Bias Correction in Localization Problem

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Assistance of Dr. Sam Drake of Australian Defence Science and Technology Organization (DSTO) with original problem formulation and provision of trial data is gratefully acknowledged
Outline

- Motivation
- Bias in Localization Problem
- Taylor-Jacobian Bias Correction Method
- Performance Evaluation and Simulation
- Conclusion
Motivation

Industry
• Process control
• Automation
• Predictive maintenance

Scientific Research
• High spatial and temporal density sampling
• Habitat monitoring
• Event detection

Health Care
• Location aware patient monitoring
• Patient vital signals

Disaster Management
• Event detection (natural disasters – fire, earthquake)
• Location awareness (fire fighters looking for survivors)
• Emergency response

Military
• Battlefield surveillance
• Target tracking
Motivation

• Accurate location of sensors plays a vital role in various network applications
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Motivation

• Many self-localization algorithms are proposed
• Generally, localization results are imprecise
  • Environment: noise, non line of sight
  • Hardware: range or angle measuring devices
  • Localization algorithms
• Many enhancement techniques have been proposed to improve the accuracy of localization
  • Geometric Constraints
  • Error Control Mechanisms (Baoqi Huang)
  • Bias Correction Methods
Motivation

- Many self-localization algorithms are proposed
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    - Localization algorithm
- Many enhanced techniques have been proposed to improve the accuracy of localization
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  - Error Control Mechanisms
  - **Bias Correction Methods**
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What is Bias

Bias is a term in estimation theory and is defined as the difference between the expected value of a parameter estimate and the true value of the parameter [1].

What is Bias

Bias is a term defined as the difference between the expected value of a parameter and the true value of the parameter.

Bias in Localization Problem

In the noisy situation, we assume $\mathbf{g} = (g_1, g_2, \ldots, g_n)$ denotes the localization mapping from the measurements to the target position estimates. We have:

$$\tilde{\mathbf{x}} = \mathbf{g}(\Theta + \delta\Theta) = \mathbf{g}(\widetilde{\Theta})$$

where $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)^T$ denotes the inaccurate estimates of the target location, $\tilde{\Theta} = (\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_N)$ denotes the noisy measurements and $\delta\Theta = (\delta\theta_1, \delta\theta_2, \ldots, \delta\theta_N)^T$ denotes the measurement noise.
Bias in Localization Problem

In practice the measurement process will repeated $M$ times. As $M \to \infty$, we would expect the estimate to go to:

$$E[\tilde{x}_i] = E[g_i(\tilde{\Theta})]$$

Because $g_i$ is nonlinear we have:

$$E[\tilde{x}_i] = E[g_i(\tilde{\Theta})]$$
$$\neq g_i(E[\tilde{\Theta}])$$
$$= g_i(\Theta)$$
$$= x_i$$

Therefore the bias appears in the estimation process:

$$Bias_{x_i} = E[\tilde{x}_i] - x_i \quad i = 1, 2, \ldots, n$$
Bias in Localization Problem

In practice the measurement process will be repeated \( M \) times. As \( M \to \infty \), we would expect the estimate to go to:

The bias will exist if two conditions are satisfied:

1. the mapping function is nonlinear
2. the measurements are noisy

Therefore the bias appears in the estimation process:

\[
Bias_{\tilde{x}_i} = E[\tilde{x}_i] - x_i \quad i = 1, 2, ..., n
\]
Significant Bias in Localization Problem

- Two sensors at (0, 8) and (0, -8)
- y value of the target is fixed at 0 while x value changes from 6 to 20
- Measurements are bearing-only
- Different variances used for measurement errors, which are zero mean.
Significant Bias in Localization Problem

• Table 1 and 2 illustrate the bias of the x component compared to the standard deviation of the error in estimating x with different level of noise.

<table>
<thead>
<tr>
<th>Value of x</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
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</thead>
<tbody>
<tr>
<td>Percentage (%)</td>
<td>16.48</td>
<td>9.57</td>
<td>6.64</td>
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<tr>
<td>Percentage (%)</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
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<tr>
<td>Percentage (%)</td>
<td>13.75</td>
<td>15.23</td>
<td>17.24</td>
<td>18.91</td>
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<th>12</th>
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<tbody>
<tr>
<td>Percentage (%)</td>
<td>20.92</td>
<td>18.3</td>
<td>14.14</td>
<td>12.5</td>
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</table>

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Bias (before removal) can be a significant fraction of the errors.
Significant Bias in Localization Problem

Normally the two conditions are satisfied easily:

• the mapping function is nonlinear
• the measurements are noisy

The bias can be a significant fraction of the error.

It is worth to analyse and remove the bias in localization.
Outline

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• Bias in Localization Problem
• Taylor-Jacobian Bias Correction Method
• Performance Evaluation and Simulation
• Conclusion
Taylor-Jacobian Bias Correction Method

Notations and Assumptions:
1. \( n \) denotes the number of dimensions of the ambient space
2. \( N \) denotes the number of obtained measurements
3. \( \mathbf{x} = (x_1, x_2, \ldots, x_n)^T \) denotes the position of the target
4. \( \Theta = (\theta_1, \theta_2, \ldots, \theta_N)^T \) denotes the measurement set
5. \( \delta\Theta = (\delta\theta_1, \delta\theta_2, \ldots, \delta\theta_N)^T \) denotes measurement errors
6. \( \mathbf{f} = (f_1, f_2, \ldots, f_N)^T \) is the mapping from the target position to the measurements
7. \( \mathbf{g} = (g_1, g_2, \ldots, g_n)^T \) is the localization mapping from the measurements to the target position
Formulation of the bias

In the noisy case, errors in measurements are inevitable. Therefore the localization problem can be formulated as follows:

\[ \mathbf{x} + \delta \mathbf{x} = \mathbf{g}(\Theta + \delta \Theta) \]

Next a Taylor series is used to expand the above equation truncating at second order:

\[
x_i + \delta x_i = g_i(\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_N) \\
= g_i(\theta_1 + \delta \theta_1, \theta_2 + \delta \theta_2, ..., \theta_N + \delta \theta_N) \\
\approx g_i(\theta_1, \theta_2, ..., \theta_N) + \sum_{j=1}^{N} \frac{\partial g_i}{\partial \theta_j} \delta \theta_j \\
+ \frac{1}{2!} \sum_{j=1}^{N} \sum_{l=1}^{N} \delta \theta_j \delta \theta_l \frac{\partial^2 g_i}{\partial \theta_j \partial \theta_l}
\]
Formulation of the bias

In the

Therefore, the approximate bias expression is immediate:

\[ E(\delta x_i) = \frac{1}{2!} \sum_{j=1}^{N} \sigma_j^2 \frac{\partial^2 g_i}{\partial \theta_j^2} \]

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+ \frac{1}{2!} \sum_{j=1}^{N} \sum_{l=1}^{N} \delta \theta_j \delta \theta_l \frac{\partial^2 g_i}{\partial \theta_j \partial \theta_l} 
\]
Formulation of the bias

However it is very difficult to compute the localization mapping $g = (g_1, g_2, \ldots, g_n)^T$ and its derivatives.

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Formulation of the bias

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$$E(\delta x_i) = \frac{1}{2!} \sum_{j=1}^{N} \sigma_j^2 \frac{\partial^2 g_i}{\partial \theta_j^2}$$

How to analytically express the bias in an easy way?
Formulation of the bias

However it is very difficult to compute the localization mapping \( g = (g_1, g_2, \ldots, g_n)^T \) and its derivatives. In contrast \( f = (f_1, f_2, \ldots, f_N)^T \) can be easily written down!

The approximate bias expression is immediate:

\[
E(\delta x_i) = \frac{1}{2!} \sum_{j=1}^{N} \sigma_j^2 \frac{\partial^2 g_i}{\partial \theta_j^2}
\]
Different Localization Techniques

Range Measurements

\[ d_1 = f_1(x, y) = \sqrt{(x - x_1)^2 + (y - y_1)^2} \]
\[ d_2 = f_2(x, y) = \sqrt{(x - x_2)^2 + (y - y_2)^2} \]

Bearing-Only Measurements

\[ \theta_1 = f_1(x, y) = \pi + \arctan\left(\frac{x - x_1}{y - y_1}\right) \mod 2\pi \]
\[ \theta_2 = f_2(x, y) = \arctan\left(\frac{x - x_2}{y - y_2}\right) \mod 2\pi \]

TDOA Measurements

\[ (t_2 - t_1) \times c = f_1(x, y) = \sqrt{(x_2 - x)^2 + (y_2 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2} \]
\[ (t_3 - t_1) \times c = f_2(x, y) = \sqrt{(x_3 - x)^2 + (y_3 - y)^2} - \sqrt{(x_1 - x)^2 + (y_1 - y)^2} \]
Different Localization Techniques

Range Measurements

\[ d_1 = f_1(x, y) \]
\[ d_2 = f_2(x, y) \]

Bearing-Only Measurements

\[ \theta_1 = f_1(x, y) = \pi + \arctan \left( \frac{x - x_1}{y - y_1} \right) \mod 2\pi \]
\[ \theta_2 = f_2(x, y) = \pi + \arctan \left( \frac{x - x_2}{y - y_2} \right) \mod 2\pi \]

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How to analytically express the bias in an easy way?

How to analytically express the bias by using \( f \) and its derivatives?
Taylor-Jacobian Bias Correction Method

Jacobian matrix and one of its property are used to calculate the derivatives of $g$ in terms of the derivatives of $f$.

$$
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_N}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_n}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial g_1}{\partial \theta_1} & \cdots & \frac{\partial g_1}{\partial \theta_N} \\
\vdots & \ddots & \vdots \\
\frac{\partial g_N}{\partial \theta_1} & \cdots & \frac{\partial g_N}{\partial \theta_N}
\end{bmatrix} = I_n
$$

By solving the above equation set, we can obtain the analytical expression for

$$
\frac{\partial g_i}{\partial \theta_j} (i = 1, 2, \ldots, n; j = 1, 2, \ldots, N)
$$
Taylor-Jacobian Bias Correction Method

Here we take $\frac{\partial g_1}{\partial \theta_1}$ for example. Assume $\frac{\partial g_1}{\partial \theta_1} = g^1_{\theta_1}$, differentiating the equation in respect to $x_1, x_2, \ldots, x_n$ respectively we can obtain the following equation set:

$$
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_1} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_1}{\partial x_n} & \cdots & \frac{\partial f_N}{\partial x_n}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial^2 g_1}{\partial \theta_1 \partial \theta_1} \\
\vdots \\
\frac{\partial^2 g_1}{\partial \theta_1 \partial \theta_N}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial g^1_{\theta_1}}{\partial x_1} \\
\vdots \\
\frac{\partial g^1_{\theta_1}}{\partial x_n}
\end{bmatrix}
$$
Taylor-Jacobian Bias Correction Method

Here we take \( \frac{\partial g_1}{\partial \theta_1} \) for example. Assume \( \frac{\partial g_1}{\partial \theta_1} = g_1^1 \), differentiating the equation in respect to \( x_1, x_2, \ldots, x_n \) respectively we can obtain the following equation set:

\[
E(\delta x_i) = \frac{1}{2!} \sum_{j=1}^{N} \sigma^2_j \frac{\partial^2 g_i}{\partial \theta_j^2} \]

Can be easily expressed analytically
Taylor-Jacobian Bias Correction Method

How to analytically express the bias in an easy way?

Solved!

1. Taylor series
2. Jacobian matrix and its property

So we call the proposed method as *Taylor-Jacobian* bias correction method.
Overdetermined Problem

**Jacobian matrix and one of its property**

\[
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_N}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_n}
\end{bmatrix}

\begin{bmatrix}
\frac{\partial g_1}{\partial \theta_1} & \cdots & \frac{\partial g_1}{\partial \theta_N} \\
\vdots & \ddots & \vdots \\
\frac{\partial g_n}{\partial \theta_1} & \cdots & \frac{\partial g_n}{\partial \theta_N}
\end{bmatrix} = I_n
\]

**Important assumption:** \(N=n\)
Overdetermined Problem

**Jacobian matrix and one of its property**

\[
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\
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\vdots & \ddots & \vdots \\
\frac{\partial g_n}{\partial \theta_1} & \cdots & \frac{\partial g_n}{\partial \theta_N}
\end{bmatrix}
= I_n
\]

**Important assumption:** \( N > n \)
Overdetermined Problem (N=n+1)

Least squares method:

\[ F_{\text{cost-function}}(x, \tilde{\Theta}) = \sum_{i=1}^{N} (f_i - \tilde{\theta}_i)^2 = \sum_{i=1}^{N} \delta \theta_i^2 \]
Overdetermined Problem (N=n+1)

Least squares method:

$$F_{\text{cost-function}}(\mathbf{x}, \tilde{\Theta}) = \sum_{i=1}^{N} (f_i - \tilde{\theta}_i)^2 = \sum_{i=1}^{N} \delta \theta_i^2$$

Minimize the distance:

$$D_{\text{min}} = \sqrt{\sum_{i=1}^{N} \delta \theta_i^2} = \varepsilon ||\mathbf{u}||$$
Overdetermined Problem ($N=n+1$)

Least squares method:

$$F_{\text{cost-function}}(x, \tilde{\Theta}) = \sum_{i=1}^{N} (f_i - \tilde{\theta}_i)^2 = \sum_{i=1}^{N} \delta \theta_i^2$$

Minimize the distance:

$$D_{\text{min}} = \sqrt{\sum_{i=1}^{N} \delta \theta_i^2} = \varepsilon ||u||$$

For the white point:

$$v_i = \left[ \frac{\partial f_1}{\partial x_i}, \frac{\partial f_2}{\partial x_i}, ..., \frac{\partial f_N}{\partial x_i} \right]^T \quad i = 1, 2, ..., n$$
Overdetermined Problem (N=n+1)

Least squares method:

\[
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\]

The normal vector:

\[
u = [u_1, u_2, ..., u_N]^T = v_1 \times v_2 \times \cdots \times v_n
\]
Overdetermined Problem (N=n+1)

Least squares method:

Finally we can obtain a new mapping:

\[ \tilde{\Theta} = F(\tilde{x}, \varepsilon) = f(\tilde{x}) + \varepsilon u \]

\[ v_i = \left[ \frac{\partial f_1}{\partial x_i}, \frac{\partial f_2}{\partial x_i}, ..., \frac{\partial f_N}{\partial x_i} \right]^T \quad i = 1, 2, ..., n \]

The normal vector:

\[ u = [u_1, u_2, ..., u_N]^T = v_1 \times v_2 \times v_n \]
Overdetermined Problem (N>n+1)

With N> n+1, the situation is similar to N=n+1 case except that the extra variable is no longer a scalar. Instead, it is a vector which can be defined as follows:

$$\mathbf{e} = [e_1, e_2, ..., e_{N-n}]^T$$

Where $e_i \ (i = 1, 2, ..., N-n)$ denotes a coefficient to minimize the moved distance in each dimension of the normal.
Overdetermined Problem (N>n+1)

Assume N=4 and n=2.
At the white point we can have:

\[ v_1 = \left[ \frac{\partial f_1}{\partial x_1}, \frac{\partial f_2}{\partial x_1}, \frac{\partial f_3}{\partial x_1}, \frac{\partial f_4}{\partial x_1} \right]^T \]

\[ v_2 = \left[ \frac{\partial f_1}{\partial x_2}, \frac{\partial f_2}{\partial x_2}, \frac{\partial f_3}{\partial x_2}, \frac{\partial f_4}{\partial x_2} \right]^T \]

These two tangent vectors define a tangent plane \( P_{tangent} \)
Contributions and key mathematic tools of our work

Contributions:
1. Express the bias in an easy way by using the function $f$ (mapping from the target position to the measurements) and its derivatives
2. Adopt a method based on least-squares idea to solve the overdetermined problem

Mathematic tools:
1. Taylor series
2. Jacobian Matrix
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Simulation

Simulation Assumption

- All simulations are done in two-dimensional space.
- The three sensors are fixed at (0, 8), (0, -8) and (8,0).
- The measurement noise for three sensors are produced by i. i. d. Gaussian with zero mean and variance $\sigma^2 = 1$.
- All the simulation results are obtained from 5000 Monte Carlo experiments.
- We compare our method with an well-cited bias-correction method GW method [1]

Simulation Results - Range Measurement

- Three sensors and a single target
- Range measurements only
- Measurement errors are $\mathcal{N}(0,1)$
- The y value of the target is fixed at 0; x value is adjusted from 6 to 20
Simulation Results – Bearing-Only Measurement

- Three sensors and a single target
- Bearing-only measurements
- Measurement errors are $N(0,1)$
- The y value of the target is fixed at 0; x value is adjusted from 6 to 20
Simulation Results - Different Level of Noise

- Truncation of Taylor series is not necessarily justified when the noise is large
- Noise level is adjusted over a large range via changing the standard deviation of measurement errors, from 0.5 to 3.5 in steps of 0.5
- Bearing-only measurements are used
Simulation Results - Different Level of Noise

necessarily justified when the range via changing the errors, from 0.5 to 3.5 in

\( S_1 (0,8) \)

\( S_2 (0,-8) \)

Target

\( (14,0) \)
Trial Data – Scan-based Measurement

- Three physical sensors and a single target, which is a radar with a mechanically rotating antenna.
- Two usable sensor measurements are obtained.
- Noise in measurements is $N(0, 0.02)$
Trial Data – Scan-based Measurement

- Three physical sensors and a single target, which is a radar

<table>
<thead>
<tr>
<th>Localization errors without bias-correction method</th>
<th>Localization errors with bias-correction method</th>
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<tbody>
<tr>
<td>0.6610</td>
<td>0.0785</td>
</tr>
<tr>
<td>0.3435</td>
<td>0.0462</td>
</tr>
<tr>
<td>0.3454</td>
<td>0.0465</td>
</tr>
</tbody>
</table>

- Noise in measurements is \( \text{N}(0,0.02) \)
Performance of the Taylor-Jacobian Method

1. The Taylor-Jacobian method is generic
   - Range Measurement
   - Bearing-only Measurement
   - Scan-based Measurement
   - The performance of the Taylor-Jacobian method is better than the GW method
   - The Taylor-Jacobian method can be more robust to the level of noise than the GW method
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Conclusion

• Bias arises due to simultaneous presence of noise and nonlinear transformations.
• In localization, the map need for computing the bias may not be analytically available; its inverse is available so the bias computation needs to be varied
• A generic Taylor-Jacobian bias correction method is proposed
• The simulation results demonstrate the performance of the proposed method
Thank you!

Publications:

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