Distributed Partitioning Strategies for Perimeter patrolling

Ruggero Carli Angelo Cenedese Luca Schenato

***Abstract*— In this work we study the problem of real-time optimal distributed partitioning for perimeter patrolling in the context of multi-camera networks for surveillance. The objective is to partition a line of fixed length into non-overlapping segments, each assigned to a different camera to patrol. Each camera has both physical mobility range and limited speed, and it must patrol its assigned segment by sweeping it back and forth at maximum speed. Here we provide the solution for the centralized optimal partitioning including the physical constrains of the cameras. Then we propose three different distributed control strategies to determine the extremes of the patrolling areas of each camera. All these strategies require only local communication with the neighboring cameras but adopt different communication schemes: synchronous, asynchronous symmetric gossip and asynchronous asymmetric gossip. For the first two schemes we provide theoretical convergence guarantees, while for the last scheme we provide numerical simulations showing the effectiveness of the proposed solution.**

I. INTRODUCTION

Current and future generations of video-surveillance systems target large scale scenarios where tens or even hundreds of smart Pan-Tilt-Zoom (PTZ) cameras coordinate one another to monitor the environment, cooperate so as to detect and track events, and perform high level decision tasks, through video content analysis algorithms.

To this aim, cameras appear as actuated sensor nodes embedded in the installation environment and connected through a communication network.

Trading-off between the complexity of the installation and the coverage performance, one canonical task of such systems is that of patrolling, meaning *the act of walking around an area in order to protect or supervise it* [1]. In this sense, a good patrolling strategy is one that minimizes the time lag between two visits to the same location, thus ensuring that all locations are constantly monitored.

In particular, there exists an interesting variety of outdoor video-surveillance scenarios where this task takes the structure of perimeter patrolling, in which the patrolling action is limited to the one dimensional boundary of the area to be protected.

To briefly review the literature, we start by pointing out how the problem of patrolling shows analogies with the problem of dynamic optimal coverage in sensor networks, as in [8], [9] where a team of mobile agents needs to coordinate to attain distributed coverage of an area while avoiding collisions. Similarly, considering coordinated robotic systems, interesting issues are raised in [10], where a multi-agent cooperative solution is studied to be robust and adaptive to perimeter changes. Also, the behavior of the agents so as to ensure efficient communication is taken into account. Again, in [5], [2] a theoretical analysis of multi-agent patrolling is carried out, where conditions that guarantee the existence of optimal strategies are studied by means of graphs analysis. As for multi-camera tracking in surveillance networks, most of the literature is concerned with computer-vision problems, for which we address the reader to the survey on intelligent surveillance systems [15]. On the other hand, in [6] a distributed algorithm for PTZ camera networks is presented, to perform coordinated task relying only on a local communication scheme to ensure system scalability.

A final and remarkable note is given, concerning the concept of equitable partitioning, studied in [13], [14] again within the scope of multiagent robotic systems: in this respect, the idea of partitioning the operational space into balanced areas of influence, while considering also the physical constraints of the agents is close in spirit to the problem we address (see also [3]), as will appear explicit in the formalization of the problem.

The remainder of the paper is organized as follows. In Section II we review the perimeter patrolling problem and its optimal solution. In Section III we formulate the problem we aim to solve in this paper. In Section IV, Section V and Section VI we propose three different solutions depending upon the communication protocol adopted by the cameras to exchange information. Specifically, in Section IV we assume the cameras communicate synchronously and we introduce the *synchronous optimal partitioning algorithm*. In Section V we assume the cameras communicate with each other through a symmetric-gossip type communication protocol and we present the *symmetric gossip partitioning algorithm*. Finally, in Section VI we assume the cameras exchange information according to an asymmetric gossip communication protocol and to deal with this scenario we introduce the *asymmetric gossip partitioning algorithm*.

II. PERIMETER PATROLLING

In this section we review the problem of patrolling a one-dimensional environment of finite length with a finite number of cameras and its optimal solution as described in [3].

Specifically let $\mathcal{L} = [-L, L]$, $L > 0$, denote the segment to be monitored and let N be the cardinality of the camera

The research leading to these results has received funding from the European Community's Seventh Framework Programme under agreement n. FP7-ICT-223866-FeedNetBack.

R. Carli, and L. Schenato are with the Department of Information Engineering, University of Padova, Via Gradenigo 6/a, 35131 Padova, Italy {carlirug|schenato}@dei.unipd.it. A.Cenedese is with the Department of management and Engineering, University of Padova, Stradella San Nicola 3, 36100 Vicenza, Italy angelo.cenedese@unipd.it

set. The cameras are labeled 1 through N and, for the sake of simplicity, we assume the following properties:

- the cameras are 1-d.o.f., meaning that the field of view (f.o.v.) of each camera is allowed to change due to pan movements only;
- the cameras have fixed coverage range, meaning that during pan movements the camera coverage range is not altered by the view perspective;
- cameras have point f.o.v..

In this context the following further definitions are introduced:

(i) $D_i = [D_{i,inf},\, D_{i,sup}] \subset \mathcal{L}$ is the total coverage range of i-th camera due to the scenario topology, the agent configuration and their physical constraints;

(ii) $v_i \in [-V_{i,max},\, +V_{i,max}]$ is the (bounded) speed of i-th camera during pan movements;

(iii) $A_i = [a_{i-1},\, a_i]$ denotes the effective coverage of the i-th camera where, clearly, it must hold $A_i \subseteq D_i, \forall\, i \in \{1, \ldots, N\}$;

(iv) $z_i(t) : \mathbb{R}^+ \to D_i$, is the continuous function mapping the position of the f.o.v. of the i-th camera as a function of the time variable t.

In our analysis, we assume that the coverage ranges D_i, $i \in \{1, \ldots, N\}$, satisfies the following *interlacing* constraint,

$$D_{i,inf} \leq D_{i+1,inf}, \qquad D_{i,sup} \leq D_{i+1,sup}.$$

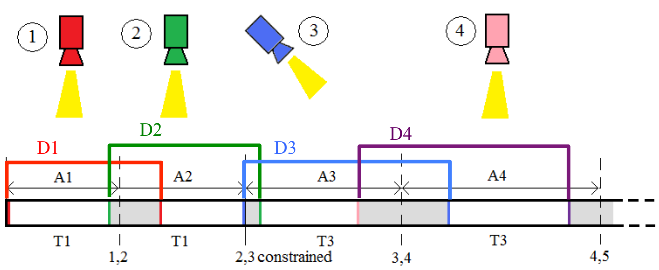

Fig. 1. Perimeter patrolled by a camera set. For the first four cameras, the physical coverages $\{D_i\}$ with some overlapping sections are shown, together with the optimal partition domains $\{A_i\}$.

In order to properly define the patrolling problem we need to introduce an appropriate cost function J and state an optimality criterium. The authors in [3] propose a functional J whose rationale is as follows: at each time instant t and position $x \in \mathcal{L}$, J is equal to 0 if location x is currently seen by any camera ($\exists z_i$ *s.t.* $z_i = x$), else it takes a positive real value as increasing as the time is passing since the last visit of $x \in \mathcal{L}$.

More simply, in this context, we reasonably assume that the cost J is a monotonic function of the time lag T_{lag} defined as the maximum (w.r.t. $x \in \mathcal{L}$) elapsed time between two visits of the same location, therefore the minimization problem for J corresponds to the computation of the smallest time lag T_o, constrained to the system dynamics

$$\dot{z}_i(t) = v_i(t) \quad s.t. \begin{cases} v_i(t) \in [-V_{i,max},\, +V_{i,max}] \\ z_i(t) \in D_i \end{cases} . \tag{1}$$

A. Optimal trajectory without coverage bounds

The optimal solution to the patrolling problem can be easily established in the absence of coverage constraint $z_i(t) \in D_i$, or equivalently, under the assumption that $D_i = \mathcal{L}$, $i \in \{1, \ldots, N\}$.

Proposition II.1 *The optimal coverage of the whole perimeter as the minimization of index J without the coverage constraint ($z_i(t) \in D_i$) as in* (1)*, is attained assuming that every camera is moving at its maximum speed $|V_{i,max}|$ with a periodical motion of period $\bar{T}$ in non-overlapping coverage interval A_i. The area length $|A_i|$ and optimal period $\bar{T}$ are obtained as*

$$|A_i| = |V_{i,max}| T_o \quad \textit{and} \quad \bar{T} = 2T_o = \frac{2L}{\sum_{i=1}^{N} |V_{i,max}|}. \tag{2}$$

The proof of the above Proposition can be found in [3].

B. Optimal trajectory with coverage bounds

In general, the optimal solution to the patrolling problem without any constraint is not equivalent to the constrained optimal solution; this happens if the unconstrained solution is feasible, that is if and only if $A_i \subseteq D_i$. In this respect, we introduce the optimal patrolling period with constraints, $T_{o,c}$. In general it holds: $T_{o,c} \geq T_o$, the equality standing only when the feasibility constraint is not violated.

To cope with the case where the unconstrained solution violates this feasibility condition, the authors in [3] suggest an optimal strategy that yields the splitting of the patrolling perimeter into smaller subintervals. This idea is here formalized in the following proposition that basically extends the results presented in [3] (violation of the first camera constraints).

Proposition II.2 *If the unconstrained solution yields $A_i \nsubseteq D_i$, the optimal coverage is attained by splitting the domain into $\mathcal{L}^l = [-L, D_{i,inf}]$ and $\mathcal{L}^r = [D_{i,sup}, L]$, and considering two separate coverage problems. Being T_o^l and T_o^r the optimal periods for the subdomains, the global coverage period is obtained as $T_{o,c} = \max\left\{T_o^l, T_o^r\right\}$.*

Proof: We start by assigning the i-th partition as $A_i = D_i$, with a corresponding patrolling period of T_i: note that $T_i < T_o$, otherwise there would not be any constraint violation. The optimal policy for the two subdomains $\mathcal{L}^l$ and $\mathcal{L}^r$ is then computed according to Prop. II.1, obtaining respectively T_o^l and T_o^r, with at least one of the two larger than T_o. In fact, if this were not the case, this very partition $\{\mathcal{L}^l, D_i, \mathcal{L}^r\}$ would attain a global patrolling period lower than the optimal solution T_o. It follows that $T_{o,c} = \max\left\{T_o^l, T_o^r\right\}$.

If both $T_i < T_o^l$ and $T_i < T_o^r$, then the optimal partition shows the three domains $\mathcal{L}^l$, D_i, $\mathcal{L}^r$. Conversely, if only $T_o^l < T_i < T_o^r$ (or $T_o^r < T_i < T_o^l$) stands, the partition is bipartite, $\{\mathcal{L}^l \cup D_i, \mathcal{L}^r\}$ (or $\{\mathcal{L}^l, D_i \cup \mathcal{L}^r\}$), with a patrolling time for the first interval that is a convex combination of T_o^l and T_i, and $T_{o,c} = T_o^r$ for the second interval (similarly, for the other case, with $T_{o,c} = T_o^l$) ■

This strategy may then be iterated for all agents violating the constraints.

Loosely speaking, Proposition II.1 and Proposition II.2 state that the optimal solution is attained by dividing the segment $\mathcal{L}$ into N separate segments, and having each camera following a periodical motion at its maximum speed in its coverage area A_i. In doing so the problem of minimizing the index J (or equivalently the time lag T_{lag}) is reduced to the optimal choice of A_i for each camera. This optimal partitioning problem will be the focus of our analysis in the rest of the paper.

We consider an "iterative" and "distributed" scenario. Specifically, we assume each camera is initialized at time $t = 0$ with a partition $A_i(0)$ that, in general, does not coincide with the optimal solution. Each camera is allowed to iteratively update A_i using only the local information coming from the neighboring cameras. The goal is to provide strategies that lead the cameras to asymptotically reach the optimal steady-state configuration for patrolling extremes.

In the next section we formally describe the setup we consider and the problem we aim to solve.

III. Distributed optimal partitioning: problem formulation

We assume that at time $t = 0$ each camera is initialized with a *dominance interval* $A_i(0)$. More precisely, for $i \in \{1, \ldots, N\}$ let $A_i(0) = [a_{i,\ell}(0), a_{i,r}(0)]$ where $a_{i,\ell}(0)$ and $a_{i,r}(0)$ are respectively the left extreme and the right extreme of $A_i(0)$. We assume that the set $\{A_1(0), \ldots, A_N(0)\}$ satisfies three constraints. Firstly, we assume a *physical constraint*, that is, for $i \in \{1, \ldots, N\}$, $A_i \subseteq D_i$. Secondly, a *covering constraint* is posed, that is,

$$\bigcup_{i\in\{1,\ldots,N\}} A_i(0) = \mathcal{L}.$$

Finally, the *interlacing constraint* is introduced, meaning that, for $i \in \{1, \ldots, N-1\}$,

$$a_{i,\ell}(0) \le a_{i+1,\ell}(0), \qquad a_{i,r}(0) \le a_{i+1,r}(0).$$

Observe that the *interlacing* and the *covering* constraints imply that $a_{1,\ell}(0) = -L$ and $a_{N,r}(0) = L$.

The goal is to design iterative algorithms that allow the cameras to update their dominance intervals using only information coming from neighboring cameras and such that

(i) the physical, the covering and the interlacing constraints are satisfied at each iteration; and
(ii) the set of dominance intervals converge to the optimal partition.

It is worth clarifying that for neighboring cameras we mean that camera i, $i \in \{2, \ldots, N-1\}$ exchange information with camera $i-1$ and camera $i+1$. If $i = 1$ (resp. $i = N$) the only neighbor of camera 1 (resp. N) is camera 2 (resp. $N-1$).

In the next sections we consider three different scenarios depending upon the communication protocol adopted by the set of cameras to exchange information with each other. Specifically, in Section IV we suppose that the cameras communicate with each other synchronously, that is, at each communication round, each camera transmits to its neighbors the information related to its current dominance interval. In this context, we introduce the *synchronous optimal partitioning algorithm*.

In Section V, we relax the synchronism of the previous section and we consider a symmetric gossip-type communication protocol; in this case, at each iteration of the partitioning algorithm only a pair of neighboring cameras communicate with each other while the other cameras do not transmit or receive any information. Accordingly, we introduce the *symmetric-gossip optimal partitioning algorithm*.

Finally, in Section VI, we assume the cameras adopt an asymmetric gossip-type communication protocol. While in the symmetric gossip at each communication round the active link is bidirectional, that is, if camera i transmits to camera $i+1$,then at the same time camera $i+1$ transmits to camera i, in the asymmetric gossip only one direction is active, that is, either camera i transmits to camera $i+1$ or camera $i+1$ transmits to camera i. In this section, we introduce the *asymmetric-gossip optimal partitioning algorithm*.

IV. Synchronous optimal partitioning algorithm

In this section, we introduce the *synchronous optimal partitioning algorithm* (denoted as *OPA* hereafter). We start our analysis by considering the case without physical constraints, or equivalently by assuming that $D_i = [-L, L]$.

For the sake of the notational convenience, in this section and in the rest of the paper we denote $V_{i,max}$ simply by v_i.

A. Implementation without physical constraints

This algorithm is formally described as follows.

Processor states: For each $i \in \{1, \ldots, N\}$, camera i keeps in memory the extremes defining its *dominance interval*, i.e., $a_{i,\ell}$ and $a_{i,r}$. Moreover, we assume also that each camera knows the maximum patrolling-speed of its neighboring cameras;

Initialization: For $i \in \{1, \ldots, N\}$ values $a_{i,\ell}(0), a_{i,r}(0)$ are given as part of the problem. We assume that the initial conditions satisfy the *covering* and *interlacing* constraints.

Transmission iteration: At each time instant $t \in \mathbb{N}$, each camera transmits to its neighbors the extremes of its dominance interval and receives from its neighbors the extremes of their neighbors' dominance regions.

Extremes' iteration: At time $t \in \mathbb{N}$, for $i \in \{2, \ldots, N-1\}$, camera i updates the value of its current left extreme $a_{i,\ell}(t)$ to the value $a_{i,\ell}(t+1) \in [a_{i-1,\ell}(t), a_{i,r}(t)]$ according to the following *neighbors' equal traveling time* criterion
"*the time required to camera i to travel at the speed v_i from $a_{i,\ell}(t+1)$ to $a_{i,r}(t)$ is equal to the time required by the camera $i-1$ to travel at speed v_{i-1} from $a_{i-1,\ell}(t)$ to $a_{i,\ell}(t+1)$*";
similarly camera i updates the value of its current right extreme $a_{i,r}(t)$ to the value $a_{i,r}(t+1) \in [a_{i,\ell}(t), a_{i+1,r}(t)]$ in such way that

"*the time required to camera i to travel at the speed v_i from $a_{i,\ell}(t)$ to $a_{i,r}(t+1)$ is equal to the time required by the camera $i+1$ to travel at speed v_{i+1} from $a_{i,r}(t+1)$ to $a_{i+1,r}(t)$*".

Formally $a_{i,\ell}(t+1)$ and $a_{i,r}(t+1)$ satisfy the conditions

$$\frac{a_{i,\ell}(t+1) - a_{i-1,\ell}(t)}{v_{i-1}} = \frac{a_{i,r}(t) - a_{i,\ell}(t+1)}{v_i} \tag{3}$$

and

$$\frac{a_{i+1,r}(t) - a_{i,r}(t+1)}{v_{i+1}} = \frac{a_{i,r}(t+1) - a_{i,\ell}(t)}{v_i}. \tag{4}$$

From Eqs. 3-4 it follows

$$a_{i,\ell}(t+1) = \frac{a_{i,r}(t)v_{i-1} + a_{i-1,\ell}(t)v_i}{v_{i-1} + v_i} \tag{5}$$

and

$$a_{i,r}(t+1) = \frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}}. \tag{6}$$

For $i = 1$ and $i = N$ we have that

$$a_{1,\ell}(t) = -L, \qquad a_{N,r}(t) = L, \qquad \text{for all } t \in \mathbb{N},$$

while $a_{1,r}(t)$ and $a_{N,\ell}(t)$ are updated similarly to (5) and (6), i.e.,

$$a_{1,r}(t+1) = \frac{a_{2,r}(t)v_1 - Lv_2}{v_1 + v_2}, \tag{7}$$

and

$$a_{N,\ell}(t+1) = \frac{a_{N-1,\ell}(t)v_N + Lv_{N-1}}{v_{N-1} + v_N}. \tag{8}$$

Observe that from (5), (6) , (8) and (7) it follows that, for $t \geq 1$,

$$a_{i,r}(t) = a_{i+1,\ell}(t) \qquad \text{for } i \in \{1, \dots, N-1\}. \tag{9}$$

This fact allows for a simpler description of the *OPA* as we next show. For $t \geq 1$, let us introduce the variables $a_0(t), a_1(t), \dots, a_N(t)$ defined by

$$a_0(t) := -L,\ a_i(t) := a_{i,r}(t) = a_{i+1,\ell}(t),\ a_N(t) := L \tag{10}$$

Then the dynamics of the *OPA* algorithm can be equivalently described by the following updating rule, for $t \geq 1$,

$$a_i(t+1) = \frac{a_{i+1}(t)v_i + a_{i-1}(t)v_{i+1}}{v_i + v_{i+1}}. \tag{11}$$

Before stating the main result of this section we observe that, according to what proved in Proposition II.2, the optimal partition $\{A_i^*\}_{i=1}^N$, $A_i^* = [a_{i-1}^*, a_i^*]$, for the case with physical constraints, can be formally described as

$$\begin{aligned} a_0^* &= -L, \quad a_N^* = L \\ a_i^* &= a_{i-1}^* + v_i T_o \end{aligned} \tag{12}$$

where T_o is defined as in (2).

The following Proposition state the main properties of the *OPA*.

Theorem IV.1 *Consider the* OPA *under the assumption that $D_i = \mathcal{L}$ for $i \in \{1, \dots, N\}$. Let $\{A_i(0)\}_{i=1}^N$ be the initial set of dominance intervals, which is assumed to satisfy the* covering *and* interlacing *constraints. Then, the evolution $t \to \{A_i(t)\}$ generated by the* OPA *satisfies:*

(i) *the* covering *and* interlacing *constraints are verified for all $t \in \mathbb{N}$; and*

(ii) *the set $\{A_i(t)\}_{i=1}^N$ converges asymptotically to the optimal solution $\{A_i^*\}_{i=1}^N$.*

Before providing the proof of the above proposition it is worth clarifying that for $\{A_i(t)\}_{i=1}^N$ converging to $\{A_i^*\}_{i=1}^N$ we mean that

$$\lim_{t\to\infty} a_{i,\ell}(t) = a_{i-1}^*, \qquad \lim_{t\to\infty} a_{i,r}(t) = a_i^*.$$

Proof: The proof of (i) follows directly from the fact that, for $t \geq 1$, the dynamics of the *OPA* can be equivalently described using the variables $a_0, \dots, a_N$ introduced in (10).

To prove fact (ii), we introduce an additional set of N auxiliary variables. For $i \in \{1, \dots, N\}$ let

$$x_i(t) := \frac{a_i(t) - a_{i-1}(t)}{v_i}. \tag{13}$$

Standard algebraic manipulations show that x_i, $i \in \{1, \dots, N\}$, satisfy the following recursive equations

$$\begin{aligned} x_1(t+1) &= \frac{v_1}{v_1 + v_2} x_1(t) + \frac{v_2}{v_1 + v_2} x_2(t) \\ x_N(t+1) &= \frac{v_{N-1}}{v_{N-1} + v_N} x_{N-1}(t) + \frac{v_{N-1}}{v_{N-1} + v_N} x_N(t) \end{aligned}$$

and, for $i \in \{2, \dots, N-1\}$,

$$\begin{aligned} x_i(t+1) = \frac{v_{i-1}}{v_i + v_{i-1}} x_{i-1} &+ \frac{v_i^2 - v_{i-1}v_{i+1}}{(v_i + v_{i+1})(v_i + v_{i-1})} x_i + \\ &+ \frac{v_{i+1}}{v_i + v_{i+1}} x_{i+1}. \end{aligned}$$

Let $x(t) = [x_1(t), \dots, x_N(t)]^T$, then we can write

$$x(t+1) = Ax(t),$$

where the elements of the $N \times N$ matrix A are defined by the previous equations. Observe that

$$\begin{aligned} &\frac{v_1}{v_1 + v_2} + \frac{v_2}{v_1 + v_2} = 1 \\ &\frac{v_{i-1}}{v_i + v_{i-1}} + \frac{v_i^2 - v_{i-1}v_{i+1}}{(v_i + v_{i+1})(v_i + v_{i-1})} + \frac{v_{i+1}}{v_i + v_{i+1}} = 1 \\ &\frac{v_{N-1}}{v_{N-1}} + \frac{v_{N-1}}{v_{N-1} + v_N} = 1, \end{aligned}$$

or equivalently that $A\mathbf{1} = \mathbf{1}$ where $\mathbf{1}$ denotes the vector with all the components equal to 1. This shows that 1 is eigenvalue of A with corresponding eigenvector the vector $\mathbf{1}$. Tedious manipulations show also that the algebraic multiplicity of A is equal to 1.

By observing that $\left|\frac{v_i^2 - v_{i-1}v_{i+1}}{(v_i+v_{i+1})(v_i+v_{i-1})}\right| < 1$, it follows from Gershgorin theorem that all the eigenvalues of A are inside the unitary circle. However one can show that equation $Ay = zy$, where $|z| = 1$, $y \in \mathbb{R}^N$ admits the unique solution $z = 1$

and $y = \mathbf{1}$, implying that all the eigenvalues of A are actually strictly inside the unitary circle, except one eigenvalue which is equal to 1.

Therefore, $\lim_{t\to\infty} x(t) = \alpha\mathbf{1}$ for some suitable α. Since the intervals $A_i = [a_{i-1}, a_i]$, $i \in \{1, \ldots, N\}$ have disjoint interiors and since $A_1 \cup \ldots \cup A_N = \mathcal{L}$, it follows that α coincides with the value of T_o defined in Proposition II.1. This concludes the proof. ■

We evaluate now the performance of the *OPA* in terms of number of iterations required to lead the set of dominance intervals $\{A_i(t)\}_{i=1}^N$ close enough to the optimal partition $\{A_i^*\}_{i=1}^N$. To make precise the concept of *being close enough* we proceed as follows. Let $\mathcal{I}_\mathcal{L}$ denote the set of all the sub-intervals of $\mathcal{L}$. Then, for $\epsilon > 0$, let us introduce the notion of ϵ*-optimal partition* task $\mathcal{T}_{\epsilon-OP} : \mathcal{I}_\mathcal{L}^N \to \{\text{true}, \text{false}\}$ defined as

$$\mathcal{T}_{\epsilon-OP}\left(\{A_i\}_{i=1}^N\right) = \begin{cases} \text{true}, & \text{if } |a_{i,r} - a_i^*| \leq \epsilon, \quad |a_{i,\ell} - a_{i-1}^*| \leq \epsilon, \quad \forall i \\ \text{false}, & \text{otherwise} \end{cases}$$

Accordingly, we introduce the notion of (worst-case) time complexity $\mathcal{TC}(\mathcal{T}_{\epsilon-OP}, \textit{OPA})$ as

$$\mathcal{TC}(\mathcal{T}_{\epsilon-OP}, \textit{OPA}) = \sup_{\{A_i(0)\}_{i=1}^N} \inf \Big\{ t \in \mathbb{N} \text{ s.t. } \mathcal{T}_{\epsilon-OP}\left(\{A_i(t')\}_{i=1}^N\right) = \text{true}, \forall t' \geq t \Big\}$$

The following proposition characterizes the time-complexity of the *OPA* under the assumption that $v_i = v$ for some $v > 0$ and for all $i \in \{1, \ldots, N\}$.

Proposition IV.2 *Consider the* OPA *under the assumption that, for all* $i \in \{1, \ldots, N\}$, $v_i = v$. *Then*

$$\mathcal{TC}(\mathcal{T}_{\epsilon-OP}, \text{OPA}) \in \Theta\left(N^2 \log(\epsilon^{-1})\right)$$

Proof: Let A be the $N \times N$ dimensional matrix as defined in the previous proof. We observe that, when $v_i = v$ for all $i \in \{1, \ldots, N\}$, $A = \text{ATrid}_N^+(1/2, 0)$ where we refer the reader to [11] and [4] for a formal definition of $\text{ATrid}_N^+(1/2, 0)$.

The proof follows by applying Theorem II.2 in [11]. ■

B. Implementation with physical constraints

In this section we suitably modify the *OPA* illustrated in the previous section in order to adapt it to a general set of physical constraints $D_1, \ldots, D_N$. Basically, the modifications we introduce are two.

The first modification refers to the additional knowledge that each camera must have about the physical constraints of its neighbors. Specifically, we assume the processor of the i-th camera keeps in memory not only $a_{i,\ell}(t)$, $a_{i,r}(t)$, v_i, v_{i-1}, v_{i+1} but also D_i, D_{i-1} and D_{i+1}.

Secondly, we have to take into account the fact that the updating rules (5), (6) might violate the physical constraints; for instance it might happen that $a_{i,r} > D_{i,sup}$ or $a_{i,\ell} < D_{i,inf}$. To deal with this situation we modify the extremes' updating rules as follows. Without loss of generality we consider only how cameras i and $i+1$ update $a_{i,r}$ and $a_{i+1,\ell}$, respectively.

We distinguish between three cases

$$(i) \quad \frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}} > D_{i,sup};$$

$$(ii) \quad \frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}} < D_{i+1,inf};$$

$$(iii) \quad \frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}} \in [D_{i+1,inf}, D_{i,sup}].$$

If case (i) is verified then

$$a_{i,r}(t+1) = a_{i+1,\ell}(t+1) := D_{i,sup};$$

if case (ii) is verified then

$$a_{i,r}(t+1) = a_{i+1,\ell}(t+1) := D_{i+1,inf};$$

if case (iii) is verified then

$$a_{i,r}(t+1) = a_{i+1,\ell}(t+1) := \frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}}.$$

One can see that the modified *OPA* satisfy the property that the *physical*, the *covering* and the *interlacing* constraints are satisfied for all $t \geq 0$. A theoretical analysis of the convergence properties of the modified *OPA* will be the subject of future research.

V. SYMMETRIC GOSSIP-TYPE IMPLEMENTATION

In this section we introduce the *symmetric gossip optimal partitioning algorithm* (denoted as *sOPA* hereafter). This algorithm is based on a symmetric gossip-type communication protocol. Basically at each iteration of the algorithm only a pair of neighboring cameras exchange information with each other, while the remaining cameras do not transmit or receive any information.

Again, we start our analysis by considering the unconstrained problem, or equivalently by assuming that $D_i = [-L, L]$.

A. Implementation without physical constraints

With the respect of the *OPA*, the **Transmission iteration** and the **Extremes' update** are changed as follows

Transmission iteration: For $t \in \mathbb{N}$, during the t-th iteration of the *sOPA*, only a pair of neighboring cameras $(i, i+1)$ communicate with each other; the communicating link is bidirectional, namely, camera i sends to camera $i+1$ the values of its extremes $a_{i,\ell}(t)$ and $a_{i,r}(t)$, and camera $i+1$ sends to camera i the values of its extremes $a_{i+1,\ell}(t)$ and $a_{i+1,r}(t)$.

Extremes' iteration: For $h \notin \{i, i+1\}$, camera h left unchanged its extremes, i.e., $a_{h,\ell}(t+1) = a_{h,\ell}(t)$ and $a_{h,r}(t+1) = a_{h,r}(t)$. Based on the information received, camera i modifies only its right extreme while camera modifies only its left extreme. Analogously to *OPA* we have that

$$a_{i,r}(t+1) = a_{i+1,\ell}(t+1) := \frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}}. \tag{14}$$

We characterizes now the convergence properties of the *sOPA*. We provide conditions ensuring both deterministic and probabilistic convergence. We start with the deterministic convergence.

Theorem V.1 *Consider the* sOPA*, under the assumption that* $D_i = \mathcal{L}$ *for* $i \in \{1, \ldots, N\}$*. Let* $\{A_i(0)\}_{i=1}^N$ *be the initial set of dominance intervals which is assumed to satisfy the* covering *and* interlacing *constraints. Moreover assume that there exists a positive integer number* T *such that, for all* $t \in \mathbb{N}$*, any pair of neighboring cameras* $(i, i+1)$*,* $i \in \{1, \ldots, N-1\}$*, communicates with each other at least once within the interval* $[t, t+T)$*. Then the evolution* $t \to \{A_i(t)\}$ *generated by the* sOPA *algorithm satisfies:*

(i) *the* covering *and* interlacing *constraints are verified for all* $t \in \mathbb{N}$*; and*
(ii) *the set* $\{A_i(t)\}_{i=1}^N$ *converges asymptotically to the optimal solution* $\{A_i^*\}_{i=1}^N$.

Proof: Observe that after T iterations of the *sOPA*, we have that $a_{i,r}(t) = a_{i+1,\ell}(t)$. Hence we can introduce the auxiliary variables $a_0, a_1, \ldots a_N$ defined in (10) and, in turn, the variables $x_1, \ldots x_N$ defined in (13).

In this context we can write that

$$x(t+1) = A(t)x(t),$$

where the matrix $A(t)$ depends on which pair of neighboring cameras communicate with each other during the t-th iteration of the *sOPA*. Let $(h, h+1)$ be such pair, for some $h \in \{1, \ldots, N-1\}$. Then

$$[A(t)]_{ij} = \begin{cases} 1 & \text{if } i = j,\ i \neq h,\ i \neq h+1 \\ \frac{v_h}{v_h + v_{h+1}} & \text{if } i = j = h \\ \frac{v_{h+1}}{v_h + v_{h+1}} & \text{if } i = j = h+1 \\ \frac{v_h}{v_h + v_{h+1}} & \text{if } i = h+1,\ j = h \\ \frac{v_{h+1}}{v_h + v_{h+1}} & \text{if } i = h,\ j = h+1 \\ 0 & \text{otherwise} \end{cases}$$

One can see that for any $t \in \mathbb{N}$ the sequence of matrices in $A(t), A(t+1), \ldots, A(T)$ satisfy the conditions given in [12] to guarantee that the previous time-varying system converges to consensus. Hence, we have that $\lim_{t\to\infty} x(t) = \alpha \mathbf{1}$ and reasoning similarly to the proof of the previous Theorem we can get the desired results. ∎

We provide now conditions ensuring probabilistic convergence.

Theorem V.2 *Consider the* sOPA *algorithm under the assumption that* $D_i = \mathcal{L}$ *for* $i \in \{1, \ldots, N\}$*. Let* $\{A_i(0)\}_{i=1}^N$ *be the initial set of dominance intervals which is assumed to satisfy the* covering *and* interlacing *constraints. Moreover assume that there exists a real number* $\bar{p}$*,* $0 < \bar{p} < 1$*, such that, for all* $t \in \mathbb{N}$ *and for all* $i \in \{1, \ldots, N-1\}$

$$\mathbb{P}\left[(i, i+1) \text{ communicates at iteration } t\right] \geq \bar{p}. \quad (15)$$

Then the evolution $t \to \{A_i(t)\}$ *generated by the* sOPA *algorithm satisfies:*

(i) *the* covering *and* interlacing *constraints are verified for all* $t \in \mathbb{N}$*; and*
(ii) *the set* $\{A_i(t)\}_{i=1}^N$ *converges almost surely to the optimal solution* $\{A_i^*\}_{i=1}^N$.

Proof: The analysis is similar to the previous Theorem, with the difference that $x(t+1) = A(t)x(t)$ is a stochastic system. One can show that, thanks to condition (15), the system $x(t+1) = A(t)x(t)$ satisfies the assumption of Corollary 3.2 of [7], which ensures that almost surely $\lim_{t\to\infty} x(t) = \alpha\mathbf{1}$, where α is as in the proof of Theorem IV.1. ∎

B. Implementation without physical constraints

In presence of general physical constraints D_i the above updating rules are modified similarly to the previous scenario.

VI. Asymmetric gossip-type implementation

In this section we introduce the *asymmetric gossip optimal partitioning algorithm* (denoted as *aOPA* hereafter). This algorithm is based on an asymmetric gossip-type communication protocol. This communication protocol is less demanding than the symmetric gossip-type communication protocol since it does not require a bidirectional exchange of information. Indeed, at each iteration of the algorithm there is only one camera sending information to one if its neighbors.

Similarly to the previous sections, we first consider the unconstrained case.

A. Implementation without physical constraints

With the respect of the *OPA* and *sOPA*, the **Transmission iteration** and the **Extremes' update** are changed as follows

Transmission iteration: For $t \in \mathbb{N}$, there is only one camera transmitting information to one of its neighbors during the t-th iteration of the *aOPA*; without loss of generality we assume that camera i transmits the values of its extremes $a_{i,\ell}(t)$ and $a_{i,r}(t)$ to camera $i+1$;

Extremes' iteration: For $h \neq i+1$, camera h left unchanged its extremes, i.e., $a_{h,\ell}(t+1) = a_{h,\ell}(t)$ and $a_{h,r}(t+1) = a_{h,r}(t)$. Based on the information received camera $i+1$ updates only the extreme "closer" to camera i. Specifically $a_{i+1,r}(t+1) = a_{i+1,r}(t)$ while $a_{i+1,\ell}(t+1)$ is updated as follows: if $\frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}} > a_{i,r}(t)$ then

$$a_{i+1,\ell}(t+1) := a_{i,r}(t) \quad (16)$$

otherwise

$$a_{i+1,\ell}(t+1) := \frac{a_{i+1,r}(t)v_i + a_{i,\ell}(t)v_{i+1}}{v_i + v_{i+1}}. \quad (17)$$

Unfortunately, we were not able to prove so far any type of convergence of the above algorithm. However we conjecture that assumptions similar to the ones stated in Theorem V.1 and in Theorem V.2 guarantee also the deterministic and the almost-surely convergence of the *aOPA*. To make more precise this last statement let us denote by $\mathcal{E}$ the set describing the feasible communications of the *aOPA*, i.e.,

$$\mathcal{E} = \{(1,2)\} \cup \{(N, N-1)\} \cup \{(i,i-1),(i,i+1)\}_{i=2}^{N-1}$$

where the pair (i,j) denotes the directional link meaning that camera i sends its information to camera j. We conjecture the following two facts:

Conjecture VI.1 *Assume that there exists a positive integer number T such that, for all $t \in \mathbb{N}$, any pair in $\mathcal{E}$ is selected at least once within the interval $[t, t+T)$. Then the evolution $t \to \{A_i(t)\}$ generated by the* aOPA*, starting from an initial condition satisfying the* covering *and* interlacing *constraints, converges asymptotically to the optimal solution $\{A_i^*\}_{i=1}^N$.*

Conjecture VI.2 *Assume a real number $\bar{p}$, $0 < \bar{p} < 1$, such that for all $t \in \mathbb{N}$ and for all edge $(i,j) \in \mathcal{E}$*

$$\mathbb{P}\left[(i,j) \text{ is selected at iteration } t\right] \geq \bar{p}.$$

Then the evolution $t \to \{A_i(t)\}$ generated by the aOPA *starting from an initial condition satisfying the* covering *and* interlacing *constraints converges almost surely to the optimal solution $\{A_i^*\}_{i=1}^N$.*

We show the effectiveness of *aOPA* in Section VII through some numerical simulation. We conclude by observing that, if $\{A_i(0)\}_{i=1}^N$ satisfies the *covering* and *interlacing* constraints, then the updating rules (16) and (17) imply $a_{i+1,\ell}(t+1) \leq a_{i,r}(t)$ for all $t \geq 0$; in other words the *covering* and *interlacing* constraints are satisfied also by $\{A_i(t)\}_{i=1}^N$ for all $t \geq 0$.

B. Implementation with physical constraints

In presence of general physical constraints D_i the above updating rules are modified similarly to the previous two scenarios.

VII. Numerical examples

In this section we provide two examples showing the effectiveness of the *aOPA*.

Example VII.1 We consider a set of 50 cameras with the goal of patrolling the interval $\mathcal{L} = [-100, 100]$. We assume that the velocities are all equal to the same value v, i.e., $v_i = v$, for all $i \in \{1, \dots, N\}$. We assume that at each iteration of the *aOPA*, an edge of $\mathcal{E}$ is randomly chosen.

To evaluate the performance of *aOPA* we consider the following functional cost

$$J(t) = \frac{1}{N}\sum_{i=1}^{10}\left(a_{i,\ell}(t) - a_{i-1}^*\right)^2 + \left(a_{i,r}(t) - a_i^*\right)^2$$

where according to (12) we have $a_i^* = -100 + 4 * i$. The obtained result is plotted in Figure 2. Observe that J goes to 0 as t increases showing the effectiveness of the *aOPA*.

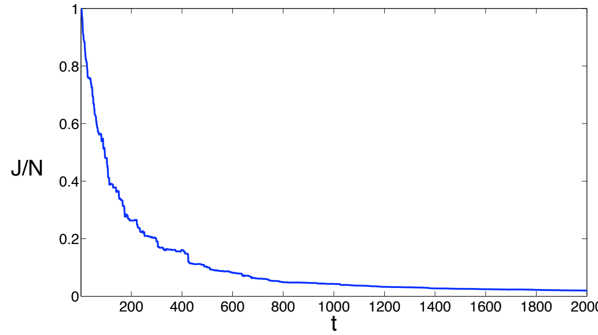

Fig. 2. Simulation of the aOPA with $N = 50$ cameras.

Example VII.2 We consider a set of $N = 6$ cameras with the goal of patrolling the interval $\mathcal{L} = [-100, 100]$. Again we assume that all the velocities v_i, $i \in \{1, \dots, N\}$, take the same value v and that at each iteration of the *aOPA*, an edge of $\mathcal{E}$ is randomly chosen. In Figure 3 we plot the behavior of $a_{i,\ell}$, $a_{i,r}$, $i \in \{1, \dots, N\}$. The simulation shows how $a_{i+1,\ell}$ and $a_{i,r}$ converge to the same value.

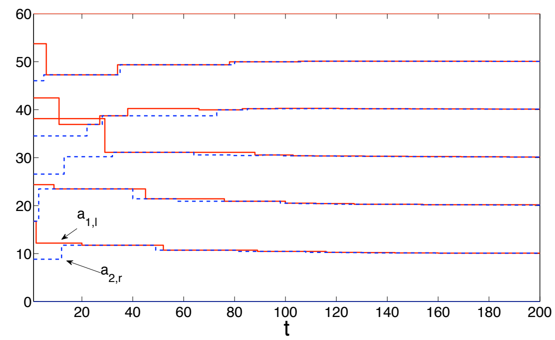

Fig. 3. Behavior of the extremes when aOPA is applied with $N = 6$ cameras.

References

[1] F. R. Abate, Ed., *The Oxford Dictionary and Thesaurus: The Ultimate Language Reference for American Readers*. Oxford University Press, 1996.

[2] A. Almeida, G. Ramalho, H. Santana, P. Tedesco, T. Menezes, V. Corruble, and C. Y., "Recent advances on multi-agent patrolling," *Lecture Notes in Computer Science*, vol. 3171, p. 474?483, 2004.

[3] M. Baseggio, A. Cenedese, P. Merlo, M. Pozzi, and L. Schenato, "Distributed perimeter patrolling and tracking for camera networks," in *Conference on Decision and Control (CDC10)*, December 15 2010, p. to appear.

[4] R. Carli and F. Bullo, "Quantized coordination algorithms for rendezvous and deployment," *SIAM Journal on Control and Optimization*, vol. 48, no. 3, pp. 1251–1274, 2009.

[5] Y. Chevaleyre, "Theoretical analysis of the multi-agent patrolling problem," sep. 2004, pp. 302 – 308.

[6] C. Costello and I.-J. Wang, "Surveillance camera coordination through distributed scheduling," dec. 2005, pp. 1485 – 1490.

[7] F. Fagnani and S. Zampieri, "Randomized consensus algorithms over large scale networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 4, pp. 634–649, 2008.

[8] I. Hussein and D. Stipanovic, "Effective coverage control using dynamic sensor networks," dec. 2006, pp. 2747 –2752.

[9] ——, "Effective coverage control using dynamic sensor networks with flocking and guaranteed collision avoidance," jul. 2007, pp. 3420 – 3425.

[10] D. Kingston, R. Beard, and R. Holt, "Decentralized perimeter surveillance using a team of uavs," *Robotics, IEEE Transactions on*, vol. 24, no. 6, pp. 1394 –1404, dec. 2008.

[11] S. Martínez, F. Bullo, J. Cortés, and E. Frazzoli, "On synchronous robotic networks – Part II: Time complexity of rendezvous and deployment algorithms," in *IEEE Conf. on Decision and Control and European Control Conference*, Seville, Spain, Dec. 2005, pp. 8313–8318.

[12] L. Moreau, "Stability of multiagent systems with time-dependent communication links," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 169–182, 2005.

[13] M. Pavone, A. Arsie, E. Frazzoli, and F. Bullo, "Equitable partitioning policies for mobile robotic networks," *IEEE Transactions on Automatic Control*, Provisionally Accepted, 2008.

[14] ——, "Equitable partitioning policies for robotic networks," in *ICRA'09: Proceedings of the 2009 IEEE international conference on Robotics and Automation*. Piscataway, NJ, USA: IEEE Press, 2009, pp. 3979–3984.

[15] M. Valera and S. Velastin, "Intelligent distributed surveillance systems: a review," *Vision, Image and Signal Processing, IEE Proceedings -*, vol. 152, no. 2, pp. 192 – 204, apr. 2005.