Control over wireless: an unfinished journey

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DEC
Plant
COD
Channel

M.a.g.i.C.
Multi Agent Intelligent Control
The XXI century: a Smart World

INTELLIGENT TRAFFIC SYSTEMS

CYBER-PHYSICAL SYSTEMS

FACTORY 4.0

SWARM ROBOTICS

SMART BUILDINGS

SMART CITIES

SMART GRIDS
The challenge cube for time-critical smart systems

- **Scale**
  - $10^{-3}$ to $10^{-6}$
  - $10^{-2}$ to $10^{-3}$
  - $10^{-1}$ to $10^1$

- **Bandwidth [Hz]**
  - $10^3$
  - $10^2$
  - $10^1$

- **Wired**
  - Fieldbus (SCADA)
  - RT Ethernet

- **Wireless**
  - WirelessHART
  - ISA100.10a

**Applications**
- Smart grids
- Buildings
- Field robotics
- Manipulation

**Wireless Network Protocols**
- Wired
- Wireless
15 years ago in Berkeley….

Assumptions:
(1) Quantization noise $\ll$ sensor noise
(2) Packet-rate limited ($\neq$ bit-rate)
(3) No transmission noise (data corrupted=dropped packet)
15 years ago in Berkeley….

\[ \hat{x}_t = \mathbb{E}[x_t | \{y_k\} \text{ available at estimator at time } t] \]

**PLANT**

\[
\begin{align*}
x_{t+1} &= Ax_t + w_t \\
y_t &= Cx_t + v_t
\end{align*}
\]

**ESTIMATOR**

\[ \gamma_t = \begin{cases} 
1 & \text{if } y_t \text{ received at time } t \\
0 & \text{otherwise}
\end{cases} \]

\[ \tilde{y}_t = \gamma_t(Cx_t + v_t) = C_t x_t + u_t \]

**Time-varying Kalman filter**

\[ \hat{x}_t = \mathbb{E}[x_t | \tilde{y}_t, \ldots, \tilde{y}_t, \gamma_t, \ldots, \gamma_1] \]
15 years ago in Berkeley…


\[
\begin{align*}
\hat{x}_{t+1|t} & = A\hat{x}_{t|t-1} + \gamma_t AK_t(y_t - C\hat{x}_{t|t-1}) \\
K_t & = f(P_{t|t-1}) \\
P_{t+1|t} & = \Phi_\gamma(P_{t|t-1})
\end{align*}
\]

\[\Phi_e(P) = APA^T + Q - (1 - \epsilon)APC^T(CPC^T + R)^{-1}CPA^T\]

- Simple to understand but not trivial
- Critical packet loss probability function of eigenvalues of A
- Some new mathematical techniques
- Estimator designed for performance not only stability
- Many open questions remained unanswered
One open question


If $n=10000$ is it better to send the quantized state rather than the quantized measurement? ==> need to include quantization
Joint work with:

Alessandro Chiuso
Andrea Zanella
Nicola Laurenti
Modeling

\[ x_{t+1} = ax_t + u_t + w_t \]
\[ y_t = x_t + v_t \]
\[ |a| > 1, \]
\[ w_t \sim N(0, \sigma_w^2), v_t \sim N(0, \sigma_v^2) \]

**Proposed approach:**
1) Separate control/estimation design from communication design.
2) Use of traditional coding with finite block-length (different from any-time coding of Sahai-Mitter 07 !!)

Ideally: \[ h_t \approx s_t \in \mathbb{R} \]
A naïve coding/decoding scheme:

[10]: symbol to be sent
[10|1]: add parity check bit
aₜ=[111|000|111]: add redundancy

Noisy Channel: recovery via majority bits

Received (bₜ)  |  Recovery  |  decoded
[101|100|011] | [10|1] | correct decoding: [10] (hₜ₟=sₜ₟)
[111|110|111] | [11|1] | erasure
[100|110|111] | [01|1] | wrong decoding: [01] (hₜ₟≠sₜ₟)

Receiver knows Δ and therefore maps [10] into the real number hₜ.
Role of code length:
$s_t^q=[10]$: 2-bits of information per period
$a_t=[111|000|111]$: 9-bit word per period over the channel

$(s_t^q,s_{t-1}^q)=[11,10] \Rightarrow a_t=[xxx|xxx|xxx|xxx|xxx|xxx|xxx]$ smarter coding
18-bit blocklength over 2 period $\Rightarrow 9$-bits/period

Longer block-length:
- Same channel rate (bits/period)
- Smaller erasure probability
- Larger delay
About quantization modeling

\[ n_t = s_t - s_{qt} \]

\[ \Delta E[n_t^2] = \frac{1}{\rho} \mathbb{E}[s_t^2], \rho: \text{SNR} \]


"Analog" channel COD/DEC model

$h_t \in \mathbb{R}$

$h_t^q$ channel

$h_t^q$ Chann DEC

$b_t$ Channel

$a_t$ Chann COD

$s_t^q$ Quantizer

$s_t$ Quantization

$n_t$: quantization noise
$\gamma_t = 0, \nu_t = \{0, 1\}$: undecoded word (erasure)
$P[\nu_t = 1] = \varepsilon_w$: undetected error probability
$\varepsilon_w \ll \varepsilon$
$E[n_t^2] = \frac{1}{\rho} E[s_t^2]$, $\rho$: SNR
$E[m_t^2] \approx E[s_t^2]$

$\gamma_t = 1, \nu_t = 0$: correctly decoded word

$\gamma_t = 1, \nu_t = 1$: wrongly decoded word

$d$: decoding delay (integer)
Problem formulation

1. Scalar dynamics
2. No transmission pre-processing
3. Estimator+ state feedback architecture

\[
x_{t+1} = ax_t + u_t + w_t
\]
\[
y_t = x_t + v_t
\]
Problem formulation (cont’d)

Linear estimator + linear controller

\[ \hat{\xi}_{t+1} = A\hat{\xi}_t + Bu_t + \gamma_{t-d+1} G \left( h_t - H\hat{\xi}_t \right) \]

\[ u_t = L\hat{\xi}_t \]

LQG performance optimization

\( (G^*, L^*) := \text{argmin}_{G, L} \mathbb{E}[y_t^2] + r\mathbb{E}[u_t^2] \)

s.t. \( \mathbb{E}[n_t^2] = \frac{1}{\rho} \mathbb{E}[y_t^2] \)
**Problem solution**

Augmented System dynamics

\[ \xi_{t+1} = A\xi_t + B(u_t + w_t) \]
\[ y_t = C\xi_t + v_t \]
\[ h_t = \gamma_{t-d+1} H(\xi_t + v_{t-d+1} + n_{t-d+1}) \]

Linear estimator + linear controller

\[ \hat{\xi}_{t+1} = A\hat{\xi}_t + B u_t + \gamma_{t-d+1} G(h_t - H\hat{\xi}_t) \]
\[ u_t = L\hat{\xi}_t \]

LQG performance optimization

\[ (G^*, L^*) := \arg\min_{G, L} J(G, L) = \mathbb{E}[y_t^2] + r\mathbb{E}[u_t^2] \]
\[ \text{s.t. } \mathbb{E}[n_t^2] = \alpha \mathbb{E}[y_t^2] \]

\[ P = (1 - \epsilon)\bar{A}_1 P \bar{A}_1^T + \epsilon \bar{A}_0 P \bar{A}_0^T + \sigma_w^2 \bar{B} \bar{B}^T + \alpha (1 - \epsilon) G\bar{C} P \bar{C}^T G^T + (1 - \epsilon)(1 + \alpha) \bar{G} \sigma_v^2 \bar{G}^T \]

\[ \mathcal{M}(P, G, L) \]

\[ P := \operatorname{Var}\left\{ \begin{bmatrix} \hat{\xi}_t \\ \xi_t - \hat{\xi}_t \end{bmatrix} \right\} \]

\[ \min_{G, L} J(P, G, L) \]
\[ \text{s.t. } P = \mathcal{M}(P, G, L) \]

\( J \) and \( \mathcal{M} \): linear in \( P \)

“quadratic” in \( G, L \)
Problem solution

Solve via Lagrangian

\[
\min_{P, \Lambda, G, L} \quad J(P, G, L) + \text{trace}(\Lambda(P - M(P, G, L))) := \mathcal{L}(P, \Lambda, G, L)
\]

s.t. \quad P \geq 0, \Lambda \geq 0

 Necessary optimal conditions

\[
\frac{\partial \mathcal{L}}{\partial P} = 0, \quad \frac{\partial \mathcal{L}}{\partial \Lambda} = 0, \quad \frac{\partial \mathcal{L}}{\partial L} = 0, \quad \frac{\partial \mathcal{L}}{\partial G} = 0
\]

 Coupled Riccati-like Equations

\[
\begin{align*}
    P &= \Phi_1(P, \Lambda) \\
    \Lambda &= \Phi_2(P, \Lambda) \\
    G &= \Psi_1(P) \\
    L &= \Psi_2(\Lambda)
\end{align*}
\]

Coupled Riccati-like Equations

\[ P = \Phi_1(P, \Lambda) \]
\[ \Lambda = \Phi_2(P, \Lambda) \]
\[ G = \Psi_1(P) \]
\[ L = \Psi_2(\Lambda) \]

For \( r = 0 \) problem equivalent to the solution of a scalar Riccati-like equation:

\[ p = a^2 p + \sigma_w^2 - \delta \frac{a^2 p^2}{p + r(d)} \]
\[ \delta := \frac{1 - \epsilon}{1 + \alpha a^{2d}} \]
Further simplification

\[ p = a^2 p + \sigma_w^2 - \delta \frac{a^2 p^2}{p + r(d)} \]
\[ \delta := \frac{1 - \epsilon}{1 + \alpha a^2 d} \]


Necessary and sufficient stability for \( r \geq 0 \):

\[ \frac{1 - \epsilon}{1 + \alpha a^2 d} > 1 - \frac{1}{a^2} \]

\( d \): decoding delay
\( \epsilon \): erasure probability
\( \alpha = \frac{1}{SNR} \): noise-to-signal ratio
Discussion w/ related works

\[ \frac{1 - \epsilon}{1 + \alpha a^2 d} > 1 - \frac{1}{a^2} \]

1) Infinite resolution (\(\alpha=0\)) and no delay (\(d=0\)):

\[ 1 - \epsilon > 1 - \frac{1}{a^2} \]


2) Infinite resolution (\(\alpha=0\)) and with delay (\(d>0\)):

\[ 1 - \epsilon > 1 - \frac{1}{a^2} \]


3) No packet loss (\(\epsilon=0\)) and no delay (\(d>0\)):

\[ \text{SNR} = \frac{1}{\alpha} > a^2 - 1 \]

Recalling the rate \( R = \frac{1}{2} \log(1 + \text{SNR}) \) and \( R < C \):

\[ C > \log |a| \]


4) No packet loss ($\varepsilon = 0$) and delay ($d = 1$):

$$SNR = \frac{1}{\alpha} > a^4 - a^2$$


5) Infinite resolution ($\alpha = 0$), packet loss as SNR-limitation + delay

$$\frac{1 - \varepsilon}{1 + \varepsilon (a^2 d - 1)} > 1 - \frac{1}{a^2}$$


Our condition less stringent and independent of delay

6) Rate-limited with delay ($d = 1$):

$$R = \frac{1}{2} \log(1 + SNR)$$

$$\mathbb{E} \left[ \left( \frac{a^2}{2^2 R_t} \right)^n \right] < 1$$

$$R_t = R \delta_t, \delta_t \sim \mathcal{B}(1 - \varepsilon)$$

$$\frac{a^2}{1 + \rho} (1 - \varepsilon) + a^2 \varepsilon < 1$$

Discussion w/ related works

\[ \frac{1 - \epsilon}{1 + \alpha a^2 d} > 1 - \frac{1}{a^2} \]

6) Relation with sequential coding (any-time capacity)

Anytime coding/decoding

Fixed-length codes (our approach)

Necessary for optimality:

What is the role of capacity?

$SNR, d, \epsilon$ are not independent

\[
a^*(C) := \max_{SNR, d, \epsilon} |a| \\
\text{s.t. } \frac{1 - \epsilon}{1 + \frac{a^2 d}{SNR}} > 1 - \frac{1}{a^2} \\
(SNR, d, \epsilon) \in \Omega(C)
\]

Feasible set which depends on channel parameters

Control over wireless: a retrospect 15 years later

- Scientific impact: one of the most active and cited area in control
- Industrial impact: marginal
- Why?
  - The right tools (model-based control) for the wrong objective (stability)
  - Legacy control systems: PIDs (modeless)
  - No real need .... yet
Control over wireless: an outlook for the future

- Industry 4.0 (reconfigurable factory)
- UAVs based applications (infrastructure maintenance)
- Theoretical challenges?
  - Multi-agent cooperation over lossy nets: stability replaced by constraint satisfaction
  - 1Khz bandwidth range (manipulation)
  - Adaptive communication for control (RT-WiFi/5G)
Proof-of-concept:
UAV manipulation over wireless

Bandwidth [Hz]

- **1000 Hz**
- **100 Hz**
- **10 Hz**

**Manipulation bandwidth requirement:** \( >1kHz \) using PID or classic control. Today only via wired communication.

**Today’s state-of-the-art**

- **Reliable WI-FI for control (no packet loss, constant delay):** \(<20Hz\). Today only for formation control.

**Tomorrow’s Project goal**

- **Cooperative manipulation at 200Hz** via distributed MPC control
- **WI-FI for control up to 200Hz** via 802.11 real-time rate adaptation

**2-order of magnitude gap!!**
Questions?

URL: http://automatica.dei.unipd.it/people/schenato.html

Chiuso, N. Laurenti, L. Schenato, A. Zanella. **LQG control over finite capacity channels: the role of data losses, delays and SNR limitations.** *Automatica (submitted)*


A. Chiuso, N. Laurenti, L. Schenato, A. Zanella. **LQG cheap control over SNR-limited lossy channels with delay.** *Conference on Decision and Control (CDC13), 2013*

A. Chiuso, N. Laurenti, L. Schenato, A. Zanella. **LQG cheap control subject to packet loss and SNR limitations.** *European Control Conference ECC13, 2013*