0 & M & N \\ 0 & N^T & Q \end{bmatrix}, \qquad (7)$$

with $M \in \mathbb{R}^{(m)\times(m)}$, $N \in \mathbb{R}^{(m)\times(n-m-1)}$, and $Q \in \mathbb{R}^{(n-m-1)\times(n-m-1)}$. The following proposition provides a approximate relation between the grid voltages and the power injected.

Proposition 3: Consider the physical model described by the set of nonlinear equations (3), (4) and (5). Then

$$\begin{bmatrix} u_0 \\ u_G \\ u_L \end{bmatrix} = e^{i\phi}\left(U_N \mathbf{1} + \frac{e^{i\theta}}{U_N} \begin{bmatrix} 0 & 0 & 0 \\ 0 & M & N \\ 0 & N^T & O \end{bmatrix} \begin{bmatrix} 0 \\ \bar{s}_G \\ \bar{s}_L \end{bmatrix} \right) + o\left(\frac{1}{U_N}\right), \qquad (8)$$

where the little-o notation means $\lim_{U_N\to\infty} \frac{o(f(U_N))}{f(U_N)} = 0$. Without loss of generality, from now on we assume $\phi = 0$.

IV. Optimal power flow problem

The goal of this paper is to design a distributed control algorithm that leads to the minimization of the power generation cost of the power supplied to the loads, that we assume to be constant power loads requiring $\hat{s}_L$. Formally the problem we are interested into can be stated as

$$\min_{s_j, j\in\mathcal{C}\cup PCC} f = \sum_{j=1}^{m} f_j(p_j) + f_0(p_0) \qquad (9a)$$

$$\text{s.t. } s_L = \hat{s}_L \qquad (9b)$$

$$s_0 = -(\mathbf{1}^T s_G + \mathbf{1}^T s_L) + \ell(s_G, s_L, U_N) \qquad (9c)$$

$$0 \leq p_v \leq p_v^M \qquad v \in \mathcal{C} \qquad (9d)$$

$$-q_v^M \leq q_v \leq q_v^M \quad v \in \mathcal{C} \qquad (9e)$$

where

- constraint in (9c) models the power conservation in the grid, that is always enforced by the PCC, being $\ell(s_G, s_L, U_N)$ the line losses in the grid;
- constraints in (9d) and (9d) model the agents generation capabilities;
- the objective function f is the sum of the cost of the power produced by the utility and injected into the microgrid through the PCC ($f_0(p_0)$), and of the microgenerators' payments for the power that they inject.

In this paper we assume all the f_j's to be proportional to the amount of power injected, and, additionally, we assume all the agents to be paid in the same way; specifically we model the f_j's as

$$f_j(p_j) = \begin{cases} c_G p_j & p_j \geq 0 \\ 0 & p_j < 0 \end{cases} \qquad \forall j \in \mathcal{C} \qquad (10)$$

In this paper we model $f_0(p_0)$ as the $f_j, j \in \mathcal{C}$,

$$f_0(p_0) = \begin{cases} c_0 p_0 & p_0 \geq 0 \\ 0 & p_0 < 0 \end{cases} \qquad (11)$$

Other realistic models might be proposed, however the one we consider, despite its simplicity, allows us to model interesting system behavior in response to energy price variations due to energy market logic or system performance necessity. Problem (9) is not convex, mainly because ℓ is not convex. However, via (8) we can approximate ℓ as

$$\ell(s_G, s_L, U_N) \simeq \ell_A(s_G, s_L, U_N) + i\ell_R(s_G, s_L, U_N) \quad (12)$$

where

$$\ell_A(s_G, s_L, U_N) = \frac{\cos(\theta)}{U_N^2}\Bigg(\begin{bmatrix} p_G^T & p_L^T \end{bmatrix} \begin{bmatrix} M & N \\ N^T & Q \end{bmatrix} \begin{bmatrix} p_G \\ p_L \end{bmatrix} + \\ + \begin{bmatrix} q_G^T & q_L^T \end{bmatrix} \begin{bmatrix} M & N \\ N^T & Q \end{bmatrix} \begin{bmatrix} q_G \\ q_L \end{bmatrix} \Bigg) \quad (13)$$

$$\ell_R(s_G, s_L, U_N) = \frac{\sin(\theta)}{U_N^2}\Bigg(\begin{bmatrix} p_G^T & p_L^T \end{bmatrix} \begin{bmatrix} M & N \\ N^T & Q \end{bmatrix} \begin{bmatrix} p_G \\ p_L \end{bmatrix} + \\ + \begin{bmatrix} q_G^T & q_L^T \end{bmatrix} \begin{bmatrix} M & N \\ N^T & Q \end{bmatrix} \begin{bmatrix} q_G \\ q_L \end{bmatrix} \Bigg) \quad (14)$$

where $\ell_A(s_G, s_L, U_N)$ approximates the active power losses while $\ell_R(s_G, s_L, U_N)$ approximates the reactive power losses. Using the above formulas we can express the active power injected by the PCC as

$$p_0 \simeq -\mathbf{1}^T p_G - \mathbf{1}^T p_L + \ell_A(s_G, s_L, U_N) \quad (15)$$

and we can approximate problem in (9) with the convex problem

$$\min_{s_j, j\in\mathcal{C}} \quad \hat{f} \quad (16a)$$

$$\text{s.t. } s_L = \hat{s}_L \quad (16b)$$

$$s_0 = -(\mathbf{1}^T s_G + \mathbf{1}^T s_L) + \ell(s_G, s_L, U_N) \quad (16c)$$

$$0 \le p_v \le p_v^M \qquad v \in \mathcal{C} \quad (16d)$$

$$- q_v^M \le q_v \le q_v^M \quad v \in \mathcal{C} \quad (16e)$$

where

$$\hat{f} = \sum_{j=1}^{m} f_j(p_j) + f_0(-\mathbf{1}^T p_G - \mathbf{1}^T p_L + \ell_A(s_G, s_L, U_N))$$

V. Gradient projected control algorithm

In this section we describe the control strategy the agents apply. It is based on a projected gradient descent of $\hat{f}$. We have

$$\frac{\partial\left(\sum_j^m f_j(p_j)\right)}{\partial p_G} = \begin{bmatrix} f_1'(p_1) \\ \vdots \\ f_m'(p_m) \end{bmatrix} := f_G'(p_G), \frac{\partial\left(\sum_j^m f_j(p_j)\right)}{\partial q_G} = 0$$

while, as far as $\frac{\partial f_0(p_0)}{\partial p_j}$, $\frac{\partial f_0(p_0)}{\partial q_j}$ are concerned, we get

$$\frac{\partial f_0(-\mathbf{1}^T p_G - \mathbf{1}^T p_L + \ell_A(s_G, s_L, U_N))}{\partial p_G} = \\ = f_0'(p_0)\Big(-\mathbf{1} + 2\frac{\cos(\theta)}{U_N^2}(Mp_G + Np_L)\Big) \quad (17)$$

$$\frac{\partial f_0(-\mathbf{1}^T p_G - \mathbf{1}^T p_L + \ell_A(s_G, s_L, U_N))}{\partial q_G} = \\ = f_0'(p_0)\Big(+ 2\frac{\cos(\theta)}{U_N^2}(Mq_G + Nq_L)\Big) \quad (18)$$

Summarizing we can write

$$\frac{\partial \hat{f}}{\partial p_G} = f_G'(p_G) + f_0'(p_0)\left(-\mathbf{1} + 2\frac{\cos(\theta)}{U_N^2}(Mp_G + Np_L)\right) \quad (19)$$

$$\frac{\partial \hat{f}}{\partial q_G} = f_0'(p_0)\left(+2\frac{\cos(\theta)}{U_N^2}(Mq_G + Nq_L)\right) \quad (20)$$

While U_N and θ can be assumed known a priori, the quantities $Mp_G + Np_L$ and $Mq_G + Nq_L$ depends on all the active power injected into the grid (also the unmonitored one of the loads) and on the topology of the grid. Again, by exploiting (8), we have that

$$u_G = \frac{e^{i\theta}}{U_N}\begin{bmatrix} M & N \end{bmatrix}\begin{bmatrix} p_G - iq_G \\ p_L - iq_L \end{bmatrix} + \mathbf{1}U_N + o\left(\frac{1}{U_N}\right) \\ = \frac{e^{i\theta}}{U_N}(Mp_G + Np_L - iMq_G - iNq_L) + \mathbf{1}U_N + o\left(\frac{1}{U_N}\right)$$

from which it follows

$$\Re(e^{-i\theta}(u_G - \mathbf{1}U_N)) = \frac{Mp_G + Np_L}{U_N} + o\left(\frac{1}{U_N}\right) \quad (21a)$$

$$-\Im(e^{-i\theta}(u_G - \mathbf{1}U_N)) = \frac{Mq_G + Nq_L}{U_N} + o\left(\frac{1}{U_N}\right) \quad (21b)$$

and, in turn,

$$\frac{\partial \hat{f}}{\partial p_G} \simeq f_G'(p_G) + f_0'(p_0)(-\mathbf{1}+ \\ + 2\frac{\cos(\theta)}{U_N}\Re(e^{-i\theta}(u_G - \mathbf{1}U_N))\Big) := \hat{g}_p \quad (22a)$$

$$\frac{\partial \hat{f}}{\partial q_G} \simeq f_0'(p_0)\left(-2\frac{\cos(\theta)}{U_N}\Im(e^{-i\theta}(u_G - \mathbf{1}U_N))\right) := \hat{g}_q \quad (22b)$$

Remarkably, from the above expressions, it turns out that the compesators can compute the gradient by taking only local voltage measurements. Indeed, $\forall k \in \mathcal{C}$, it holds

$$\Big[\Re(e^{-i\theta}(u_G - \mathbf{1}U_N))\Big]_k = |u_k|\cos(\angle u_k - \theta) \\ - |u_N|\cos(\angle u_N - \theta) \quad (23)$$

$$\Big[\Im(e^{-i\theta}(u_G - \mathbf{1}U_N))\Big]_k = |u_k|\sin(\angle u_k - \theta) \\ - |u_N|\sin(\angle u_N - \theta) \quad (24)$$

and hence, each compensator, in order to evaluate the corresponding component of $\hat{f}$'s gradient, needs only the knowledge of its voltage, of the PCC voltage and of $f_0'(p_0)$. Based on the above observations, we propose the following algorithm, where we assume that the agents can update their state variables q_h and λ_h, $h \in \mathcal{C}\backslash\{0\}$, synchronously.

Let γ_p and γ_q be positive scalar parameter. At every synchronous iteration of the algorithm, each agent $h \in \mathcal{C}\backslash\{0\}$ executes the following operations in order:

1) senses the system obtaining its voltage phasorial measurement u_h;
2) receives the PCC voltage phasorial measurement $u_0 = U_N$;

3) computes the gradient directions approximations

$$[\hat{g}_p]_h = f_h'(p_h) + f_0'(p_0)(-1+ \tag{25a}$$
$$+ 2\frac{\cos\theta}{U_N}\left(|u_k|\cos(\angle u_k - \theta) - |u_N|\cos(\angle u_N - \theta)\right)\Big)$$
$$[\hat{g}_q]_h = f_0'(p_0)(-2\frac{\cos\theta}{U_N}\big(|u_k|\cos(\angle u_k - \theta)+ \tag{25b}$$
$$-|u_N|\cos(\angle u_N - \theta)\big)\Big)$$

4) computes the active and reactive power to be injected in the grid performing the following gradient descent steps

$$p_h \leftarrow p_h - \gamma_p[\hat{g}_p]_h \tag{26a}$$
$$q_h \leftarrow q_h - \gamma_q[\hat{g}_q]_h \tag{26b}$$

5) projects p_h and q_h into the feasible region and actuates the projected values

$$p_h \leftarrow \text{proj}(p_h, 0, p_h^M) \tag{27a}$$
$$p_h \leftarrow \text{proj}(q_h, -q_h^M, p_h^M). \tag{27b}$$

Based on the above description, it is clear what is the feedback scheme that underlies the procedure we propose: during each iteration each agent senses the grid, communicates with the PCC, computes the power set-point and then actuates it. Concerning the convergence properties of the above distributed control algorithm, we have the following result.

Proposition 4: Consider problem (16), and assume

$$\sum_{h\in\mathcal{C}} p_h^M \leq -\mathbf{1}^T p_L \tag{28}$$

Then the algorithm in (25), (26) and (27) converges asynptotically to the optimal solution of (16) if

$$\gamma_p \leq \gamma_{\max}, \qquad \text{and} \qquad \gamma_q \leq \gamma_{\max}$$

where $\gamma_{\max} = 2\frac{f'(p_0)\cos\theta}{U_N^2}\rho(M)$ being $\rho(M)$ the spectral radius of M.

Remark 5: Condition (28) incorporates the fact that the agents total generation capacity is not able to satisfy the whole loads power requirement. This situation might be encountered in those grids where the agents are all small-size renewable generators. We plan to study situations where condition (28) is not satisfied as future research.

VI. Simulations

The algorithm has been tested on the testbed IEEE 37 [4], whose scheme is on Fig. 1, on a nonlinear exact solver of the grid [5]. The load buses are a blend of constant-power, constant current, and constant-impedance loads, with a total power demand of almost 2 MW of active power and 1 MVAR of reactive power. The impedances of the power lines are different for different edges. However, the inductance/resistance ratio exhibits a smaller variation, ranging from $\angle z_e = 0.47$ to $\angle z_e = 0.59$ (justifying in some sense Assumption 1). We considered the scenario in which 5 microgenerators have been deployed in this distribution grid. We ran the algorithm with different values of the ratio $\alpha = c_0/c_G$, and with $\gamma_p = \gamma_q = \gamma^M/4$. The asymptotical active power values (reported in Table I) of each simulation reached the optimum values. In Fig. 2 instead is reported the cost generation trajectory of the ran with $\alpha = 1$. We can see that the feedback control strategy at every iteration makes the cost to reduce, until it reaches the optimum value.

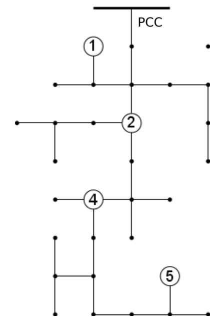

Fig. 1.

TABLE I
Resulting Active Power Allocation [kW]

node	$\alpha = 1$	$\alpha = 0.96$	$\alpha = 0.50$	p^M
PCC	409	1319	1994	
1	85	0	0	85
2	415	0	0	415
3	368	78	0	368
4	178	155	0	178
5	490	407	0	490

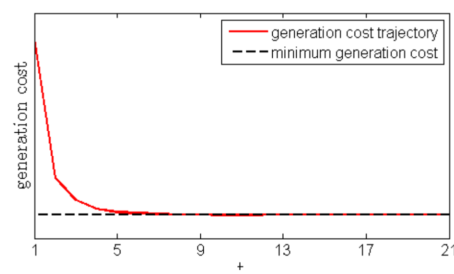

Fig. 2. Trajectory of the generation cost with $\alpha = 1$

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