Mapping and Coverage Control in Robotics Networks

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April 1, 2016
Why Localization and Mapping?
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Why Multirobot?

1. Better localization (error $\frac{\sigma}{\sqrt{N}}$),
2. Map building can be $N$ time faster.

But there are some difficulties:

1. Coordination
2. Integration of the information
3. Limited communication
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2. Map building can be $N$ time faster.

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How to localize the robots?

1. Sensors
2. Sensor fusion
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1. Sensors
2. Sensor fusion

A. Carron et al. “Multi-Robot Localization via GPS and Relative Measurements in the Presence of Asynchronous and Lossy Communication”. In: ECC 16
M. Todescato et al. “Distributed Localization from Relative Noisy Measurements: a Robust Gradient Based Approach”. In: ECC 15
A. Carron et al. “Adaptive consensus-based algorithms for fast estimation from relative measurements”. In: IFAC NecSys 13
Thesis Contributions

Localization:
1. efficient
2. distributed
3. heterogeneous measurements

Mapping:
1. efficient
2. applied to coverage control
3. time-varying functions
Multi-robots Client-Server Gaussian Estimation and Coverage Control with Lossy Communications


J. Choi, J. Lee, and S. Oh. “Swarm intelligence for achieving the global maximum using spatio-temporal Gaussian processes”. In: *ACC 08*. 2008

Contributions

1. Estimation from noisy measurements
2. Bounds on the estimation error
3. Robustness
Voronoi Partitions and Coverage

\[ \text{Environment } \mathcal{X} \]
Voronoi Partitions and Coverage

Environment $\mathcal{X}$

Agents $x_1, \ldots, x_N$
Voronoi Partitions and Coverage

Environment $\mathcal{X}$

Agents $x_1, \ldots, x_N$

Voronoi Partitions

$\mathcal{P} = \mathcal{W}(x_1, \ldots, x_N)$
Voronoi Partitions and Coverage

- Environment $\mathcal{X}$
- Agents $x_1, \ldots, x_N$
- Voronoi Partitions $\mathcal{P} = \mathcal{W}(x_1, \ldots, x_N)$
- Density Function $\mu$ and Centroids $c(\mathcal{P}, \mu)$
Goal

Dispatch the N robots to **optimally cover** the environment \( \mathcal{X} \), namely we want to have many robots where \( \mu(x) \) is large and few where it is small.
Coverage Goal and the Lloyd Algorithm

Goal

Dispatch the N robots to \textbf{optimally cover} the environment $\mathcal{X}$, namely we want to have many robots where $\mu(x)$ is large and few where it is small.

\[
\min_{\mathcal{P}} H(\mathcal{P}, \mu) = \min_{\mathcal{P}} \sum_{i=1}^{N} \int_{\mathcal{P}_i} \| q - c_i(\mathcal{P}_i) \|^2 \mu(q) dq
\]
Goal

Dispatch the N robots to **optimally cover** the environment $\mathcal{X}$, namely we want to have many robots where $\mu(x)$ is large and few where it is small.

\[
\min_{\mathcal{P}} H(\mathcal{P}, \mu) = \min_{\mathcal{P}} \sum_{i=1}^{N} \int_{\mathcal{P}_i} ||q - c_i(\mathcal{P}_i)||^2 \mu(q) dq
\]

Solution: Classical Lloyd algorithm

1. compute the centroids of the current partition, e.g. $c(\mathcal{P})$
2. update $\mathcal{P}$ to the partition $\mathcal{W}(c(\mathcal{P}))$

Or more briefly $\mathcal{P}^L(k + 1) = \mathcal{W}(c(\mathcal{P}^L(k)))$. 

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**Sensory Function**

- **Unknown** function $\mu : \mathcal{X} \subset \mathbb{R}^2 \mapsto \mathbb{R}_+$

- $\mu$ is a zero-mean Gaussian random field with covariance $K : \mathcal{X} \times \mathcal{X} \mapsto \mathbb{R}_+$

- Radial Mercer Kernels

- $K(x, x) = \lambda$

---

**Figure:** Gaussian Process

**Figure:** Gaussian Kernel
Minimum Variance Estimate

The set $I_k = \{x_i, y_i\}_{i=0}^{m_k}$ represents the complete information set available at the BS at iteration $k$ and $m_k = |I_k|$ is its cardinality.
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$$\hat{\mu}_k(x) = \mathbb{E}[\mu(x)|I_k] = \sum_{i=1}^{m_k} c_i K(x_i, x), \ x \in \mathcal{X}$$
Minimum Variance Estimate

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$$\hat{\mu}_k(x) = \mathbb{E} [\mu(x)|I_k] = \sum_{i=1}^{m_k} c_i K(x_i, x), \ x \in \mathcal{X}$$

An index of the accuracy of the estimate is given by the posterior variance

$$V_k(x) = \text{Var} [\mu(x)|I_k]$$
Problem Formulation

Base Station

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## Exploration and Exploitation Dilemma

### Goal

The ultimate goal is to position the $N$ robots in the centroids of a good partition that minimizes $H(P, \mu)$. To do so we need to:

1. have a good estimate $\hat{\mu}$ of the sensory function \( \rightarrow \) exploration
2. minimize the cost function $H(P, \mu)$ \( \rightarrow \) exploitation
Exploration and Exploitation Dilemma

Goal

The ultimate goal is to position the $N$ robots in the centroids of a good partition that minimizes $H(\mathcal{P}, \mu)$. To do so we need to:

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Strategy

1. initially promote exploration
2. when the estimate is more accurate transit to exploitation
3. random approach based on the maximum of the posterior variance
rEC - Robots

Collects Measurements

Measurement transmission

Waits new target

Moves to new target

Robot
rEC - Base Station

Collects Measurements

Computes $\mu$ and $V$

Computes $P_i, c_i$ and $M_i$

Explore or Exploit?
Proposition 1 - Sensory Function Convergence

If:

1. $F(M) : [0, 1] \rightarrow [0, 1]$ continuous and monotonically increasing,
2. $F(M) > 0$ for $M > 0$,
3. $\mathbb{P}[\beta_{i,k} = 1] = \bar{\beta} > 0$,
4. $\mathbb{P}[\gamma_{i,k} = 1] = \bar{\gamma} > 0$.

Then

$$\hat{\mu}_k \xrightarrow{P} \mu.$$
Online Gaussian Estimation

What is the most expensive operation?
Online Gaussian Estimation

What is the most expensive operation?

\[(\bar{K}_{k+1} + \sigma^2 I)^{-1} = \left( \begin{bmatrix} \bar{K}_k & \bar{K}_{k+1,12} \\ \bar{K}^T_{k+1,12} & \bar{K}_{k+1,22} \end{bmatrix} + \sigma^2 I \right)^{-1} \]

How much is its computational complexity?
Online Gaussian Estimation

What is the most expensive operation?

\[
(\bar{K}_{k+1} + \sigma^2 I)^{-1} = \left( \begin{bmatrix} \bar{K}_k & \bar{K}_{k+1,12} \\ \bar{K}_{k+1,12}^T & \bar{K}_{k+1,22} \end{bmatrix} + \sigma^2 I \right)^{-1}
\]

How much is its computational complexity?

**Naive:** \( (\bar{K}_k + \sigma^2 I)^{-1} \rightarrow O(k^3) \)

**Schur:** \( (\bar{K}_{k+1,22} - \bar{K}_{k+1,12}^T (\bar{K}_k + \sigma^2 I)^{-1} \bar{K}_{k+1,12})^{-1} \rightarrow O(k^2) \)
Consider

\[ \mathcal{X}_{\text{grid}} := \{ x_{\text{grid},1}, \ldots, x_{\text{grid},m} \} \subset \mathcal{X}. \]

Given the scalar \( \Delta > 0 \), \( \mathcal{X}_{\text{grid}} \) forms a \textit{sampled space} of resolution \( \Delta \) if

\[
\min_{i=1,\ldots,m} \| x_{\text{grid},i} - x \| \leq \Delta, \quad \forall x \in \mathcal{X}.
\]
Consider

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\[ \min_{i=1,\ldots,m} \| x_{\text{grid},i} - x \| \leq \Delta, \quad \forall x \in \mathcal{X}. \]

We define the projector operator

\[ \mathcal{X} \mapsto \mathcal{X}_{\text{grid}} : \quad x \mapsto \text{proj}(x) = \arg \min_{a \in \mathcal{X}_{\text{grid}}} \| x - a \|. \]
Collect Measurements

Compute $\mu$ and $V$

Compute $P_i$, $c_i$ and $M_i$

Explore or Exploit?
If the assumptions of Proposition 3 holds then:

$$\lim_{k \to \infty} V_k(x) = \lambda - k_{\text{grid}}(x)K^{-1}_{\text{grid}} k_{\text{grid}}(x)^\top.$$
Proposition 2 - Posterior Variance

If the assumptions of Proposition 3 holds then:

$$\lim_{k \to \infty} V_k(x) = \lambda - k_{\text{grid}}(x) K_{\text{grid}}^{-1} k_{\text{grid}}(x)^\top.$$ 

The following simple $\Delta$ dependent bound holds

$$\lim_{k \to \infty} V_k(x) \leq \lambda - \frac{K^2(\Delta)}{\lambda}.$$
Convergence Analysis - Sensory Function

Proposition 2 - Posterior Variance

If the assumptions of Proposition 3 holds then:

\[ \lim_{k \to \infty} V_k(x) = \lambda - k_{\text{grid}}(x) K_{\text{grid}}^{-1} k_{\text{grid}}(x)^\top. \]

The following simple \( \Delta \) dependent bound holds

\[ \lim_{k \to \infty} V_k(x) \leq \lambda - \frac{K^2(\Delta)}{\lambda}. \]

If \( K \) is the Gaussian kernel with \( K(a, b) = \lambda e^{-\frac{||a-b||^2}{\zeta^2}} \), for small \( \Delta \) we have

\[ \lim_{k \to \infty} V_k(x) \leq \lambda - \frac{K^2(\Delta)}{\lambda} \approx \frac{\lambda \Delta^2}{\zeta^2}. \]
Simulations Setup

- Team of $N = 8$ robots
- Domain $\mathcal{X} = [0, 1] \times [0, 1]$
- Gaussian kernel $K(x, x') = e^{-\frac{\|x-x'\|^2}{0.002}}$
- Exploration-Exploitation trade-off: $F_\alpha(M) = M^\alpha$
- Sensory function

$$\mu(x) = 5 \left( e^{\frac{-\|x-\mu_1\|^2}{0.04}} + e^{\frac{\|x-\mu_2\|^2}{0.04}} \right)$$

where

$$\mu_1 = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix}$$
Voronoi partitions computed using the r-EC Algorithm (black lines) for different sensory function $\mu(x)$. Blue dots indicate the locations of the centroids obtained with the r-EC algorithm.
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Average Energy

![Average Energy Graph](image-url)

- **Lloyd**
- **r-EC α=0.01**
- **r-EC α=1**
- **r-EC α=10**

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Mapping and Coverage
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Comparison between the original r-EC algorithm and the grid based approximation for different total number of points $p^2$. The table reports the steady state value after 400 iterations and the execution times obtained using the grid based approximation and classic algorithms.

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rEC in action!

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Conclusions and on-going works

The r-EC/r-EC-grid are shown to be:

1. capable to converge to the optimal estimate of $\mu$,
2. robust to packet losses,
3. efficient.

What else can be done?

1. consider time varying $\mu$,
2. consider localization errors.
Competitive - Cooperative RHC game

GOAL: minimize a cost function which depends on your state, your input and the opponent input.

RESULTS: closed form solution given the control parameters and stability analysis.

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Thank you for your attention!